Lesson 5: Integral Exponents

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Prime Factorization and Exponents Lesson #5:
Integral Exponents

The Negative Exponent

a) Complete the patterns below.

$10^3 = 1000$ $\quad 3^3 = 27$ $\quad a^0 = 1$

$10^2 = 100$ $\quad 3^2 = 9$

$10^1 = 10$ $\quad 3^1 = 3$

$10^0 = 1$ $\quad 3^0 = 1$

$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$ $\quad 3^{-1} = \frac{1}{3} = \frac{1}{3^1}$ $\quad a^{-1} = \frac{1}{a^1}$

$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$ $\quad 3^{-2} = \frac{1}{9} = \frac{1}{3^2}$ $\quad a^{-2} = \frac{1}{a^2}$

$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$ $\quad 3^{-3} = \frac{1}{27} = \frac{1}{3^3}$ $\quad a^{-3} = \frac{1}{a^3}$

b) Write the following with positive exponents.

i) $10^{-7} = \frac{1}{10^7}$ **ii)** 3^{-5} $\frac{1}{3^5}$ **iii)** base, exponent: $a^{-n} = \frac{1}{a^n}$

Using the Exponent Laws to Define the Negative Exponent

Consider the expression $5^4 \div 5^7$.

a) Evaluate the expression as an exact value using a calculator.

b) Complete the following to evaluate the expression.

$$5^4 \div 5^7 = \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{5^{\boxed{3}}} = \frac{1}{125}$$

c) Use the quotient law to complete the following.

$5^4 \div 5^7 = 5^{\boxed{4} - \boxed{7}} = 5^{\boxed{-3}}$

d) The results in a) to c) are examples of a general rule when a base is raised to a negative exponent. Complete: $a^{-p} = \frac{1}{a^p}$

e) Write the following with positive exponents and evaluate.

i) $2^{-1} = \frac{1}{2^1} = \boxed{\frac{1}{2}}$ **ii)** $3^{-2} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$ **iii)** 4^{-3} $\frac{1}{4^3} = \frac{1}{64}$

The Negative Exponent in the Denominator

Use the rule for division of fractions to show that $\frac{1}{4^{-3}} = 4^3$. Use a calculator to confirm.

$\frac{1}{4^{-3}} = 64$

Negative Exponent Law

A base (not including zero) raised to a negative exponent has the following properties:

$$a^{-n} = \frac{1}{a^n}, \quad a \neq 0 \quad \text{and} \quad \frac{1}{a^{-n}} = a^n, \quad a \neq 0$$

Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^5 \times 4^{-3}$ $4^{5+(-3)} = 4^2 = 16$

b) $3^2 \times 3^{-5}$ $3^{2+(-5)} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

c) $\frac{1}{2^{-5}}$ $= 2^5 = 32$

d) $\frac{6^{-7}}{6^{-5}}$ a) $6^{-7-(-5)} = 6^{-7+5} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ b) $\frac{6^5}{6^7} = 6^{5-7} = 6^{-2}$

e) $(2^3)^{-1}$ a) $\frac{1}{2^3} = \frac{1}{8}$ b) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Identify the following as true or false.

a) $\frac{8^3}{8^{-1}} = 8^4$ TRUE

$= 8^{3-(-1)}$

$= 8^{3+1} = 8^4$

b) $\frac{8^3}{4^{-1}} = 2^4$ False

- need the same bases for exponent laws

c) $a^{-3} = \frac{1}{a^3}$ True

Explain why $2p^{-3} \neq \frac{1}{2p^3}$.

coefficient ↑ base so only the p is effected by the exponent

$2p^{-3} = \frac{2}{p^3}$

a) $a^{-4} \times a^{-3}$

$= a^{-4+(-3)}$

$= a^{-7}$

$= \boxed{\frac{1}{a^7}}$

b) $6x^2 \div 2x^7$ $= \frac{6 \cdot x \cdot x}{2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$

$= 3x^{2-7}$

$= 3x^{-5} = \boxed{\frac{3}{x^5}}$ $\frac{3}{x^5}$

c) $\frac{1y^6}{2y^{-5}}$

$= \frac{1}{2} y^{6-(-5)}$

$= \frac{1}{2} y^{11} = \boxed{\frac{y^{11}}{2}}$

d) $(-2x)^{-3}$

$= \frac{1}{(-2x)^3} = \boxed{\frac{1}{-8x^3}}$

$= \frac{1}{(-2x)^3} = \frac{1}{(-2x)(-2x)(-2x)}$

$= \frac{1}{(-2)(-2)(-2)(x)(x)(x)}$

$= \frac{1}{-8x^3}$

e) $\frac{8a^{-5}}{4b^{-3}} = \boxed{\frac{2b^3}{a^5}}$

f) $\frac{(5p)^{-2}}{5q^4} = \frac{1}{5q^4(5p)^2}$

$= \frac{1}{5q^4 \cdot 5^2 \cdot p^2}$

$= \frac{1}{5q^4 \cdot 25 \cdot p^2}$

$= \boxed{\frac{1}{125q^4p^2}}$

#4-6, 9-11, 12bd, 13bdf

Simplifying a Fractional Base with a Negative Exponent

Consider the expression $\left(\frac{2}{3}\right)^{-4}$.

a) Complete the following $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^{\square}} = \frac{1}{\quad} = 1 \times \quad =$

b) Evaluate $\left(\frac{3}{2}\right)^4$.

c) Classify the following statement as true or false. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$

d) Suggest a quick method for evaluating $\left(\frac{5}{2}\right)^{-3}$ without using a calculator.

In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad a, b \neq 0.$

Complete Assignment Questions #1 - #15

Assignment

1. Write the following with positive exponents.

a) x^{-3} **b)** y^{-9} **c)** 4^{-1} **d)** $\dfrac{1}{a^{-5}}$ **e)** $\dfrac{1}{6^{-2}}$

2. Without using a calculator show that $\dfrac{3}{5^{-2}} = 75$.

3. Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^3 \times 4^{-4}$ **b)** $3^0 \times 3^{-3}$ **c)** $\dfrac{1}{7^{-2}}$ **d)** $\dfrac{10^{-3}}{10}$ **e)** $(3^2)^{-2}$

4. Express with positive exponents.

a) n^2m^{-5} **b)** $c^{-2}x^{-5}$ **c)** $16h^{-1}$ **d)** $\dfrac{2}{3}b^{-8}$ **e)** $(y^{-4})^{-2}$

f) $\dfrac{t^{-5}}{4}$ **g)** $\dfrac{1}{4x^{-9}}$ **h)** $\dfrac{4}{x^{-9}}$ **i)** $\dfrac{a^2}{b^{-7}}$ **j)** $\dfrac{a^{-2}}{b^7}$

5. Evaluate the following without using a calculator.

a) -3^{-2} **b)** $(-3)^{-2}$ **c)** $-7^2 \cdot 8^{-2}$ **d)** $(-8.3)^0$ **e)** $[-(3.9)^0]^{-2}$

6. Use a calculator to find the exact value of the following.

a) -4^{-4} **b)** $(-7)^{-3}$ **c)** $(0.75)^{-3}$ **d)** $(-0.025)^{-2}$ **e)** $\left(\dfrac{4}{7}\right)^{-3}$

7. State whether the following are true or false.

a) $6x^{-3} = \dfrac{6}{x^3}$ **b)** $5a^{-4} = \dfrac{1}{5a^4}$ **c)** $\dfrac{4}{b^{-6}} = 4b^6$ **d)** $\dfrac{x^{-3}}{2} = \dfrac{2}{x^3}$

e) $\dfrac{1}{5y^{-1}} = 5y$ **f)** $\dfrac{1}{\frac{1}{4}p} = \dfrac{1}{4}p^{-1}$ **g)** $(3x)^5 = \dfrac{1}{(3x)^{-5}}$ **h)** $\dfrac{1}{\left(\frac{1}{7}a\right)^{-2}} = 49a^2$

8. Simplify and write the answer with positive exponents.

a) $x^{10} \cdot x^{-5}$ **b)** $m^5 \div m^8$ **c)** $b^{-1} \cdot b^{-3}$ **d)** $-w^0 \div w^5$

9. Simplify and write the answer with positive exponents.

a) $a^8 \times a^{-10}$ **b)** $10x^2 \div 2x^{-1}$ **c)** $\dfrac{6y^{-6}}{2y^{-4}}$ **d)** $\dfrac{2a^{-5}}{4b^6}$

e) $-7x^{-2}$ **f)** $-(7x)^{-2}$ **g)** $(-7x)^{-2}$ **h)** $\dfrac{(-7x)^{-2}}{-7x^{-2}}$

10. Simplify each expression, writing the answer with positive exponents.

a) $a^{-3}a^{-3}$ **b)** $(5b^8b^{-12})(-10b^3b^{-12})$ **c)** $(-7x^3x^{-5})(x^2x^{-3})$

d) $(-2a^3)^{-3} \cdot 3a^{12}$ **e)** $\dfrac{16a^6b^{-3}}{-4a^6b^3}$ **f)** $(-3a^5b^{-3}c^0)^{-2}$

11. Simplify. Write the final answer with positive exponents.

a) $\dfrac{32a^2b^{-4}}{4a^{-8}b^{-2}} \times \dfrac{-8a^{-2}}{-3b^{-3}}$

b) $\dfrac{10(p^3q^2r^0)^{-3}}{(8p^{-3}q^5r^3)^{-2}}$

c) $(-2x^5y^3z^8)^{-2}(-2x^2y^{-8}z^{12})^3$

d) $(5a^3b^2)(-2a^{-2}b)^{-3} \div (-5a^8b^{-9})^{-2}$

12. Evaluate the following without using a calculator.

a) $\left(\dfrac{2}{3}\right)^{-3}$

b) $\left(\dfrac{1}{5}\right)^{-2}$

c) $\left(\dfrac{8}{5}\right)^{-1}$

d) $\left(\dfrac{3}{2}\right)^{-4}$

13. Simplify. Write the final answers with positive exponents.

a) $\left(\dfrac{c}{d}\right)^{-3}$

b) $\left(\dfrac{x}{4}\right)^{-3}$

c) $\left(\dfrac{p^2}{r^4}\right)^{-3}$

d) $\left(\dfrac{a^{-2}}{b^{-5}}\right)^{-3}$

e) $\left(\dfrac{-12x^{-3}}{6y^{-8}}\right)^{-1}$

f) $\left(\dfrac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

Multiple Choice **14.** The value of $\dfrac{1^{-3}+3^{0}}{2^{-1}}$ is

A. 1
B. 4
C. 8
D. 12

15. Which of the following statements are true?

i) $3a^{-3}=\dfrac{1}{3a^{3}}$ ii) $8x^{4}\div 4x^{7}=\dfrac{1}{2x^{3}}$ iii) $\dfrac{1}{2a}=2a^{-1}$

A. i) only
B. ii) only
C. iii) only
D. none of the statements are true

Answer Key

1. a) $\dfrac{1}{x^{3}}$ **b)** $\dfrac{1}{y^{9}}$ **c)** $\dfrac{1}{4}$ **d)** a^{5} **e)** 6^{2}

2. $\dfrac{3}{5^{-2}}=3\times 5^{2}=3\times 25=75$

3. a) $\dfrac{1}{4^{1}}=\dfrac{1}{4}$ **b)** $\dfrac{1}{3^{3}}=\dfrac{1}{27}$ **c)** $7^{2}=49$ **d)** $\dfrac{1}{10^{4}}=\dfrac{1}{10\,000}$ **e)** $\dfrac{1}{3^{4}}=\dfrac{1}{81}$

4. a) $\dfrac{n^{2}}{m^{5}}$ **b)** $\dfrac{1}{c^{2}x^{5}}$ **c)** $\dfrac{16}{h}$ **d)** $\dfrac{2}{3b^{8}}$ **e)** y^{8}

f) $\dfrac{1}{4t^{5}}$ **g)** $\dfrac{x^{9}}{4}$ **h)** $4x^{9}$ **i)** $a^{2}b^{7}$ **j)** $\dfrac{1}{a^{2}b^{7}}$

5. a) $-\dfrac{1}{9}$ **b)** $\dfrac{1}{9}$ **c)** $-\dfrac{49}{64}$ **d)** 1 **e)** 1

6. a) $-\dfrac{1}{256}$ **b)** $-\dfrac{1}{343}$ **c)** $\dfrac{64}{27}$ **d)** 1600 **e)** $\dfrac{343}{64}$

7. a) T **b)** F **c)** T **d)** F **e)** F **f)** F **g)** T **h)** F

8. a) x^{5} **b)** $\dfrac{1}{m^{3}}$ **c)** $\dfrac{1}{b^{4}}$ **d)** $-\dfrac{1}{w^{5}}$

9. a) $\frac{1}{a^2}$ **b)** $5x^3$ **c)** $\frac{3}{y^2}$ **d)** $\frac{1}{2a^5b^6}$

e) $-\frac{7}{x^2}$ **f)** $-\frac{1}{49x^2}$ **g)** $\frac{1}{49x^2}$ **h)** $-\frac{1}{343}$

10. a) $\frac{1}{a^6}$ **b)** $-\frac{50}{b^{13}}$ **c)** $-\frac{7}{x^3}$ **d)** $-\frac{3}{8}a^{3}$ **e)** $-\frac{4}{b^6}$ **f)** $\frac{b^6}{9a^{10}}$

11. a) $\frac{64}{3}a^8b$ **b)** $\frac{640q^4r^6}{p^{15}}$ **c)** $-\frac{2z^{20}}{x^4y^{30}}$ **d)** $-\frac{125a^{25}}{8b^{19}}$

12. a) $\frac{27}{8}$ **b)** 25 **c)** $\frac{5}{8}$ **d)** $\frac{16}{81}$

13. a) $\frac{d^3}{c^3}$ **b)** $\frac{64}{x^3}$ **c)** $\frac{r^{12}}{p^6}$ **d)** $\frac{a^6}{b^{15}}$ **e)** $-\frac{x^3}{2y^8}$ **f)** $\frac{4y^{12}}{9x^8}$

14. B **15.** D