Lesson 3: Function Notation - Part Two

Friday, August 31, 2018 2:40 AM

Functions Lesson #3:
Function Notation - Part Two

Graphing a Function

Consider the function $f(x) = 3x + 1$. The values of x represent the inputs and make up the domain of the function. The values of $f(x)$ represent the outputs and make up the range of the function.

In previous lessons, we have used y to represent the outputs and the range of a relation. We can therefore write the function $f(x) = 3x + 1$ in x-y notation as $y = 3x + 1$.

The function $f(x) = 3x + 1$ can be written in x-y notation as shown.

Function notation	x-y notation
$f(x) = 3x + 1$	$y = 3x + 1$

- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**. x
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**. f(x) y

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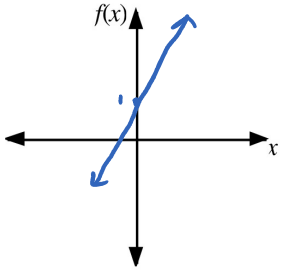
Use a ~~graphing calculator~~ to sketch the graph of the function $f(x) = 3x + 1$.



a) In each case, express the relation given in function notation as an equation in two variables.

i) $f(x) = 7x - 23$

$y = 7x - 23$

ii) $g(t) = t^2 - 2t + 35$

$y = t^2 - 2t + 35$

b) Express the relation $y = 11x - 15$ in function notation.

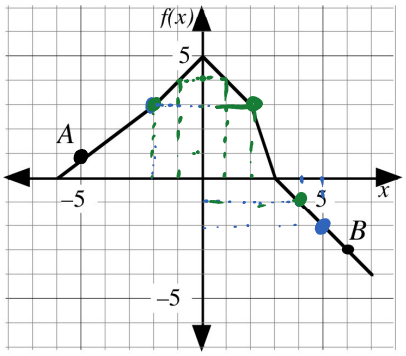
$f(x) = 11x - 15$

c) The graph of the function defined by $y = f(x)$ has equation $y = 4 - 3x$. Express the equation in function notation.

$f(x) = 4 - 3x$

$y = f(x)$

The graph of a function f is shown.



a) Complete

i) $f(5) = -2$ (x, y) **ii)** $f(-2) = 3$ (x, y) **iii)** $f(4) = -1$

b) Write the ordered pairs associated in **a)**.

i) $(5, -2)$ **ii)** $(-2, 3)$ **iii)** $(4, -1)$

c) State the value(s) of x if

i) $f(x) = -1$ (y) **ii)** $f(x) = 3$ (y) **iii)** $f(x) = 4$

$x = 4$ $x = -2, 2$ $x = \pm 1$

d) Use the notation in **a)** to make a statement about the points A and B on the graph.

$A(-5, 1) \rightarrow f(-5) = 1$ $B(6, -3)$ $f(6) = -3$

e) Write the x- and y- intercepts of the graph using function notation.

x-int, $y = 0$
$f(-6) = 0$
$f(3) = 0$

y-int, $x = 0$
$f(0) = 5$

Reminder
$f(6) = -3$
input output
$(6, -3)$
input output

f) Complete the following statements.

- The domain of f is $\{x \mid \underline{-6} \le x \le \underline{7},\ x \in R\}$
- The range of f is $\{f(x) \mid \underline{-4} \le f(x) \le \underline{5},\ f(x) \in R\}$

Complete Assignment Questions #1 - #12

Assignment

1. In each case, express the relation given in function notation as an equation in two variables.

a) $f(x) = 10 - 3x$ **b)** $g(x) = 12x^2 - 5$ **c)** $P(t) = 2t + 9$

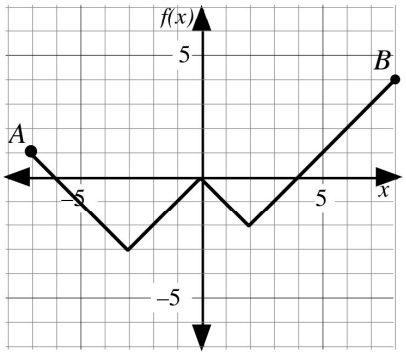
2. Express the following relations in function notation.

a) $y = 17x - 9$ **b)** $y = 4v + 25$ **c)** $x + 2y + 6 = 0$

3. **a)** The graph of the function defined by $y = f(x)$ has equation $y = 0.5x - 0.25$
Express the equation in function notation.

b) The graph of the velocity function defined by $v = f(t)$ has equation $v = 4.9t^2$.
Express the equation in function notation.

4. The graph of a function f is shown.



a) Complete

i) $f(3) =$ **ii)** $f(-3) =$ **iii)** $f(-6) =$

b) Write the ordered pairs associated with **a)**.

i) **ii)** **iii)**

c) State the value(s) of x if

i) $f(x) = 3$ **ii)** $f(x) = -2$ **iii)** $f(x) = -4$

d) Use the notation in a) to make a statement about the points A and B on the graph.

e) Write the x- and y- intercepts of the graph using function notation.

f) Complete the following statements.

- The domain of f is $\{x \mid$ _____ $\leq x \leq$ _____ , $x \in R\}$
- The range of f is $\{f(x) \mid$ _____ $\leq f(x) \leq$ _____, $f(x) \in R\}$

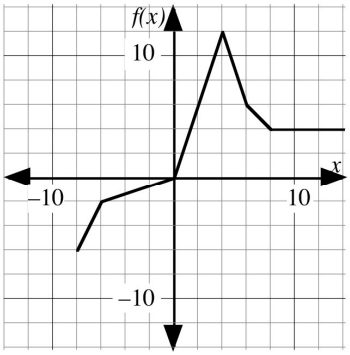
5. The function $g(x) = 3x^2 - 4$ has a domain $\{-2, -1, 0, 1, 2\}$.

a) State the range of g. **b)** Solve the equation $g(x) = -1$.

6. Consider the graph of the function f shown below.

a) Complete the table.

b) Explain why the solution to the equation $f(x) = 4$ has an infinite number of solutions.



x	$f(x)$	Ordered Pair
		(2,)
	0	
–6		
8		
	–6	
10		

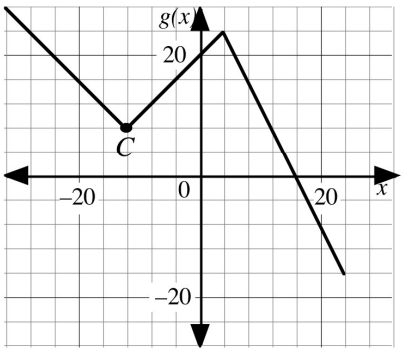
7. Given that $f(x) = 9 - 2x$

a) evaluate $f(-3)$

b) find the value of $f(t) + f(-t)$

c) calculate the x-intercept and the y-intercept on the graph of f.

8. The graph of a function is shown.



a) A student is asked to make a statement about point C on the graph. The student states that $f(-3) = 2$.

i) Explain **two** errors in the student's statement.

ii) Write a correct statement using function notation about point C.

b) Give the solution to the following equations.

i) $g(x) = -8$ **ii)** $g(x) = 16$.

c) State the value of **i)** $g(-8)$ **ii)** $g(16)$

d) State the domain and range of the function.

e) The equation $g(a) = b$ has **exactly two** solutions. Explain clearly how to use the graph to determine values of a and b, and provide two sets of answers to the problem.

9. Consider the function $f(x) = 1 - x^2$, where x is an integer.

a) Evaluate $f(2) - f(-1)$ **b)** Given that $f(a) = -8$, calculate all possible values of a.

Multiple Choice

10. The graph of the function $f(x) = 4^x$, $x \in R$, intersects the y-axis at

A. $(0, 0)$
B. $(0, 1)$
C. $(0, 4)$
D. no point

Use the following information to answer the next question.

Function P is such that $P(5) = -1$.
Two students each make a statement about the function P.

- Rose states "When the domain value is 5, the related range value is -1."
- Susan states "The point $(-1, 5)$ is on the graph of $y = P(x)$."

11. Which of the following is true?

A. Both statements are correct.
B. Both statements are incorrect.
C. Rose is correct and Susan is incorrect.
D. Susan is correct and Rose is incorrect.

Numerical Response **12.** Consider the graph of the function $f(x) = 5x - 11$. The x-intercept of the graph of f is located at $(a, 0)$. The value of a is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y = 10 - 3x$ **b)** $y = 12x^2 - 5$ **c)** $y = 2t + 9$

2. **a)** $f(x) = 17x - 9$ **b)** $f(v) = 4v + 25$ **c)** $f(x) = -\frac{1}{2}x - 3$

3. **a)** $f(x) = 0.5x - 0.25$ **b)** $f(t) = 4.9t^2$

4. **a)** **i)** -1 **ii)** -3 **iii)** 0

b) **i)** $(3, -1)$ **ii)** $(-3, -3)$ **iii)** $(-6, 0)$

c) **i)** 7 **ii)** $-4, -2, 2$ **iii)** no solution **d)** A is $f(-7) = 1$, B is $f(8) = 4$

e) x-intercepts can be represented in function notation by; $f(-6) = 0, f(0) = 0, f(4) = 0$
y-intercept can be represented in function notation by $f(0) = 0$

f) $-7 \le x \le 8$, $-3 \le f(x) \le 4$

5. **a)** Range $= \{-4, -1, 8\}$ **b)** $x = \pm 1$

6. See table below.

x	$f(x)$	Ordered Pair
2	6	(2, 6)
0	0	(0, 0)
–6	–2	(–6, –2)
8	4	(8, 4)
–8	–6	(–8, –6)
10	4	(10, 4)

b) The horizontal line where $f(x) = 4$ has an infinite number of input values between 8 and 14.

7. **a)** 15 **b)** 18 **c)** x-int $= \frac{9}{2}$, y-int $= 9$

8. **a)** **i)** The name of the function is g not f. The scale is 4 units per box, not 1 unit per box.
ii) $g(-12) = 8$

b) **i)** $x = 20$ **ii)** $x = -20, -4, 8$

c) **i)** 12 **ii)** 0

d) Domain $= \{x \mid -32 \le x \le 24, x \in R\}$, $\{g(x) \mid -16 \le g(x) \le 28\}, g(x) \in R$

e) A horizontal line must intersect the graph at exactly two points.
This occurs when $g(x) = 24$ and when $g(x) = 8$.
<u>Solution 1</u>: $b = 24$ when $a = -28$ or 4.
<u>Solution 2</u>: $b = 8$ when $a = -12$ or 12

9. **a)** -3 **b)** ± 3 **10.** B **11.** C **12.**

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Functions Lesson #4:
Function Notation and Problem Solving

Using Function Notation

In the previous unit we solved problems about relations defined by an equation.
In this lesson we will solve problems where function notation is used to define the relation.

On page 256, assignment question #6, we had the following scenario.

" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours."

The relation between height and time is described by an **equation.**

The relation is a function because for each input there is only one output, and so it can be described using the **function notation** below.

" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours."

$h(t) = h$

In this example, the notation $h(4)$ is a simplified way of representing the height of the candle after four hours. ← the height at 4 hours

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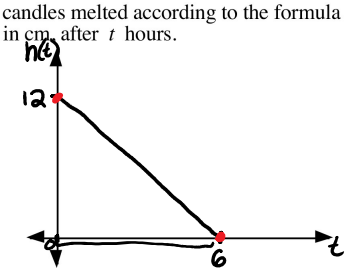
A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours.

a) Use a ~~graphing calculator~~ to sketch the graph of the function and show the graph on the grid



b) Determine the value of $h(5)$.

$h(5) = -2(5) + 12$
$= -10 + 12 = 2$

c) Write in words the meaning of $h(5)$.

The height of the candle after 5 hrs is 2cm

d) Evaluate the following, and explain the significance of each.

i) $h(0)$ $t = 0$
$= 12$
$h(0) = -2(0) + 12$
$= 12$
Starting height is 12cm

ii) $h(6)$ $t = 6$
$= 0$
candle is gone at 6 hours.

iii) $h(8)$
$h(8) = -2(8) + 12$
$= -16 - 12$
$= -4$
has no meaning in the context

e) How long will it take for the candle to burn down to a height of 7 cm?

$h(t) = 7$
$7 = -2t + 12$ *solve for t
$-12 \quad -12$
$\frac{-5}{-2} = \frac{-2t}{-2}$
$t = 2.5$

f) Suggest an appropriate domain and range for the function.

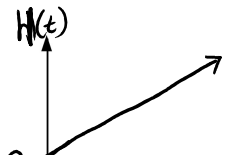
D: $0 \le x \le 6$, $x \in \mathbb{R}$
R: $0 \le y \le 12$, $y \in \mathbb{R}$

Complete Assignment Questions #1 - #4

Assignment

1. Ivory the botanist treated a 2 cm plant with a special growth fertilizer. With this fertilizer, the plant grew at a rate modelled by the function $H(t) = \frac{5}{3}t + 2$, where $H(t)$ represents the height of the plant in cm after t days.

a) Use a ~~graphing calculator~~ to sketch the graph of the function and show the graph on the grid.



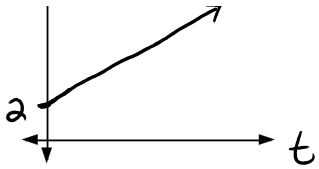
b) Determine the value of $H(3)$.

L3 #(1,2)ac, 4, 6 /10

L4 #1 /5

a) Use a ~~graphing calculator~~ to sketch the graph of the function and show the graph on the grid.

b) Determine the value of $H(3)$.



/5

c) Write in words the meaning of $H(3)$.

d) Evaluate the following.

i) $H(0)$ **ii)** $H(6)$ **iii)** $H(21)$

e) How long will it take for the plant to reach a height of 21 cm?

f) It takes 27 days for the plant to mature (to reach maximum height). State the domain and range of the function $H(t)$.

2. The cost to Inner Technology of producing IT graphing calculators can be modelled by the function $C(n) = 11750 + 32n$, where $C(n)$ represents the cost in dollars of producing n calculators.

a) Sketch the graph of the function for a maximum of 4000 calculators.

b) Determine the value of $C(30)$.

c) Write in words the meaning of $C(30)$.

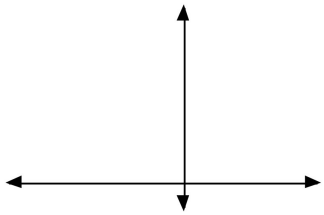
d) Evaluate $C(0)$ and explain its significance.

e) How many calculators can be produced for $31 270?

f) Last month IT produced 2 600 calculators and spent $14 000 on advertising. If there are other fixed monthly costs of $24 500, and each calculator sells for $165, how much profit would be made if all the calculators are sold?

3. Over the last 10 years, data was recorded for the number of cups of hot chocolate sold at BGB Senior High School. It was found from the data that the warmer the weather, the less cups of hot chocolate were sold. The data can be modelled by the formula $N(t) = 150 - 10t$, where $N(t)$ is the daily number of cups of hot chocolates sold when the average daily temperature is t °C.

a) Sketch the graph of the function on the grid provided.



b) Determine the value of $N(-5)$.

c) Write in words the meaning of $N(-5)$.

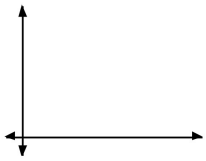
d) What was the average temperature if 190 cups of hot chocolate were sold?

e) Explain how to estimate the lower limit of the domain of the relation.

f) Suggest an appropriate domain and range for the function $N(t)$ if BGB High School is located in the Okanagan, British Columbia.

4. A special type of weather balloon follows a path which can be represented by the formula $h(t) = -9t^2 + 900t$, where $h(t)$ is the height in cm after t minutes.

a) Sketch the graph of the function on the grid.



b) Determine the value of $h(30)$ and $h(70)$.

c) Does $h(30) = h(70)$? Do they mean the same thing? Explain.

d) Evaluate the following, and explain their significance in the context of the question.

i) $h(0)$ **ii)** $h(100)$ **iii)** $h(110)$

e) What is the highest point the balloon will reach?

f) When will the balloon land?

g) Suggest an appropriate domain and range for the function $h(t)$?

Answer Key

1. b) 7 **c)** After 3 days the height is 7 cm. **d)** i) 2 ii) 12 iii) 37 **e)** 11.4 days
f) domain $\{t \mid 0 \le t \le 27, t \in R\}$ range $\{H(t) \mid 2 \le H(t) \le 47, H(t) \in R\}$

2. b) 12710 **c)** It costs \$12 710 to produce 30 calculators.
d) $C(0) = 11\,750$. There are fixed costs of \$11750 before any calculators are produced.
e) 610 **f)** \$295 550

3. b) 200 **c)** 200 cups are sold when the average temperature is –5°C. **d)** –4°C
e) Estimate the minimum average daily temperature.
f) Answers may vary. domain $\{t \mid -20 \le t \le 15, t \in R\}$ range $\{N(t) \mid 0 \le N(t) \le 350, N(t) \in W\}$

4. b) both = 18 900
c) They are equal but do not represent the same thing. $h(30)$ is the height after 30 minutes. and $h(70)$ is the height after 70 minutes
d) i) 0 Initial height = 0 m ii) 0 After 100 min the balloon has landed on the ground.
iii) –9900 this has no meaning since the balloon has already landed
e) 22 500 cm = 225 m **f)** after 100 min
g) domain $\{t \mid 0 \le t \le 100, t \in R\}$ range $\{h(t) \mid 0 \le h(t) \le 22\,500, h(t) \in R\}$