Lesson 3: Multiplication of Two Binomials

Friday, August 31, 2018 2:35 AM

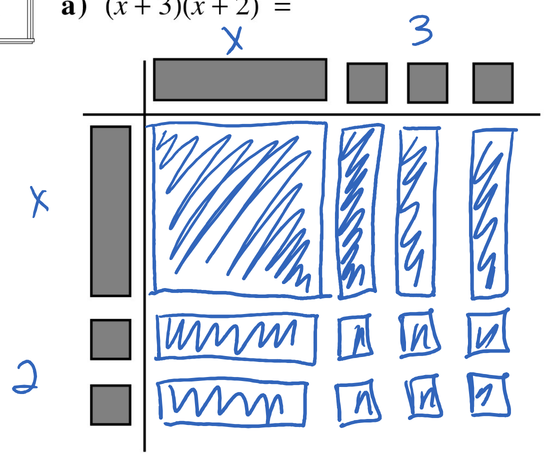
Polynomial Operations Lesson #3:
Multiplication of Two Binomials

Multiplying Two Binomials using Area Diagrams

In the last lesson, we multiplied a monomial by a polynomial. In this lesson, we extend the process to the product of two binomials.

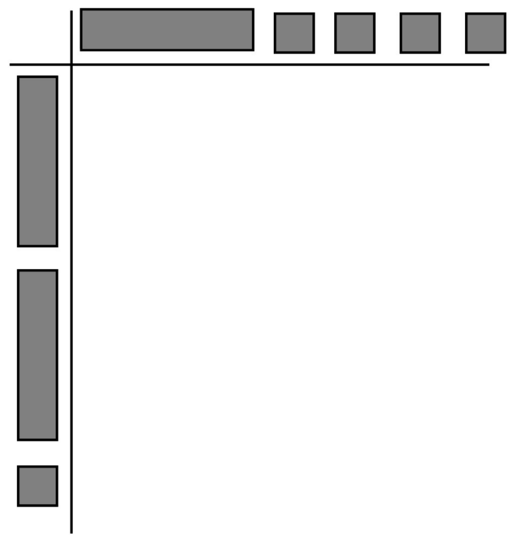
Complete the algebra tile diagrams and determine the binomial products.

a) $(x + 3)(x + 2)$ =

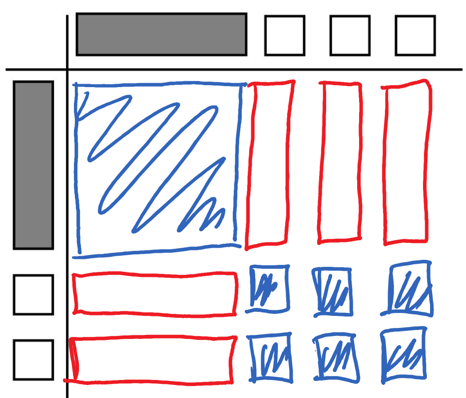


$x^2 + 5x + 6$

b) (   )(   ) =

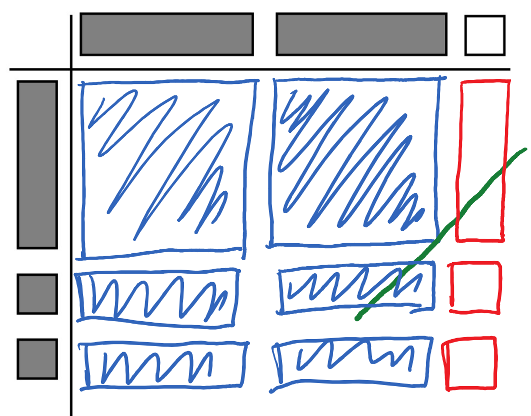


c) ($x - 3$)($x - 2$) =



$x^2 - 5x + 6$

d) ($2x - 1$)($x + 2$) =



$2x^2 + 3x - 2$

In class example 1a), we used an algebra tile diagram to show that the product $(x+3)(x+2)$ could be expressed in simplified expanded form as $x^2 + 5x + 6$.

The algebra tile diagram used to model $(x+3)(x+2)$ can be modified into the following area diagram which shows that the product of two binomials is equivalent to four monomial products.

	x	3
x	x^2	$3x$
2	$2x$	6

$(x+3)(x+2)$
$= x^2 + 5x + 6$

Use an area diagram like the one above to determine the product of each of the following binomials.

a) $(5x - 6)(2x + 1)$

	$5x$	-6
$2x$	$10x^2$	$-12x$
1	$5x$	-6

$= 10x^2 - 12x + 5x - 6$
$= 10x^2 - 7x - 6$

b) $(a^2 - 5)(a^2 - 8)$

	a^2	-5
a^2	a^4	$-5a^2$
-8	$-8a^2$	40

$= a^4 - 13a^2 + 40$

c) $(3p + 2q)(p + 9q)$

	$3p$	$2q$
p	$3p^2$	$2pq$
$9q$	$27pq$	$18q^2$

$= 3p^2 + 29pq + 18q^2$

d) $(a + b)(c + d)$

An area diagram can be used to show that the multiplication of two, two-digit numbers can be performed as four separate products.
For example the product 32×34 can be determined without a calculator, by long multiplication or by an area diagram as follows:

Long Multiplication

```
   3 2
 × 3 4
 -----
 1 2 8
 9 6
 -----
1 0 8 8
```

Area Diagram

	30	2
30	900	60
4	120	8

32 x 34
= 900 + 120 + 60 + 8
= 1088

Use an area diagram and no calculator to determine the following products.

a) 43×51

	40	3
50	2000	150
1	40	3

2000 + 150 + 40 + 3 = 2193

b) 76×82

	70	6
80	5600	480
2	140	12

5600 + 480 + 140 + 12 = 6232

Complete Assignment Questions #1 - #3

Multiplying Two Binomials using the Distributive Property

In the area diagram modelling $(x+3)(x+2)$, we noted that there were four separate monomial products involved in the expansion.
These products are simply the extension of the distributive property to binomial products.

Distributive property for binomials $(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd$

FOIL

Class Ex. #4

Use the distributive property to determine the following products.

a) $(x+3)(x+2)$
$= x(x+2)+3(x+2)$
$= x^2+2x+3x+6$
$= x^2+5x+6$

b) $(a-7)(2a-1)$
$= a(2a-1)-7(2a-1)$
$= 2a^2-a-14a+7$
$= 2a^2-15a+7$

c) $(p-8)(q-8)$
$= p(q-8)-8(q-8)$
$= pq-8p-8q+64$

d) $(x+4y)(x-5y)$

e) $(9a^2-1)(5a^3+6)$

The method used in the distributive property can be simplified by noticing that the four monomial products $(a+b)(c+d) = ac + ad + bc + bd$ can be memorized using the acronym FOIL .

F – first term in each bracket	ie	ac
O – outside terms	ie	ad
I – inside terms	ie	bc
L – last term in each bracket	ie	bd

Class Ex. #5

Use FOIL to determine each product.

a) $(x+6)(x+4)$
F: $x \cdot x = x^2$
O: $x \cdot 4 = 4x$
I: $6 \cdot x = 6x$
L: $6 \cdot 4 = 24$
$= x^2+10x+24$

b) $(y-7)(y+2)$
$= y^2+2y-7y-14$
$= y^2-5y-14$

c) $(3x+1)(x-5)$
$= 3x^2-15x+x-5$
$= 3x^2-14x-5$

d) $(6a-5b)^2$
$= (6a-5b)(6a-5b)$
$= 36a^2-30ab-30ab+25b^2$
$= 36a^2-60ab+25b^2$

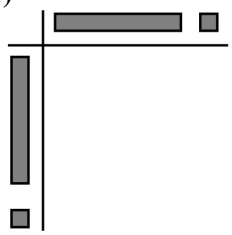
Complete Assignment Questions #4 - #9

#2ace, 4acegi, 5acegi, 7acegi

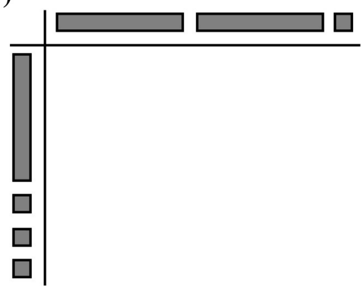
Assignment

1. Complete the algebra tile diagrams and determine the binomial products.

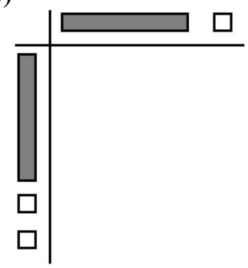
a)



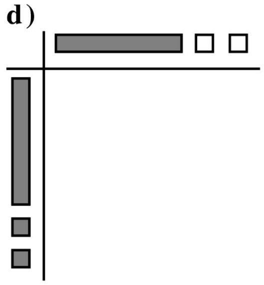
b)



c)



d)



e)

2. Use an area diagram to determine the product of each of the following binomials.

a) $(x + 6)(x - 2)$ **b)** $(2x + 3)(2x + 7)$ **c)** $(y - 3)(4y + 1)$

d) $(3d - 5)(6d - 9)$ **e)** $(2x - y)(4x + y)$ **f)** $(3p - 8q)(p - 5q)$

g) $(a^2 + 8)(a^2 - 8)$ **h)** $(t^3 + 2s)(t^3 + 2s)$ **i)** $(a + b)(a + c)$

3. Without a calculator, use an area diagram to determine the following products.

a) 23×21 **b)** 34×12 **c)** 74×32

d) 65×73 **e)** 49×55 **f)** 86×86

4. Use the distributive property to determine the following products.

a) $(x + 4)(x + 7)$ **b)** $(a + 7)(3a - 5)$ **c)** $(p - 2)(p - 8)$

d) $(x + 6y)(x - 2y)$ **e)** $(4a + 9b)(2a + 3b)$ **f)** $(6 - y)(1 + 4y)$

g) $(2a - 1)(6b - 1)$ **h)** $(7x^2 - 3)(7x^2 - 3)$ **i)** $(2y^2 - 3)(5y^5 + 1)$

5. Use FOIL to determine each product.

a) $(x + 3)(x + 6)$ **b)** $(y + 4)(y + 9)$ **c)** $(x + 1)(x - 8)$ **d)** $(a - 7)^2$

e) $(x + 2)(5x + 4)$ **f)** $(3y - 5)(2y + 9)$ **g)** $(6x + 1)(x - 6)$ **h)** $(6 - 5b)(6 - 5b)$

i) $(x + 3\text{y})(x + 4\text{y})$ **j)** $(a - 7\text{b})(3a + 4\text{b})$ **k)** $(5x + z)(5x - z)$ **l)** $(9 - a^2)(5 - a^2)$

6. A rectangle has length $(2a + 5)$ cm and width $(a + 4)$ cm.
Determine the area of the rectangle (in cm^2) by completing each of the following solutions.

Area = length × width = (   )(   )

(i) *use a diagram*

(ii) *use the distributive property*
$(2a + 5)(a + 4)$
$= 2a(a + \quad) +$

(iii) *use FOIL*
$(2a + 5)(a + 4)$

7. Expand and simplify where possible.

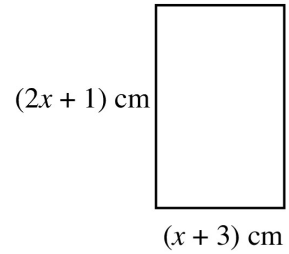
a) $(7x - 2)(3x + 5)$ **b)** $(2h - 3)(2h - 1)$ **c)** $(3z + 4)(3z + 5)$

d) $(4x - 3)(3x - 4)$ **e)** $(8x - 3\text{y})(2x + y)$ **f)** $(1 + 3b)^2$

g) $(x - 2)(6y - 1)$ **h)** $(1 + 3y^2)(1 - 3y^2)$ **i)** $(x^2 + 7y^2)(2x^2 - 5y^2)$

Numerical Response

8. The area of the rectangle shown can be written in the form $px^2 + qx + r$, where $p, q,$ and r are natural numbers.



Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.

(Record your answer in the numerical response box from left to right)

9. The expansion of $(3x - c)(x - 3)$, where c is a whole number, results in a polynomial in x with a leading coefficient of 3 and a constant term of 12.
The value of c is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $(x + 1)(x + 1) = x^2 + 2x + 1$ **b)** $(2x + 1)(x + 3) = 2x^2 + 7x + 3$
c) $(x - 1)(x - 2) = x^2 - 3x + 2$ **d)** $(x - 2)(x + 2) = x^2 - 4$
e) $(1 + 3x)(1 - x) = 1 + 2x - 3x^2$

2. **a)** $x^2 + 4x - 12$ **b)** $4x^2 + 20x + 21$ **c)** $4y^2 - 11y - 3$
d) $18d^2 - 57d + 45$ **e)** $8x^2 - 2xy - y^2$ **f)** $3p^2 - 23pq + 40q^2$
g) $a^4 - 64$ **h)** $t^6 + 4st^3 + 4s^2$ **i)** $a^2 + ab + ac + bc$

3. **a)** 483 **b)** 408 **c)** 2368 **d)** 4745 **e)** 2695 **f)** 7396

4. **a)** $x^2 + 11x + 28$ **b)** $3a^2 + 16a - 35$ **c)** $p^2 - 10p + 16$
d) $x^2 + 4xy - 12y^2$ **e)** $8a^2 + 30ab + 27b^2$ **f)** $6 + 23y - 4y^2$
g) $12ab - 2a - 6b + 1$ **h)** $49x^4 - 42x^2 + 9$ **i)** $10y^7 - 15y^5 + 2y^2 - 3$

5. **a)** $x^2 + 9x + 18$ **b)** $y^2 + 13y + 36$ **c)** $x^2 - 7x - 8$
d) $a^2 - 14a + 49$ **e)** $5x^2 + 14x + 8$ **f)** $6y^2 + 17y - 45$
g) $6x^2 - 35x - 6$ **h)** $36 - 60b + 25b^2$ **i)** $x^2 + 7xy + 12y^2$
j) $3a^2 - 17ab - 28b^2$ **k)** $25x^2 - z^2$ **l)** $45 - 14a^2 + a^4$

6. Area $= (2a + 5)(a + 4) = 2a^2 + 13a + 20$

7. **a)** $21x^2 + 29x - 10$ **b)** $4h^2 - 8h + 3$ **c)** $9z^2 + 27z + 20$
d) $12x^2 - 25x + 12$ **e)** $16x^2 + 2xy - 3y^2$ **f)** $1 + 6b + 9b^2$
g) $6xy - x - 12y + 2$ **h)** $1 - 9y^4$ **i)** $2x^4 + 9x^2y^2 - 35y^4$

8.

2	7	3	

9.

4			