Polynomial Operations Lesson #2:
Multiplying a Polynomial by a Monomial

Using Algebra Tiles

positive
negative

In previous math courses, we learned how to multiply
i) two monomials, and **ii)** a monomial and a binomial or trinomial.

We can use algebra tiles to illustrate the process of multiplying a monomial by a polynomial.

Shaded tiles represent positive quantities and unshaded tiles represent negative quantities.

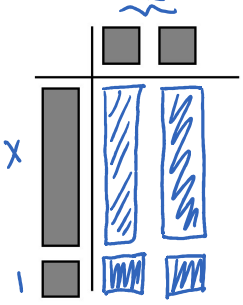
= 1

= x

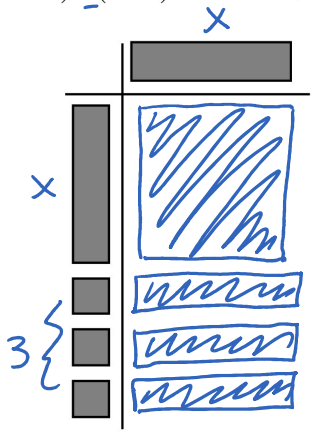
= x^2

Complete the diagram to determine the product.

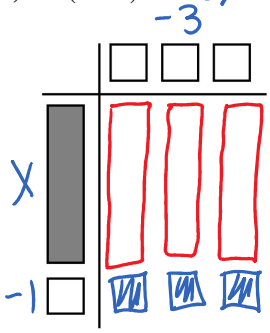
a) $2(x+1) =$ 2x+2



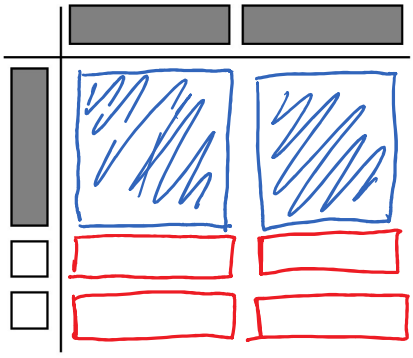
b) $x(x+3) =$ x^2+3x



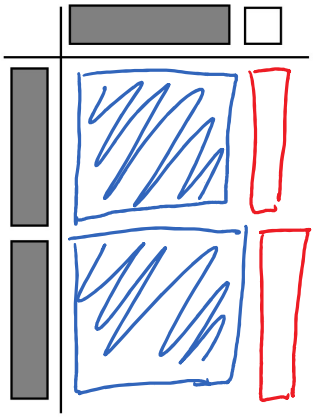
c) $-3(x-1) =$ -3x+3



d) $2x(x-2) =$ $2x^2-4x$



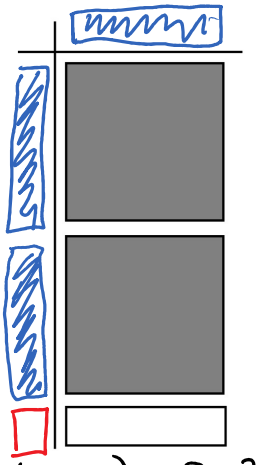
e) $(x-1)(2x) =$ $2x^2-2x$



Each diagram below illustrates the result of the product of a monomial and a binomial.

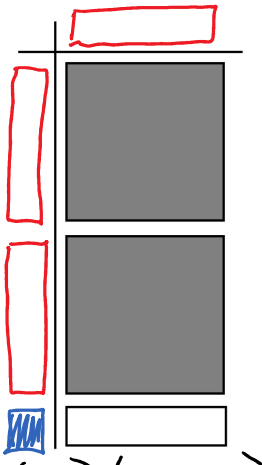
$2x^2 - x$

Diagram 1



$x(2x-1) = 2x^2 - x$

Diagram 2



$(-x)(-2x+1) = 2x^2 - x$

a) State the polynomial represented in each of the diagrams.

b) Complete the left side and the top of Diagram 1 and write the polynomial product.

c) Complete Diagram 2 to illustrate and write a different polynomial product than in b).

d) Write each product as a sum or difference of terms.

e) Verify the polynomial products in d) when $x = 3$.

Complete Assignment Questions #1 - #3

The Distributive Property

In Class Example #1 we have shown that:

$2(x+1) =$ $2x+2$, $x(x+3) =$ x^2+3x, $2x(x-2) =$ $2x^2-4x$,

$-3(x-1)$ $-3x+3$, and $(x-1)(2x) =$ $2x^2-2x$.

These above are examples of the **distributive property**

$$a(b+c) = ab + ac \qquad \text{or} \qquad (b+c)(a) = ba + ca \;\Rightarrow\; ab + ac.$$

The distributive property can be extended to any number of terms.

Using Numerical Values to Verify the Distributive Property

Consider the expression $-2(3-5)$.

i) Evaluate $-2(3-5)$ by calculating the value inside the brackets first and then multiplying by -2.

ii) Evaluate $-2(3-5)$ by using the distributive property.

iii) Comment on your results from i) and ii).

Use the distributive property to determine the following products.

a) $4(3x+1)$
$12x+4$

b) $-5(2x^2+x-6)$
$-10x^2-5x+30$

c) $(x^3-2)x^2$
x^5-2x^2

d) $-3x(7x-2y+z)$
$-21x^2+6xy-3xz$

In the example above we have written a product of polynomials as a sum or difference of terms.

In this process we **expanded** the polynomial expressions by using the distributive property, $a(b+c) = ab + ac$ and the exponent rule, $x^a \times x^b = x^{a+b}$.

Expand and simplify.

a) $6 - 4(8x + 1)$

$= 6 - 32x - 4$

$= -32x + 2$

b) $4(2x - 3) - 2(x - 6)$

$= 8x - 12 - 2x + 12$

$= 6x$

c) $5x(3x^2 - 7x + 1) - (4x + 3x^2)$

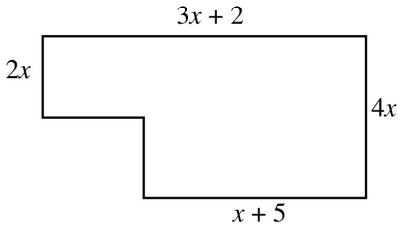
$= 15x^3 - 35x^2 + 5x - 4x - 3x^2$

$= 15x^3 - 38x^2 + x$

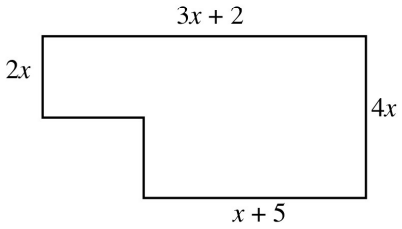
(1,2)de, 4acegi, 5acegi, 7acegi

Determine a simplified expression for the area of the given shape by

i) adding the areas of two rectangles.



ii) subtracting the areas of two rectangles.

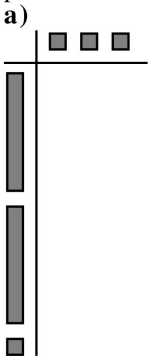


Complete Assignment Questions #4 - #11

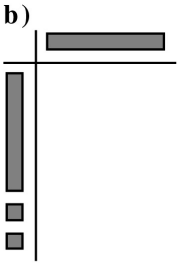
Assignment

1. In each case complete the diagram, state the polynomial product in x, and express the product as a sum or difference of terms.

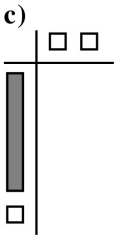
a)



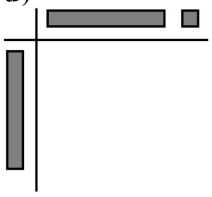
b)



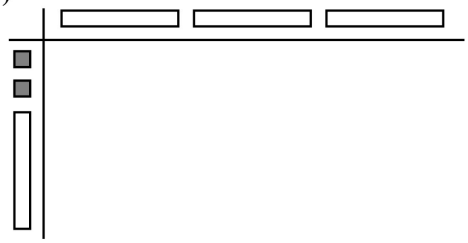
c)



d)

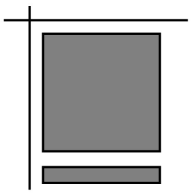


e)

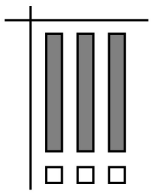


2. In each case state the polynomial product in x which is indicated by the algebra tile diagram. Express the product as a sum or difference of terms.

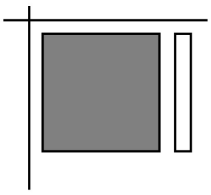
a)



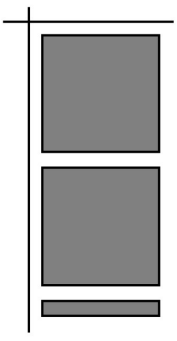
b)



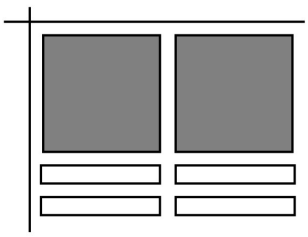
c)



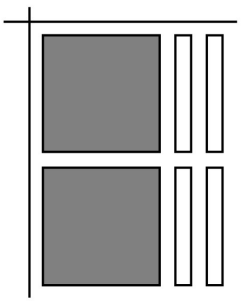
d)



e)



f)



3. For each of the following:

i) Draw an algebra tile diagram to model the product.
ii) Express the product as a sum or difference of terms.
iii) Verify the polynomial product when $x = 4$.

a) $2x(2x - 1)$ **b)** $-3x(2 - x)$

4. Expand.

a) $6(7x - 3)$ **b)** $-4(4x + 9)$ **c)** $4x(2y + 8z)$ **d)** $-x(x - 5y)$

e) $3(x - 2y + 3z)$ **f)** $-2a(b - c + 5d)$ **g)** $(x + 3)3x$

h) $2x(x - 5y + 4z)$ **i)** $x(x - 2x^2 + 3x^3)$ **j)** $(2x^2 + x - 6)(-4x)$

5. Expand and simplify.

a) $3(x + 5) - 7$

b) $8 - 2(5x + 11)$

c) $6(x - 2) + x$

d) $2(x + 3) + 4(2x - 1)$

e) $2(x + 3) - 4(2x - 1)$

f) $-2(x + 1) + 7(3x - 2)$

g) $5(-x + 12) + 5(x - 8)$

h) $(2 - x) - 2(2x - 10)$

i) $6(-x + 4) - (x - 15)$

6. Identify the errors in the following and provide the correct simplification.

a) $3x(2x + y) = 6x + 3xy$

b) $x^2(x^3 - 2x + 7) = x^6 - 2x^3 + 7x^2$

c) $4(x - 2) - 2(x - 3)$
$= 4x - 8 - 2x - 6$
$= 2x - 14$

d) $2(2t - 3) - 4(t + 5)$
$= 4t - 3 - 4t - 5$
$= -8$

e) $5(a + b) - (a + b)$
$= 5a + 5b - a + b$
$= 4a + 6b$

7. Expand and simplify.

a) $2a(a+3)-4a(2a-1)$ **b)** $4(x^2+3)-(2x^2-1)$ **c)** $2(x+3)-x-1$

d) $z(z^3+3)-(3z+7)$ **e)** $5(8x-3y)+2(4y+x)$

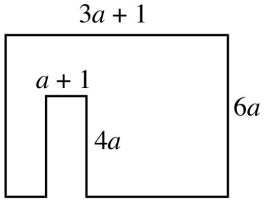
f) $-2x(x^4+3x^3)-7x(2x^4-x^3)$ **g)** $3a(2a^2b-ab+b^2)-6b(a^3+3ab-5b^2)$

h) $3x(x-3)-2x(x-1)+x(2x-2)$ **i)** $(p^2-3p)(4p)-(3+5p)(-2p^2)$

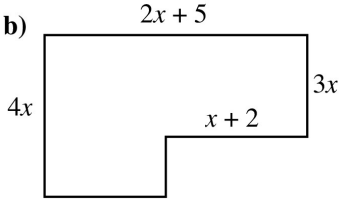
j) $a(b-c)+b(c-a)+c(a-b)$ **k)** $20x^3y^3-4x^3y^2(3x+5y-xy)$

8. Determine a simplified expression for the area of the given shape.

a)

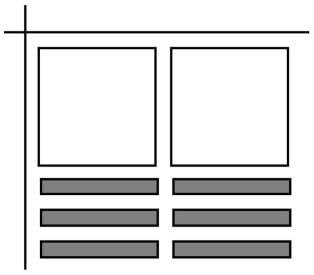


b)



Multiple Choice

9. The algebra tile diagram represents the expansion of:



A. $2x(x + 3)$
B. $-2x(x + 3)$
C. $2x(x - 3)$
D. $-2x(x - 3)$

10. Which of the following expansions is incorrect?

A. $-2x^2(3x + 2) = -6x^3 - 4x^2$
B. $-4x(2 - x) = -8x + 4x^2$
C. $-5x(x^2 - 3) = -5x^3 - 15x$
D. $7x^2(x^2 + 3) = 7x^4 + 21x^2$

Numerical Response

11. The expression $2x(4 - 3x) + 5x(2x - 1) - 3(4x + 2)$ can be written in the form $ax^2 + bx + c$. The value of $a + b - c$ is _____.

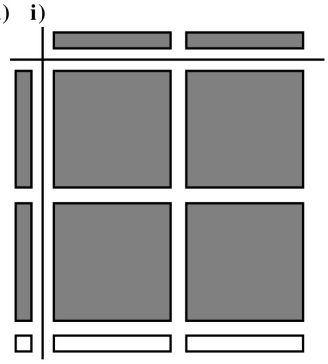
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $3(2x + 1) = 6x + 3$ **b)** $x(x + 2) = x^2 + 2x$ **c)** $-2(x - 1) = -2x + 2$
d) $(x + 1)(x) = x^2 + x$ **e)** $-3x(2 - x) = -6x + 3x^2$

2. **a)** $x(x + 1) = x^2 + x$ **b)** $3(x - 1) = 3x - 3$ **c)** $(x - 1)(x) = x^2 - x$
d) $x(2x + 1) = 2x^2 + x$ **e)** $2x(x - 2) = 2x^2 - 4x$ **f)** $(x - 2)(2x) = 2x^2 - 4x$

3. **a)** **i)**

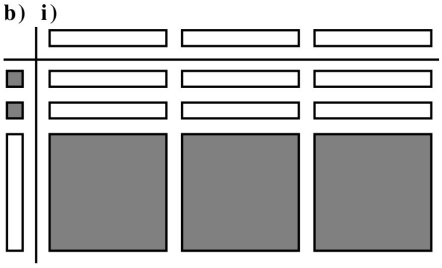


ii) $2x(2x - 1) = 4x^2 - 2x$

iii)

Left Side	Right Side
(2 x 4) ((2 x 4) – 1)	4 (4^2) – 2(4)
= (8) ((7)	= 64 – 8
= 56	= 56

b) **i)**



ii) $-3x(2 - x) = -6x + 3x^2$

iii)

Left Side	Right Side
(–3 x 4) (2 – 4)	(–6 x 4) + 3(4^2)
= (–12) ((–2)	= –24 + 48
= 24	= 24

4. **a)** $42x - 18$ **b)** $-16x - 36$ **c)** $8xy + 32xz$
d) $-x^2 + 5xy$ **e)** $3x - 6y + 9z$ **f)** $-2ab + 2ac - 10ad$
g) $3x^2 + 9x$ **h)** $2x^2 - 10xy + 8xz$ **i)** $x^2 - 2x^3 + 3x^4$ **j)** $-8x^3 - 4x^2 + 24x$

5. **a)** $3x + 8$ **b)** $-10x - 14$ **c)** $7x - 12$
d) $10x + 2$ **e)** $-6x + 10$ **f)** $19x - 16$
g) 20 **h)** $-5x + 22$ **i)** $-7x + 39$

6. **a)** $3x(2x) = 6x^2$, not $6x$. $3x(2x + y) = 6x^2 + 3xy$
b) $x^2(x^3) = x^5$ not x^6. $x^2(x^3 - 2x + 7) = x^5 - 2x^3 + 7x^2$
c) $-2(-3) = 6$, not -6. $4(x - 2) - 2(x - 3) = 4x - 8 - 2x + 6 = 2x - 2$
d) The monomials 2 and –4 multiply <u>both</u> terms in the binomials.
$2(2t - 3) - 4(t + 5) = 4t - 6 - 4t - 20 = -26$.
e) The negative multiplies both a and b. $5(a + b) - (a + b) = 5a + 5b - a - b = 4a + 4b$.

7. **a)** $-6a^2 + 10a$ **b)** $2x^2 + 13$ **c)** $x + 5$ **d)** $z^4 - 7$
e) $42x - 7y$ **f)** $-16x^5 + x^4$ **g)** $-3a^2b - 15ab^2 + 30b^3$ **h)** $3x^2 - 9x$
i) $14p^3 - 6p^2$ **j)** 0 **k)** $-12x^4y^2 + 4x^4y^3$

8. **a)** $14a^2 + 2a$ **b)** $7x^2 + 18x$ **9.** D **10.** C **11.**

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