Lesson 1: Review and Preview

Friday, August 31, 2018 2:32 AM

Polynomial Operations Lesson #1:
Review and Preview

Overview of Unit

In this unit we study algebraic expressions called polynomials. We review the classification of polynomials, addition and subtraction of polynomials, and multiplication by a monomial. We introduce the product of two binomials (concretely, pictorially, and symbolically) and extend this to the multiplication of polynomials. We also solve problems involving polynomial expressions.

Review

In algebra, a letter that represents one or more numbers is called a **variable**.

Expressions like $2a - b + 4$ or $\frac{5}{x} + 3$ are called **algebraic expressions**.

Certain algebraic expressions are called **polynomials** as explained below.

A **monomial** is a number or a variable or the product of numbers and variables. (Note that the exponent of any variable must be a positive integer in the numerator of the monomial.)

eg. 6, x, $6x$, $-\frac{1}{2}xy$, $0.25x^3$, abc, $2p^4q^2$ etc. are all monomials.

The number that multiplies the variable is called the **numerical coefficient**.

A **polynomial** is a monomial or a sum or difference of monomials.

- 6, x, $6 + x$, $2y + 7z$, $x^2 - 5x - 9$ etc. are all examples of polynomials.

Explain why $\frac{5}{x} + 3$ is not a polynomial.

$5x^{-1} + 3$ - negative exponent means not a polynomial

Classifying Polynomials

Polynomials may be classified in two different ways as shown below.

Ways to Classify Polynomials

- number of terms in the polynomial
- degree of the polynomial

Continued on the next two pages

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Classifying Polynomials by The Number of Terms

A polynomial may be classified by the number of **terms** it contains.

- A term can be a number, a variable, or the product of a number and variable(s).
- When there is more than one term, the terms are connected by + or – signs.

A polynomial with 1 term is a **monomial** (eg. $4x$).
A polynomial with 2 terms is a **binomial** (eg. $x+4$).
A polynomial with 3 terms is a **trinomial** (eg. x^2+x+4).
A polynomial with 4 or more terms is simply called a polynomial when classifying by the number of terms.

Consider the following algebraic expressions. In each case:

- State whether the expression represents a polynomial or not.
- If the expression does not represent a polynomial, explain why.
- If the expression does represent a polynomial, state whether the polynomial is a monomial, a binomial, or a trinomial.

a) $\frac{1}{4}xy-10$

- yes, polynomial
- binomial

b) $3pq^{\frac{1}{2}}$

- not a polynomial
- exponent is not a positive integer

c) $\sqrt{7}x^4-x^3+1$

- yes a polynomial
- trinomial

d) $3x^2+9x-4x^{0.2}$

- not a polynomail
- exponent is not a positive integer

e) $\frac{7}{a}=7a^{-1}$

- not a polynomial
- exponent is not a positive integer

Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by # Terms
$4x+3yz$	x, y, z	2	binomial
$2a-4b+7c$	a, b, c	3	trinomial
x^2+3x+4	x	3	trinomial
$\sqrt{2}x$	x	1	monomial
$2x^3+3x^2y+3y^2-8$	x, y	4	polynomial

Classifying Polynomials by The Degree of The Polynomial

Polynomials can also be classified according to **degree** of the polynomial .

The **degree of a monomial** is the sum of the exponents of its variable(s).

eg. $2x^5$ has degree 5 $-\frac{2}{3}ab^3c^2$ has degree 6 (1+3+2)

Recall from the lesson "Whole Number Exponents" on page 20 that a variable raised to the power zero is equal to 1.

For example, the monomial $7x^0$ can be written as $7(1)$ or 7.
Therefore the degree of a monomial with no variable present is 0.

The **degree of a polynomial** is given by the **term** or **monomial** with highest degree.

eg. $3x^2y^2 - 2x^4 + xy^4 - 2$ has degree 5 (4, 4, 5)

If a polynomial has a term with no variable present, this term is called a **constant term**.

In the polynomial $3x^2y^2 - 2x^4 + xy^4 - 2$, the constant term is -2.

State the degree of the following polynomials.

a) $3x^2 - 10x^4 - 9$ (2, 4, 0) degree 4

b) $7p^2q^3 - 8p^7q - 2q^7$ (5, 8, 7) degree 8

Give an example of

a) a binomial of degree 1 in one variable. $2x+8$

b) a trinomial in two variables with a constant term. $6x+4y+10$

c) a monomial of degree 6 with a (numerical) coefficient of 9. $9x^6$

d) a binomial of degree 8 with each term containing two variables. $2x^6y^2+3ab^4$

The following list classifies polynomials by using the degree of the polynomial.

- A **Constant** Polynomial has a degree of 0 eg. 8
- A **Linear** Polynomial has a degree of 1 eg. $x + 3$
- A **Quadratic** Polynomial has a degree of 2 eg. $x^2 - 2x + 5$
- A **Cubic** Polynomial has a degree of 3 eg. $x^3 - 8x^2 + x + 1$
- A **Quartic** Polynomial has a degree of 4 eg. $x^4 - 61x + 9$
- A **Quintic** Polynomial has a degree of 5 eg. $x^5 - 17$

There are names for polynomials of higher degree that are beyond the scope of this course.

Complete the following table.

Polynomial Expression	Degree	Classification by Degree	Constant Term
$4xy - 6$	2	Quadratic Poly	-6
$9y^2 - 8y^3$	3	Cubic poly	0

Polynomials in a Single Variable

Polynomials in a single variable are usually arranged in **ascending** or **descending** order of the powers of the variable.

The **leading coefficient** of a polynomial in a single variable is the coefficient of the term with highest power of the variable.

Class Ex. #7

Consider the polynomial expression $2x - 4x^3 - 7 + \frac{6x^2}{5}$.

a) Write the polynomial in descending powers of x. $-4x^3 + \frac{6x^2}{5} + 2x - 7$

b) Write the polynomial in ascending powers of x. $-7 + 2x + \frac{6x^2}{5} - 4x^3$

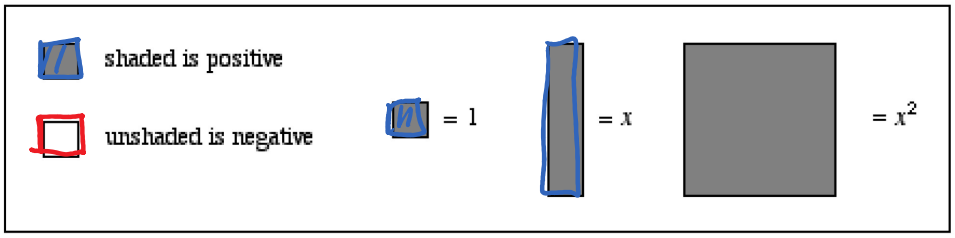
c) State the leading coefficient and the constant term. -4 or -7, constant -7

d) State the numerical coefficient of the term in x^2. $\frac{6}{5}$

Complete Assignment Questions #1 - #9

Representing Polynomials Using Algebra Tiles

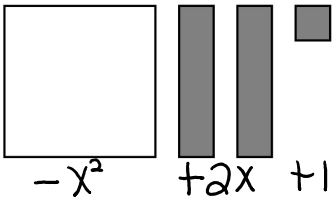
The following legend will be used for algebra tiles in this workbook.



Class Ex. #8

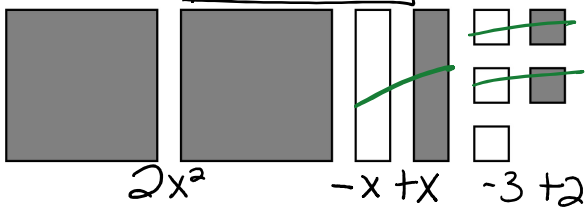
State the polynomial expression which describes each diagram.

a)



$-x^2 + 2x + 1$

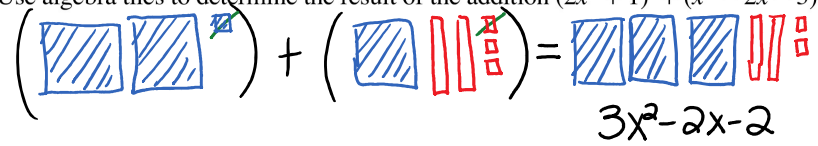
b) $2x^2 - 1$



$2x^2 - x + x - 3 + 2$

Addition and Subtraction Using Algebra Tiles

Use algebra tiles to determine the result of the addition $(2x^2 + 1) + (x^2 - 2x - 3)$.



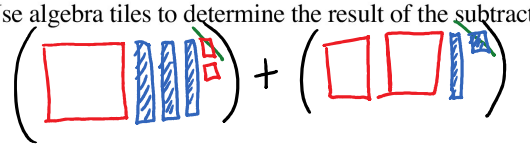
$3x^2-2x-2$

Subtracting a polynomial is equivalent to adding the inverse polynomial,

eg. $(4x + 3) - (2x - 5)$ is equivalent to $(4x + 3) + (-2x + 5)$

Use algebra tiles to determine the result of the subtraction $(-x^2 + 3x - 2) - (2x^2 - x - 1)$.

$+(-2x^2+x+1)$



$=-3x^2+4x-1$

#1, 2, 3, 5, 7ac, 9, 11a, 12a

Addition and Subtraction of Polynomial Expressions

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Like terms are terms with the same variable raised to the same exponent.

eg. $3a$, $7a$ and a are like terms. $2x^3$, $\frac{1}{5}x^3$ and $-4x^3$ are like terms.

Unlike terms have different variables or the same variable raised to different exponents.

eg. $2x^3$, $\frac{1}{5}x^2$ and $-4x$ are unlike terms. $4x$ and $4y$ are unlike terms.

Like terms can be added or subtracted to produce a single term.

Simplify the following polynomials by collecting like terms.

a) $(3a - 4b + c) + (3b - 5c - 3a)$

$3a-4b+c+3b-5c-3a$

$0-b-4c$

$=-b-4c$

b) $(4x^2 - 9x + 6) - (2x^2 - 3x - 1)$

$2x^2-6x+7$

Simplify

a) $4x - 2x^2 + 3 - 6x^2 + 5 - x$

$= -8x^2 + 3x + 8$

b) $a^2b - ab^2 + 4a^3b - 7ab^2 + 5a^2b$

$= 6a^2b - 8ab^2 + 4a^3b$

$= 4a^3b + 6a^2b - 8ab^2$

Complete Assignment Questions #10 - #20

Assignment

1. Identify as a monomial, a binomial, or a trinomial.

a) $x + 1$ **b)** $3x^3$ **c)** $2x^2 + 2x + 2$

2. State the degree of each monomial.

a) 5a **b)** $3x^3y$ **c)** 10 **d)** $-2a^2b^2$ **e)** $3xy^2z^3$

3. State whether or not the following are polynomial expressions. If they are not polynomial expressions, explain why not.

a) $\frac{1}{2}x^2 - 3x$ **b)** $8m^{-2}$ **c)** $\sqrt{6}$

d) $\frac{7}{x^3}$ **e)** $\frac{8x^2}{3}$ **f)** $x^4 + 3x^{1.5}$

4. Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by Number of Terms	Degree
$2y^3 + y^4 - y + 13$				
$9ab - 4x + 11c$				
25				
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$				

5. Complete the following table for the single variable polynomials.

Polynomial Expression	Leading Coefficient	Constant Term	Degree	Classification by Degree
$y^4 - y + 13$				
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$				
$\sqrt{7} - x^5$				
$\pi x^2 - 7 - 3x$				
$-\frac{1}{10}$				
$9x + 12$				

6. Give an example of

a) a trinomial of degree 2 in one variable.

b) a binomial in four variables with a constant term of 6.

c) a monomial of degree 3 in two variables with a negative numerical coefficient.

d) a monomial with a degree of 0.

7. Arrange the following in descending powers of the variable.

a) $6w^2 - 9w + 5 + 2w^3$

b) $\frac{1}{4}a^2 - \frac{2}{3}a^3 - 1 - a$

c) $z - 3 - 4z^6 + z^3$

8. Arrange the following in ascending powers of the variable.

a) $6w^2 - 9w + 5 - 2w^3$

b) $3x^2 - 4x^5 - 2x^4 - 4x^3 + 9x - 7$

c) $8x^3 - 8x + 8$

9. State which of the following are true and which are false.

a) -54 is a polynomial.

b) The degree of the polynomial $3x^3y^3$ is 9.

c) The numerical coefficient of $\frac{6x}{5}$ is 6.

d) A polynomial may have 1000 terms.

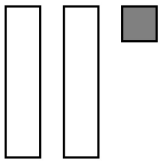
e) $\frac{2}{a^3} - 1$ is a binomial.

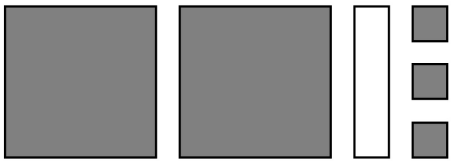
f) The degree of the polynomial 0 is 0.

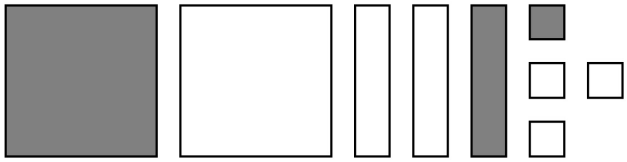
g) The polynomial $x^3 + 2x^2 + 3x + 4$ is written in ascending powers of x.

h) The polynomials $3x^2 - 9x + 1$ and $1 - 9x + 3x^2$ are equivalent.

10. State the polynomial expression which describes each diagram.

a) 

b) 

c) 

11. Use algebra tiles to determine the result of the addition of :

a) $(x^2 - x - 3) + (x^2 - 2x - 3)$ **b)** $(3x + 1) + (2x^2 - 3x - 2)$

12. Use algebra tiles to determine the result of the subtraction of:

a) $(x^2 - 3) - (2x^2 + 4x + 1)$ **b)** $(2 - x - x^2) - (1 - 2x + x^2)$

13. Simplify

a) $6p - 7q - 3q - 2p$ **b)** $5x - 3x^2 + 2x - 8x^2$ **c)** $\frac{1}{2}x - 3 + \frac{3}{2}x + 18$

d) $4a^3 + 7a - 2a^2 - 6a - 4a^3 - a^2$ **e)** $3 - 2x + 7y + 4y - 2x + 8z - 9$

14. Simplify the following polynomial expressions by collecting like terms.

a) $(5a - 9b - 2c) + (c - 7b - 3a)$ **b)** $(3 - a - 2a^2) + (9 - 4a + 5a^2)$

c) $(2x^2 + 5x - 1) + (3x - 6 - 6x^2) + (4 - 5x + x^2)$ **d)** $(4a - 6b) - (5a - 2b)$

e)
$$\begin{array}{rl} & (5x^2 - 8x + 3) \\ - & \underline{(3x^2 - 3x - 1)} \end{array}$$

f)
$$\begin{array}{rl} & (7x^2 + 2x - 1) \\ - & \underline{(-5x^2 - 3x - 1)} \end{array}$$

g)
$$\begin{array}{rl} & (-4x^2 + 2x - 6) \\ - & \underline{(3x + 6 - 2x^2)} \end{array}$$

15. **a)** Subtract $3x^2 - 2x + 7$ from $6x^2 - 5x - 2$.

b) Subtract the sum of $2x^3 - 7x^2 - 6x + 1$ and $8 - 3x + 5x^2 - 4x^3$ from $2x^3 - 7x + 9$.

16. A triangle has a perimeter of $(6m + n)$ cm. One side measures $(2m - 3n)$ cm and another side measures $(3n + 2m)$ cm.

a) Write and simplify an expression for the length of the third side of the triangle.

b) Determine the measure of each side when $m = 4$ and $n = -1$.

Multiple Choice

17. Which of the following is a polynomial expression of degree 4?

A. $4x^4 - 4x^7$

B. $5x^4 - 3x^3 + 2x^{-2} + x - 1$

C. $\dfrac{4x^4 - 3x}{x}$

D. $9 + 3x - \frac{1}{3}x^2 - x^3 + \frac{2}{5}x^4$

18. Which of the following polynomial expressions, when simplified, is equal to $5x$?

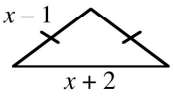
A. $(3x^2 - 3x) - (2x + 3x^2)$

B. $5x - (2x^2 - 2x) + (2x^2 + 2x)$

C. $8 + (4 - 2x) - (12 - 7x)$

D. $(2x^2 - 2x + 6) - (2x^2 - 2x) + (9x - 6)$

19. The perimeter of the isosceles triangle shown can be represented by



A. a monomial

B. a binomial

C. a trinomial

D. none of the above

Numerical Response **20.** If the polynomial $4 - 7x + 2x^2 - 5x^3$ has degree a, leading coefficient b, and constant term c, then the value of $3a - 2b - c$ is _____.

(Record your answer in the numerical response box from left to right) | | | | |

Answer Key

1. **a)** binomial **b)** monomial **c)** trinomial **2.** **a)** 1 **b)** 4 **c)** 0 **d)** 4 **e)** 6

3. **a)** yes **b)** no, negative exponent **c)** yes

d) no, $\frac{7}{x^3} = 7x^{-3}$, which is a negative exponent. **e)** yes

f) no, the exponent 1.5 is not a positive integer.

4.

Polynomial expression	*# variables*	*# terms*	*Classification by Number of Terms*	*degree*
$2y^3 + y^4 - y + 13$	1	4	polynomial	4
$9ab - 4x + 11c$	4	3	trinomial	2
25	0	1	monomial	0
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$	3	2	binomial	9

5.

Polynomial expression	*leading coefficient*	*constant term*	*degree*	*Classification by Degree*
$y^4 - y + 13$	1	13	4	Quartic
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$	0.2	-0.5	3	Cubic
$\sqrt{7} - x^5$	-1	$\sqrt{7}$	5	Quintic
$\pi x^2 - 7 - 3x$	π	-7	2	Quadratic
$-\frac{1}{10}$	$-\frac{1}{10}$	0	0	Constant
$9x + 12$	9	12	1	Linear

6. answers may vary **a)** $x^2 - x + 30$ **b)** $abcd + 6$ **c)** $-2xy^2$ **d)** 10

7. **a)** $2w^3 + 6w^2 - 9w + 5$ **b)** $-\frac{2}{3}a^3 + \frac{1}{4}a^2 - a - 1$ **c)** $-4z^6 + z^3 + z - 3$

8. **a)** $5 - 9w + 6w^2 - 2w^3$ **b)** $-7 + 9x + 3x^2 - 4x^3 - 2x^4 - 4x^5$ **c)** $8 - 8x + 8x^3$

9. **a)** true **b)** false **c)** false **d)** true **e)** false **f)** true **g)** false **h)** true

10. **a)** $-2x + 1$ **b)** $2x^2 - x + 3$ **c)** $-x - 2$ **11.** **a)** $2x^2 - 3x - 6$ **b)** $2x^2 - 1$

12. **a)** $-x^2 - 4x - 4$ **b)** $1 + x - 2x^2$

13. **a)** $4p - 10q$ **b)** $-11x^2 + 7x$ **c)** $2x + 15$ **d)** $-3a^2 + a$ **e)** $-4x + 11y + 8z - 6$

14. **a)** $2a - 16b - c$ **b)** $3a^2 - 5a + 12$ **c)** $-3x^2 + 3x - 3$ **d)** $-a - 4b$

e) $2x^2 - 5x + 4$ **f)** $12x^2 + 5x$ **g)** $-2x^2 - x - 12$

15. **a)** $3x^2 - 3x - 9$ **b)** $4x^3 + 2x^2 + 2x$

16. **a)** $(2m + n)$ cm **b)** 11 cm, 5cm, and 7 cm

17. D **18.** C **19.** A **20.**

1	5		