Lesson 1: Trigonometric Ratios

Friday, August 31, 2018 2:21 AM

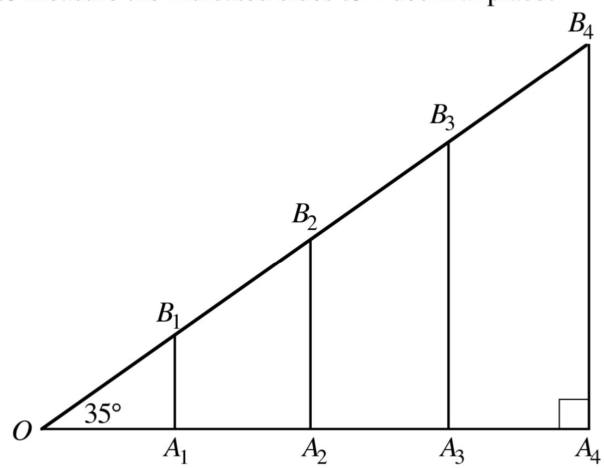
Trigonometry Lesson #1:
Trigonometric Ratios

Overview of Unit

Trigonometry (from the Greek trigonon = three angles and metro = measure) is a branch of mathematics dealing with angles, triangles and trigonometric functions such as sine, cosine and tangent. In this unit we study relationships between sides and angles in right triangles.

Investigation #1

The diagram shows a series of similar right triangles, $\triangle OA_1B_1$, $\triangle OA_2B_2$, etc.
Complete the work below using a ruler to measure the indicated sides to 1 decimal place. Calculate each ratio to 1 decimal place.



$\frac{A_1B_1}{OA_1} = \frac{1.4}{2} = 0.7$

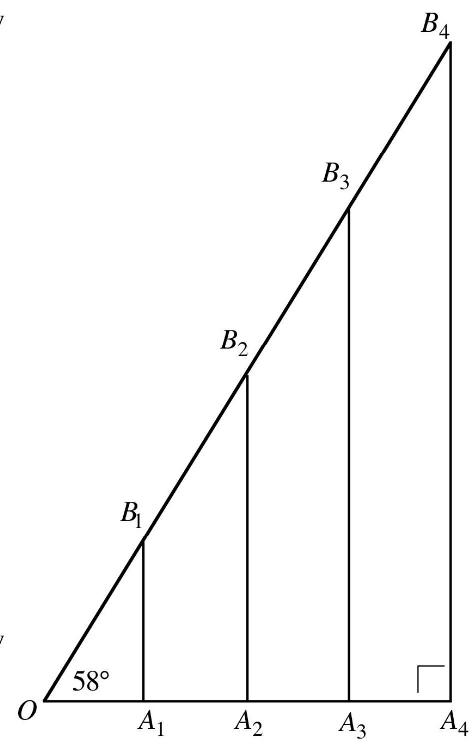
$\frac{A_2B_2}{OA_2} = \frac{}{4} =$

$\frac{A_3B_3}{OA_3} = \quad =$

$\frac{A_4B_4}{OA_4} = \quad =$

We can conclude that the ratio ______ is somehow connected to the angle of 35°.

Investigation #2



$\frac{A_1B_1}{OA_1} = \frac{2.4}{1.5} = 1.6$

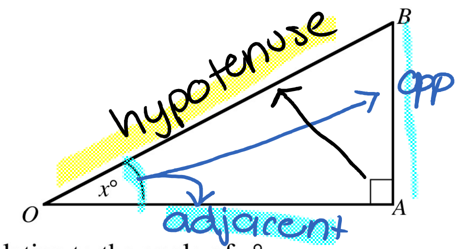
$\frac{A_2B_2}{OA_2} = \frac{}{3} =$

$\frac{A_3B_3}{OA_3} =$

$\frac{A_4B_4}{OA_4} =$

We can conclude that the ratio ______ is somehow connected to the angle of 58°.
We will investigate these ratios further in the rest of the lesson.

Ratios of Sides in a Right Triangle



Consider the right triangle AOB shown.

Let angle $AOB = x°$.

Each of the sides of the triangle is given a special name relative to the angle of $x°$.

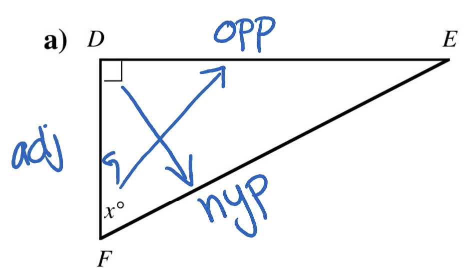
The longest side, OB, is called the HYPOTENUSE (hyp).

The side opposite the angle of $x°$, AB, is called the OPPOSITE (opp).

The third side of the triangle, OA, is called the ADJACENT (adj).

Mark on each of these triangles the hypotenuse (hyp), the opposite (opp), and the adjacent (adj) relative to the angle of $x°$.

a)



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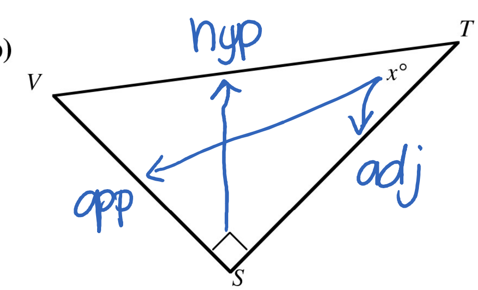


- The ratio $\frac{AB}{OA}$ in the diagram at the top of this page is the same ratio we used in Investigations #1 and #2.
- This ratio can be written as $\frac{\text{opposite}}{\text{adjacent}}$ or $\frac{\text{opp}}{\text{hyp}}$ and is known as the tangent ratio.
- There are five other ratios possible using two of the sides of triangle AOB. All six ratios are listed below.

$\frac{AB}{OB} = \frac{\text{opposite}}{\text{hypotenuse}}$ = the sine ratio

$\frac{OA}{OB} = \frac{\text{adjacent}}{\text{hypotenuse}}$ = the cosine ratio

$\frac{AB}{OA} = \frac{\text{opposite}}{\text{adjacent}}$ = the tangent ratio

The three ratios above left are the primary trigonometric ratios and will be studied in this course.

$\frac{OB}{AB} = \frac{\text{hypotenuse}}{\text{opposite}}$ = the cosecant ratio

$\frac{OB}{OA} = \frac{\text{hypotenuse}}{\text{adjacent}}$ = the secant ratio

$\frac{OA}{AB} = \frac{\text{adjacent}}{\text{opposite}}$ = the cotangent ratio

The three ratios above right are the reciprocal trigonometric ratios and will be studied in **higher level math courses** .

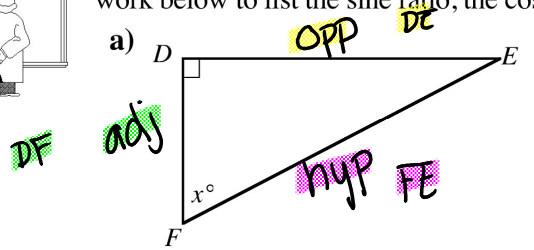
SOH CAH TOA

sine = opp/hyp cosine = adj/hyp tan = opp/adj

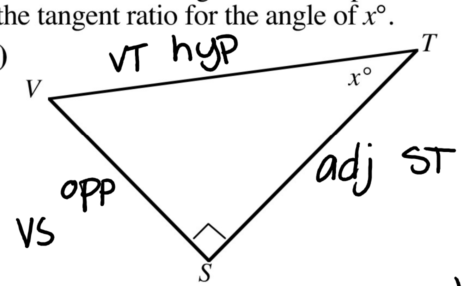
The rules for determining the sine ratio, cosine ratio and tangent ratio for an angle in a right triangle can be memorized by using the acronym SOH CAH TOA.

The diagrams shown below are from Class Example #1. Use these diagrams to complete the work below to list the sine ratio, the cosine ratio, and the tangent ratio for the angle of $x°$.

a)



b)



sine ratio for the angle of $x°$ = $\frac{\text{opposite}}{\text{hypotenuse}}$ = $\frac{DE}{FE}$

cosine ratio for the angle of $x°$ = $\frac{\text{adjacent}}{\text{hypotenuse}}$ = $\frac{DF}{FE}$

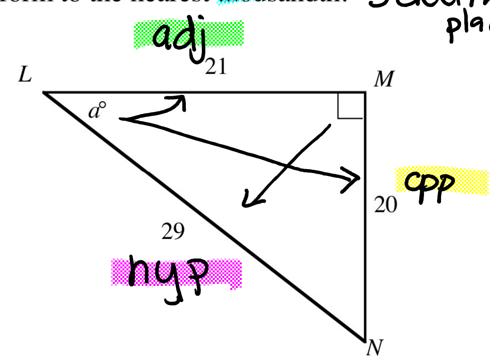
tangent ratio for the angle of $x°$ = $\frac{\text{opposite}}{\text{adjacent}}$ = $\frac{DE}{DF}$

sine ratio for the angle of $x°$ = $\frac{opp}{hyp} = \frac{VS}{VT}$

cosine ratio for the angle of $x°$ = $\frac{adj}{hyp} = \frac{ST}{VT}$

tangent ratio for the angle of $x°$ = $\frac{opp}{adj} = \frac{VS}{ST}$

Complete the work below to express the three primary trigonometric ratios, relative to angle $a°$, as rational numbers and in decimal form to the nearest thousandth. (0.000 — 3 decimal places)



sine ratio relative to angle $a°$ = $\frac{opp}{hyp} = \frac{20}{29}$

$20 \div 29 = 0.68965\ldots = 0.690$

cosine ratio relative to angle $a°$ = $\frac{adj}{hyp} = \frac{21}{29}$

$21 \div 29 = 0.72413\ldots = 0.724$

tangent ratio relative to angle $a°$ = $\frac{opp}{adj} = \frac{20}{21}$

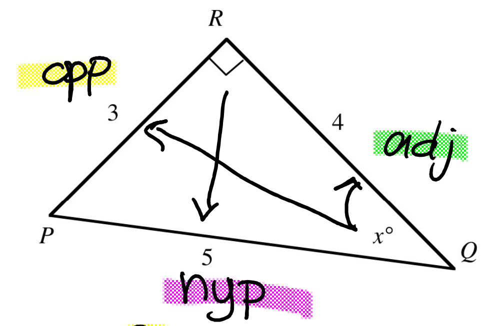
$20 \div 21 = 0.95238\ldots = 0.952$

Complete Assignment Questions #1 - #3

Using sin, cos, tan

Class Ex. #4

Consider right triangle PQR shown.



The sine ratio for the angle of $x°$ is $\frac{PR}{PQ} = \frac{3}{5}$

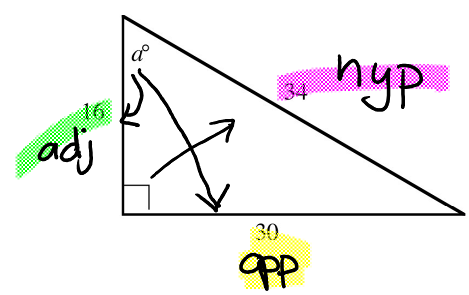
In short we write $\sin x° = \frac{3}{5}$.

- Similarly $\cos x° = \frac{4}{5}$ and $\tan x° = \frac{3}{4}$

Note Sometimes the trigonometric ratios for an angle are given in terms of the letter at which the angle is located. For example, in $\triangle PQR$ above we could write $\sin Q = \frac{3}{5}$.

Class Ex. #5

Complete the following, writing the ratio in simplest rational form.



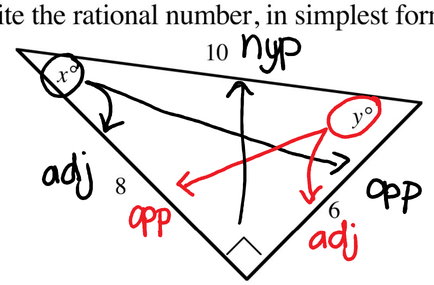
a) $\sin a° = \frac{30}{34} = \frac{15}{17}$ SOH

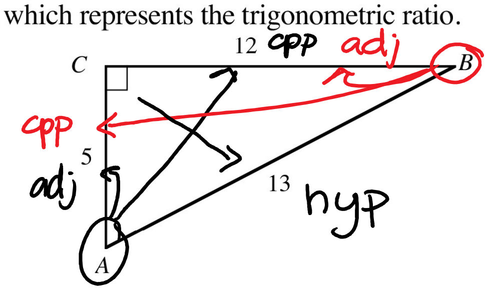
b) $\cos a° = \frac{16}{34} = \frac{8}{17}$ CAH

c) $\tan a° = \frac{30}{16} = \frac{15}{8}$ TOA

Class Ex. #6

Write the rational number, in simplest form, which represents the trigonometric ratio.





i) $\sin x° = \frac{6}{10} = \frac{3}{5}$ SOH

ii) $\tan y° = \frac{8}{6} = \frac{4}{3}$ TOA

iii) $\cos A = \frac{5}{13}$ CAH

iv) $\tan B = \frac{5}{12}$ TOA

v) $\sin y° = \frac{8}{10} = \frac{4}{5}$ SOH

vi) $\cos x° = \frac{8}{10} = \frac{4}{5}$ CAH

vii) $\cos B = \frac{12}{13}$ CAH

viii) $\sin A = \frac{12}{13}$ SOH

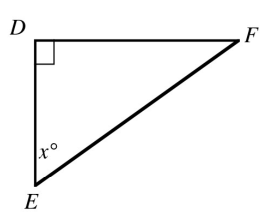
Complete Assignment Questions #4 - #11 #1aceg, 2b, 3, 4, 6ace, 7

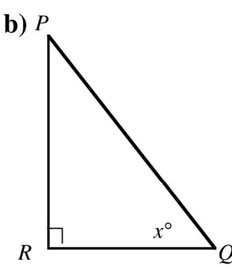
Assignment

1. Consider the eight triangles below. For each triangle, complete the tables for the angle $x°$.

Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)		DE	EF
b)			
c)			
d)			
e)			
f)			
g)			
h)			

Triangle	sine ratio	cosine ratio	tangent ratio
a)		$\frac{DE}{EF}$	
b)			
c)			
d)			
e)			
f)			
g)			
h)			

a)


b)


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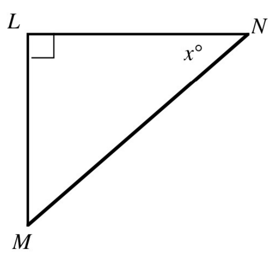

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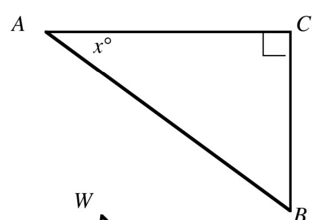

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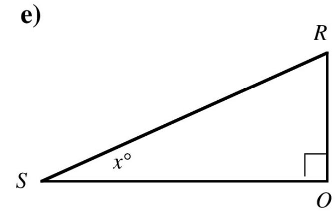

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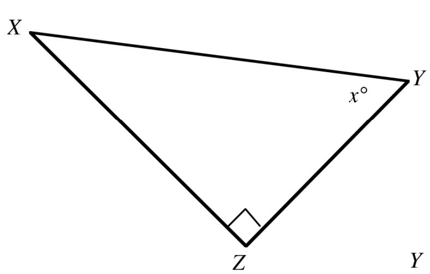
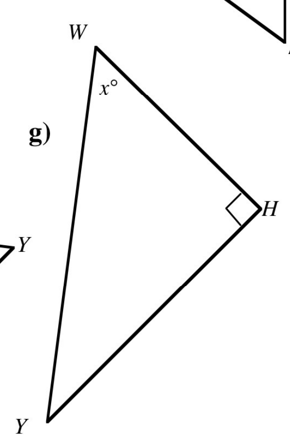


g)


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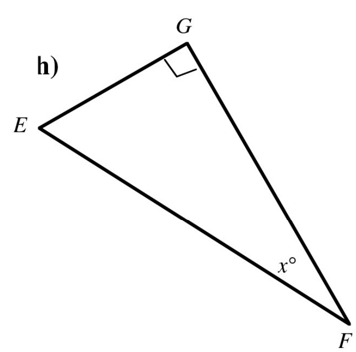
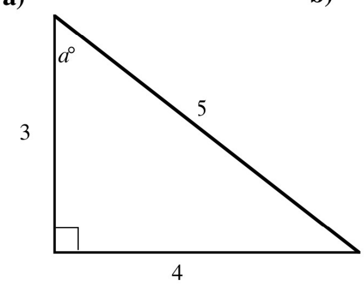


2. On each triangle mark the (hyp), (opp) and (adj) relative to angle $a°$ and determine the trigonometric ratios associated for angle $a°$.

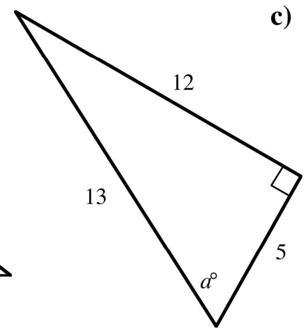
Complete the table.

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{4}{5}$		
b)			
c)			

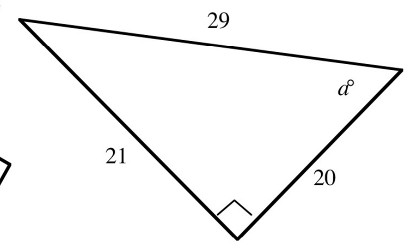
a)



b)

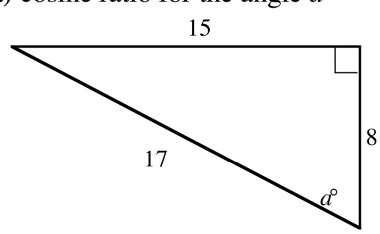


c)

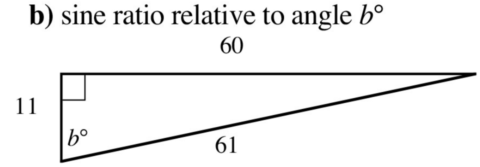


3. Determine, as a fraction in simplest form, the value of the trigonometric ratio indicated.

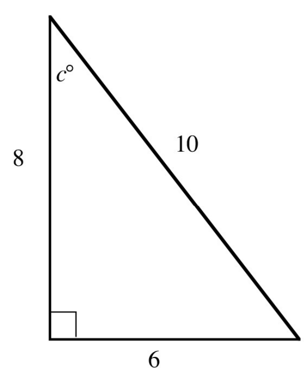
a) cosine ratio for the angle $a°$



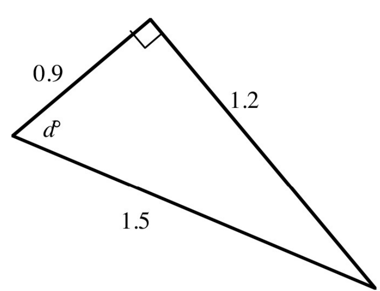
b) sine ratio relative to angle $b°$



c) tangent ratio relative to angle $c°$

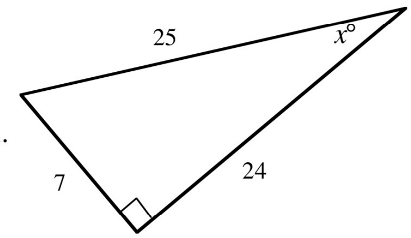


d) sine ratio for the angle $d°$

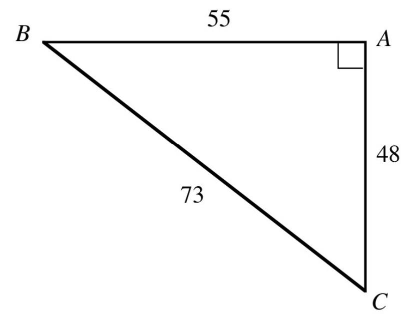


4. On the triangle, mark the hyp, opp, and adj for the angle $x°$ and determine the values of $\sin x°$, $\cos x°$, and $\tan x°$.

Write each answer as a decimal to the nearest hundredth.



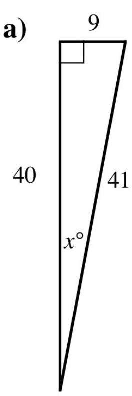
5. In the triangle, determine the exact values of $\sin B$, $\cos B$, and $\tan B$.



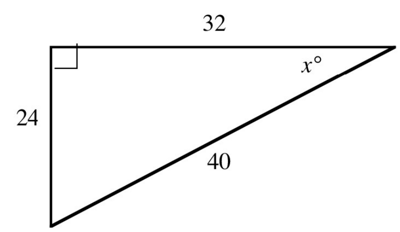
6. In each triangle, determine the values of $\sin x°$, $\cos x°$, and $\tan x°$ in simplest rational form.

Triangle	sin $x°$	cos $x°$	tan $x°$
a)			
b)			
c)			
d)			
e)			
f)			

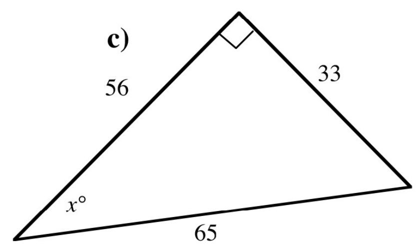
a)



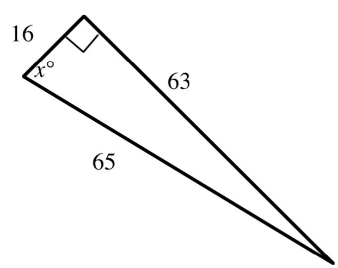
b)



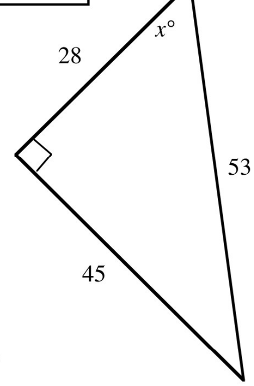
c)



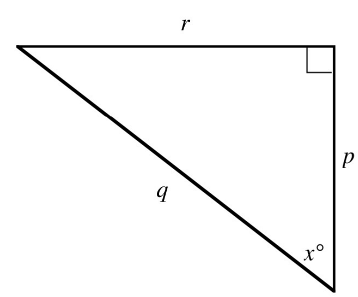
d)



e)

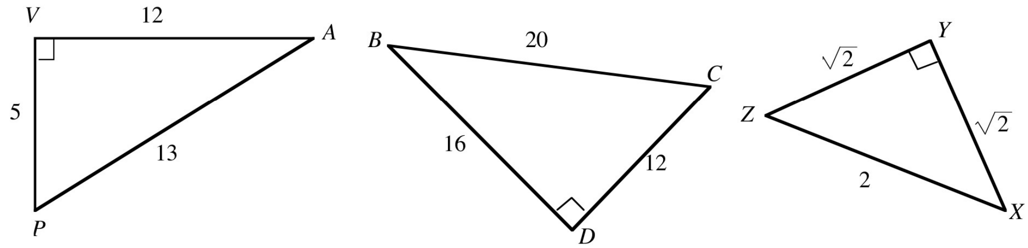


f)



7. In each case, write the rational number which represents the trigonometric ratio.

a) $\sin A =$ **b)** $\cos B =$ **c)** $\tan X =$



Multiple Choice

8. In right triangle ABC, $AB = 52$ units, $AC = 48$ units and $BC = 20$ units. The value of $\cos B$ and $\sin B$ are respectively

A. $\frac{5}{13}$ and $\frac{12}{13}$

B. $\frac{12}{13}$ and $\frac{5}{13}$

C. $\frac{5}{12}$ and $\frac{12}{5}$

D. $\frac{5}{13}$ and $\frac{12}{5}$

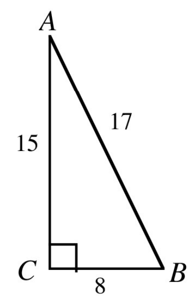
9 For the right angled triangle ABC, only one of the following ratios is correct. The correct ratio is

A. $\sin A = \frac{8}{15}$

B. $\cos A = \frac{8}{17}$

C. $\tan B = \frac{8}{15}$

D. $\sin B = \frac{15}{17}$



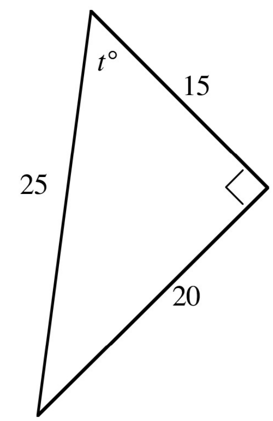
10. In a right triangle $\tan x° = \frac{7}{5}$. A student claims this indicates that in the right triangle the side <u>opposite</u> to the angle $x°$ is 7 units and the side <u>adjacent</u> to the angle $x°$ is 5 units.

The student's claim

A. is always true   **B.** is always false

C. may be true or false.   **D.** depends on the value of $x°$

Numerical Response **11.** In the diagram, to the nearest tenth, the value of $\frac{\sin t°}{\cos t°}$ is _______.



(Record your answer in the numerical response box from left to right)

Answer Key

1 .

Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)	*DF*	*DE*	*EF*
b)	*PR*	*QR*	*PQ*
c)	*LM*	*LN*	*MN*
d)	*BC*	*AC*	*AB*
e)	*OR*	*OS*	*RS*
f)	*XZ*	*YZ*	*XY*
g)	*HY*	*HW*	*WY*
h)	*EG*	*FG*	*EF*

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{DF}{EF}$	$\frac{DE}{EF}$	$\frac{DF}{DE}$
b)	$\frac{PR}{PQ}$	$\frac{QR}{PQ}$	$\frac{PR}{QR}$
c)	$\frac{LM}{MN}$	$\frac{LN}{MN}$	$\frac{LM}{LN}$
d)	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$
e)	$\frac{OR}{RS}$	$\frac{OS}{RS}$	$\frac{OR}{OS}$
f)	$\frac{XZ}{XY}$	$\frac{YZ}{XY}$	$\frac{XZ}{YZ}$
g)	$\frac{HY}{WY}$	$\frac{HW}{WY}$	$\frac{HY}{HW}$
h)	$\frac{EG}{EF}$	$\frac{FG}{EF}$	$\frac{EG}{FG}$

2 .

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$
b)	$\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{5}$
c)	$\frac{21}{29}$	$\frac{20}{29}$	$\frac{21}{20}$

3 . **a)** $\frac{8}{17}$ **b)** $\frac{60}{61}$ **c)** $\frac{3}{4}$ **d)** $\frac{4}{5}$

4 . $\sin x° = 0.28$ $\cos x° = 0.96$ $\tan x° = 0.29$

5 . $\sin B = \frac{48}{73}$ $\cos B = \frac{55}{73}$ $\tan B = \frac{48}{55}$

6 .

Triangle	$\sin x°$	$\cos x°$	$\tan x°$
a)	$\frac{9}{41}$	$\frac{40}{41}$	$\frac{9}{40}$
b)	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$
c)	$\frac{33}{65}$	$\frac{56}{65}$	$\frac{33}{56}$
d)	$\frac{63}{65}$	$\frac{16}{65}$	$\frac{63}{16}$
e)	$\frac{45}{53}$	$\frac{28}{53}$	$\frac{45}{28}$
f)	$\frac{r}{q}$	$\frac{p}{q}$	$\frac{r}{p}$

7 . **a)** $\frac{5}{13}$ **b)** $\frac{4}{5}$ **c)** 1

8 . A **9 .** D **10.** C

11.

1	.	3	