Characteristics of Linear Relations Lesson #1:
Line Segments on a Cartesian Plane

Lesson 1 and Lesson 2 of this unit are not required for this curriculum, but are included because

1. they are important characteristics of linear relations not covered elsewhere
2. this information is required in higher level math courses such as Calculus

Unit Overview

The graph of a linear relation is represented by a straight line. The line can be infinite or finite depending on the domain and range of the linear relation. In some cases we are only interested in a portion of a line. This portion is called a **line segment**.

We have already studied some of the characteristics of the graph of a linear relation: intercepts, domain, and range. In this unit we study some characteristics of line segments: namely, length, midpoint, distance, and slope. We demonstrate an understanding of slope with respect to rise and run, the slope formula, and rate of change. We then discuss the slopes of parallel and perpendicular lines.

Line Segment

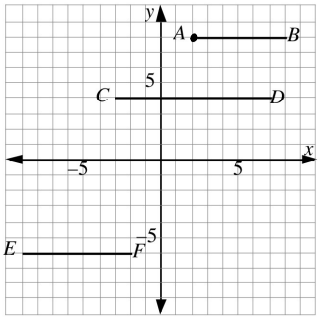
A line segment is the portion of a line between two points on the line.

If the endpoints of a line segment are A and B, we refer to it as line segment AB.

NOTE: Line segment AB may also be written as $\overline{AB}$.

Length of a Horizontal Line Segment

Consider the line segments shown on the grid.



a) Find the length of each line segment by counting.

- length of AB is 6 units.
- length of CD is 10 units.
- length of EF is 7 units.

b) Determine the coordinates of the endpoints of each line segment.

- $AB \rightarrow A(2, 8)$ $B(8, 8)$
- $CD \rightarrow C(-3, 4)$ $D(7, 4)$
- $EF \rightarrow E(-9, -6)$ $F(-2, -6)$

c) Complete the following.

- The difference in the x-coordinates, $x_B - x_A$, is $8-2$ = 6.
- The difference in the x-coordinates, $x_D - x_C$, is $7-(-3)$ = 10.
- The difference in the x-coordinates, $x_F - x_E$, is $-2-(-9)$ = 7.

d) How can the coordinates of the end points of a horizontal line segment be used to find the length of the line segment?

subtract the x value of the left endpoint from the x value of the right endpoint.

length = right − left

bigger − small

a) Line segment AB has endpoints $A(2, 8)$ to $B(-5, 8)$. Determine the length of $\overline{AB}$.

right left

$2-(-5)=7$

b) Determine the length of the line segment from $P(a-2, b)$ to $Q(a+4, b)$.

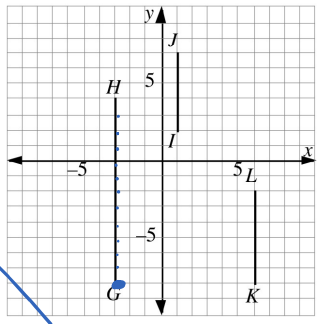
left right

$a+4-(a-2)$

$a+4-a+2 = 6$ units

Length of a Vertical Line Segment

Consider the line segments shown on the grid.



a) Find the lengths of each line segment by counting.

- length of GH is 12 units.
- length of IJ is 5 units.
- length of KL is 6 units.

b) Determine the coordinates of the endpoints of each line segment.

- $GH \rightarrow G(-3, -8)$ $H(-3, 4)$
- $IJ \rightarrow I(1, 2)$ $J(1, 7)$
- $KL \rightarrow K(6, -8)$ $L(6, -2)$

c) Complete the following.

- The difference in the y-coordinates, $y_H - y_G$, is $4-(-8)$ 12.
- The difference in the y-coordinates, $y_J - y_I$, is $7-2$ 5.
- The difference in the y-coordinates, $y_L - y_K$, is $-2-(-8)$ 6.

d) How can the coordinates of the end points of a vertical line segment be used to find the length of the line segment?

Subtract the y-value of the lower end point from the y-value of the higher end point

higher lower

length = higher − lower

bigger − smaller

a) Line segment RS has endpoints $R(1, -4)$ to $S(1, -9)$. Determine the length of $\overline{RS}$.

$-4-(-9)=5$

b) Determine the length of the line segment from $P(a, b)$ to $Q(a, b+10)$.

lower higher

$b+10-b=10$

Complete Assignment Questions #1 - #5

Midpoint

Midpoint

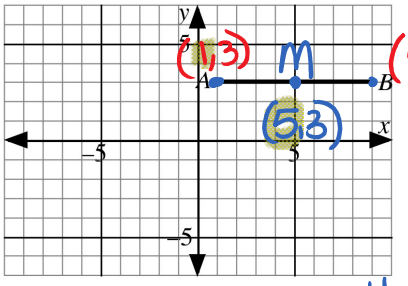
The **midpoint, *M*,** of a line segment on the graph of a linear relation is the point at the centre of the line segment.

Midpoint of a Horizontal Line Segment

Consider the line segment AB shown on the grid.

a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

b) List the coordinates of point A and point B on the grid. How can the x-coordinates of points A and B be used to find the coordinates of the midpoint of a horizontal line?



(1,3) M (9,3) (5,3)

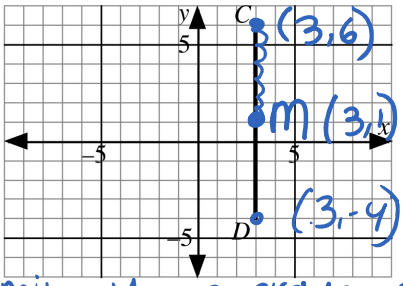
$M = \frac{1+9}{2} = \frac{10}{2} = 5$ Determine the average of x values.

Midpoint of a Vertical Line Segment

Consider the line segment CD shown on the grid.

a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

b) List the coordinates of point C and point D on the grid. How can the y-coordinates of points C and D be used to find the coordinates of the midpoint?



(3,6) M (3,1) (3,-4)

length $6-(-4)=10$

$\frac{6+(-4)}{2} = \frac{2}{2} = 1$ Determine the average of y values

Midpoint of an Oblique (Diagonal) Line Segment

Consider the line segment EF shown on the grid.

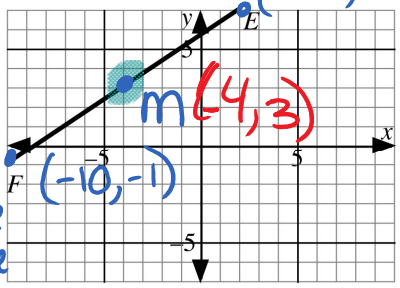
a) Use the results from above to determine the midpoint of EF.

b) Express in words how to find the midpoint, M, of the line segment joining the points (x_1, y_1) and (x_2, y_2).

- determine average x value
- determine average y value

c) Complete the formula to express the relationship in **b)**. $x_M = \frac{x_1 + x_2}{2}$ $y_M = \frac{y_1 + y_2}{2}$

midpoint x coordinate



(2,7) M (-4,3) (-10,-1)

$x_m: \frac{2+(-10)}{2} = \frac{-8}{2} = -4$

$y_m: \frac{7+(-1)}{2} = \frac{6}{2} = 3$

Midpoint of a Line Segment

Consider line segment PQ with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$.

The midpoint, M, of the line segment has coordinates.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Line segment PQ can also be written as PQ.

Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $P(4, 7)$, $Q(12, 3)$ **b)** $E(-5, 7)$, $F(-11, -2)$ **c)** $A(w + 3, 2w)$, $C(5w - 1, 7w + 1)$

$M\left(\frac{4+12}{2}, \frac{7+3}{2}\right)$ $M\left(\frac{-5+11}{2}, \frac{7+(-2)}{2}\right)$ $M\left(\frac{w+3+5w-1}{2}, \frac{2w+7w+1}{2}\right)$

$= M(8, 5)$ $= \left(\frac{-16}{2}, \frac{5}{2}\right) = (-8, 2.5)$ $= \left(\frac{6w+2}{2}, \frac{9w+1}{2}\right)$

$= \left(3w+1, \frac{9w+1}{2}\right)$

Ruby was doing a question in her coordinate geometry homework, and her little brother Max wrote over part of the question as a prank.

$P(5, y_1)$ $Q(-11, -10)$

Midpoint $(x_m, -6)$

Calculate the missing coordinates.

$x_m = \frac{5 + (-11)}{2} = \frac{-6}{2}$

$= -3$

$y_m = \frac{y_1 + y_2}{2}$

* sub in what we know

$-6 = \frac{y_1 + (-10)}{2}$ ·2

* solve for y_1

$-12 = y_1 - 10$ +10

$-2 = y_1$

Complete Assignment Questions #6 - #17

(1,2) ab, 6, 7, 9 11

Assignment

1. Determine the length of each line segment.

a) $A(2, 7)$ to $B(5, 7)$

b) $C(-5, 3)$ to $D(-5, 12)$

c) $I(-3, -8)$ to $J(-3, -3)$

d) $K(7, -10)$ to $L(-35, -10)$

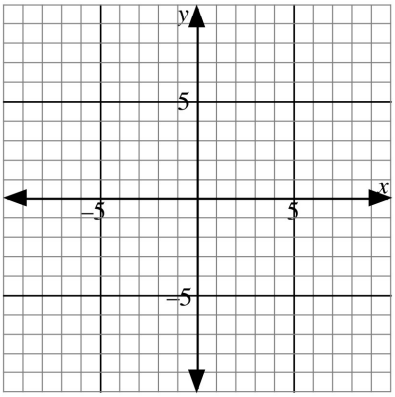
2. Determine whether each line segment is horizontal or vertical, and write an expression for its length.

a) $A(p, q)$ to $B(p - 4, q)$

b) $C(m - 3, n + 5)$ to $D(m - 3, n + 12)$

3. A triangle has vertices $P(-4,-3)$, $Q(9, -3)$, and $R(1, 5)$.

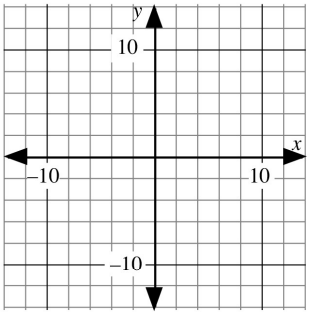
- Sketch the triangle on the grid.
- Calculate the area of the triangle.



4. On the grid, plot the points $P(-6, 6)$, $Q(-6, -10)$, and $R(6,-10)$.

a) Determine the distance from P to R using the Pythagorean Theorem.

b) Calculate the area and perimeter of $\triangle PQR$.



5. Rebecca uses quadrant I in a Cartesian plane to describe the location of the bases in a game of high school softball. The four bases form a square. The origin is at home plate. First base is at $(18, 0)$, and the distance between each base is 18 m. The pitcher's mound is located between home plate and second base.

a) State the coordinates of second base.

b) The pitcher stands on the mound 12 m from home plate. If she has to throw a ball to second base, what distance to the nearest tenth of a metre, would she throw the ball?

6. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $A(2, 6),\ C(4, 16)$ **b)** $X(-3, -8),\ Y(-11, 0)$ **c)** $K(15, -17),\ L(-11, 3)$

7. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $C(3x, 8y),\ D(7x, -4y)$ **b)** $S(a + b, a + 7b),\ T(a + b, a - 3b)$

8. Otto was given two points: $A(-6, 4)$ and $B(12, -8)$. He was asked to divide $\overline{AB}$ into four equal parts. State the coordinates of the points which will divide $\overline{AB}$ into four equal parts.

9. In each case M is the midpoint of $\overline{AB}$. Determine the value of x.

a) $A(2,6)$, $B(6,x)$, $M(4,-1)$ **b)** $A(3,6)$, $B(x,0)$, $M(0,3)$

Multiple Choice **10.** $ABCD$ is a square with vertices $\left(\sqrt{5},0\right)$, $\left(0,\sqrt{5}\right)$, $\left(-\sqrt{5},0\right)$, and $\left(0,-\sqrt{5}\right)$ respectively. The area of the square, in unit^2, is

A. 5
B. 10
C. 20
D. 100

11. $P(4,-8)$ and $Q(-2,10)$ are the endpoints of a diameter of a circle. The coordinates of the centre of the circle are

A. $(-3,9)$
B. $(2,2)$
C. $(3,-9)$
D. $(1,1)$

12. AB is a diameter of a circle; the centre is C. If $A(8,-6)$ and $C(5,-2)$ then B is the point

A. $(2,2)$
B. $(6.5,-4)$
C. $(11,-10)$
D. $(13,-8)$

13. Which statement is always true?

A. Two line segments of equal length have the same midpoint.
B. Two line segments with the same midpoint are of equal length.
C. A point equidistant from the endpoints of a line segment is the midpoint.
D. None of the above statements is always true.

Numerical Response

14. To the nearest tenth, the perimeter of $\triangle PQR$ with vertices, $P(3, 8)$, $Q(3, 0)$, and $R(-1, 8)$ is _____.

(Record your answer in the numerical response box from left to right)

15. The midpoint of line segment ST is $M\left(\frac{1}{2}, -4\right)$. If the coordinates of T are $(-3, 3)$, and the coordinates of S are (x, y), the value of x is _____.

(Record your answer in the numerical response box from left to right)

16. The point $M(a, 6)$ is the midpoint of $\overline{GH}$ with $G(22, b)$ and $H(6, -8)$. The value of $a + b$ is _____.

(Record your answer in the numerical response box from left to right)

17. The midpoint of line segment AB lies on the y-axis. A lies on the x-axis, and B has coordinates $(-4, 5)$. The length of AB, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) 3 **b)** 9 **c)** 5 **d)** 42 **2. a)** horizontal, 4 **b)** vertical, 7

3. 52 units^2 **4. a)** 20 units **b)** area – 96 units^2, perimeter = 48 units

5. a) (18,18) **b)** 13.5 m **6. a)** (3, 11) **b)** (−7, −4) **c)** (2, −7)

7. a) $(5x, 2y)$ **b)** $(a + b, a + 2b)$ **8.** $\left(-\frac{3}{2}, 1\right)$, (3, −2), $\left(\frac{15}{2}, -5\right)$

9. a) −8 **b)** −3 **10.** B **11.** D **12.** A **13.** D

14. 2 0 . 9 **15.** 4

16. 3 4 **17.** 9 . 4