Name:

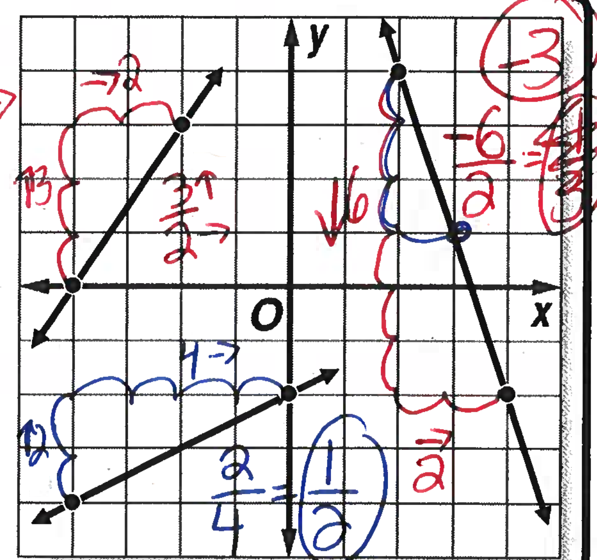
Slope (vertical change over horizontal change) is represented by the letter "m."

$$m = \frac{\text{"rise"}}{\text{"run"}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope represents the **rate of change**.
Slope should be written as a fraction in simplest form.

Find the slope of each line below.



SLOPE

The slope of a line can be determined from a table, by counting units on a coordinate plane, or by subtracting coordinates.

Find the slope between the two points.

1. (3, -2) and (4, 4)
   $\frac{4 - -2}{4 - 3} = \frac{6}{1}$ **6**
2. (6, 0) and (-8, -1)
   $\frac{-1 - 0}{-8 - 6}$ **$\frac{1}{14}$**

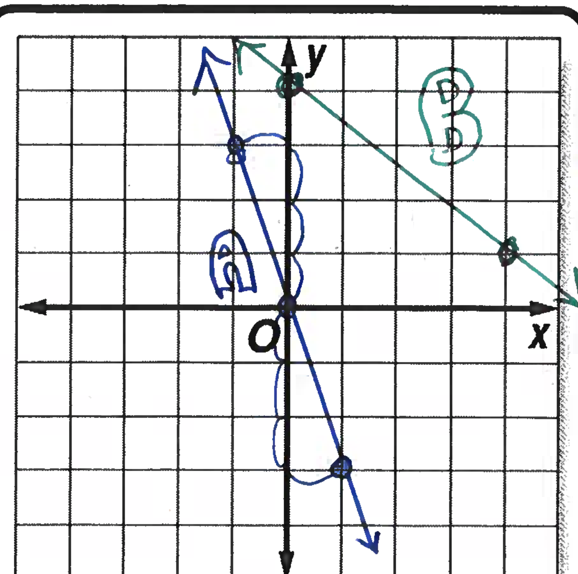
The slope of a horizontal line is 0.

The slope of a vertical line is undefined.

Remember:
UP and RIGHT are positive movements;
DOWN and LEFT are negative movements.

Plot a line that starts at the origin and has a slope of -3. Label it "a."

Plot a line that starts at (0, 4) and has a slope of $-\frac{3}{4}$. Label it "b."

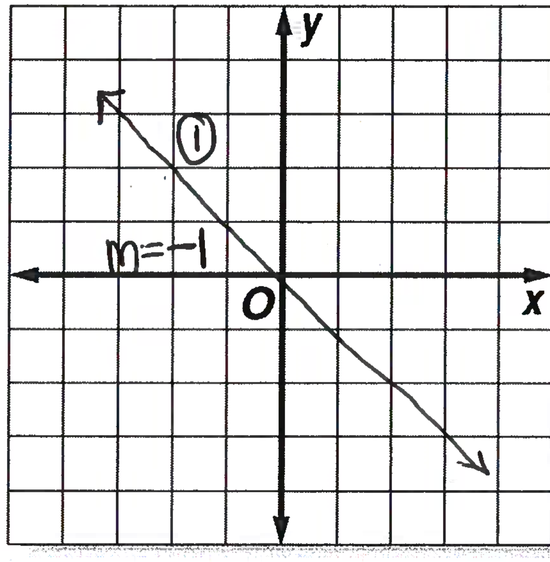


Steeper slopes have greater ________.

STEEP

Name:

try-it

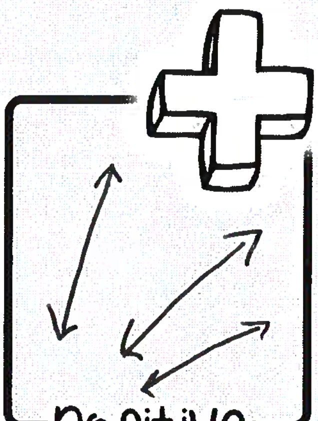


Graph four different lines, all with different negative slopes. Show each slope and compare steepness.

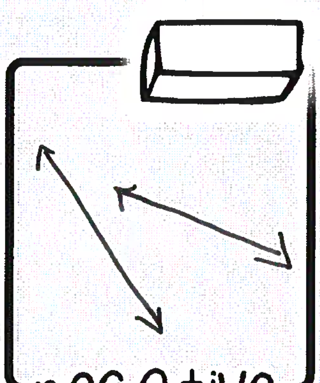
Slopes will be represented with fractions with a greater | absolute value |

Sketch a sample (or a few) of each type of slope. Add a skier if you want! It may help you remember the direction and whether the values are increasing or decreasing.

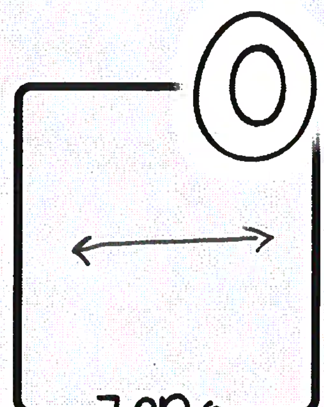
sketch it



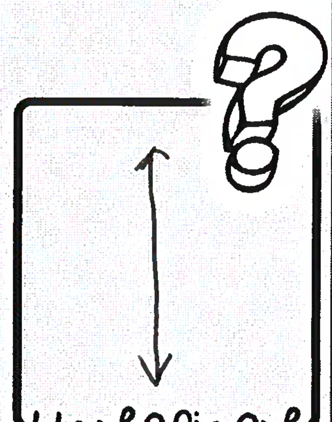
positive slope



negative slope



zero slope



undefined slope

order from steepest to least steep: 1/3, 3, 3/2, 3/4

$3, \frac{3}{2}, \frac{3}{4}, \frac{1}{3}$

Characteristics of Linear Relations Lesson #4: The Slope Formula

Review

Complete the following statements.

a) Slope is the measure of the steepness of a line.

b) Slope is the ratio of the vertical change (called the rise) over the horizontal change (called the run).

c) A line segment which rises from left to right has a positive slope.

d) A line segment which falls from left to right has a negative slope.

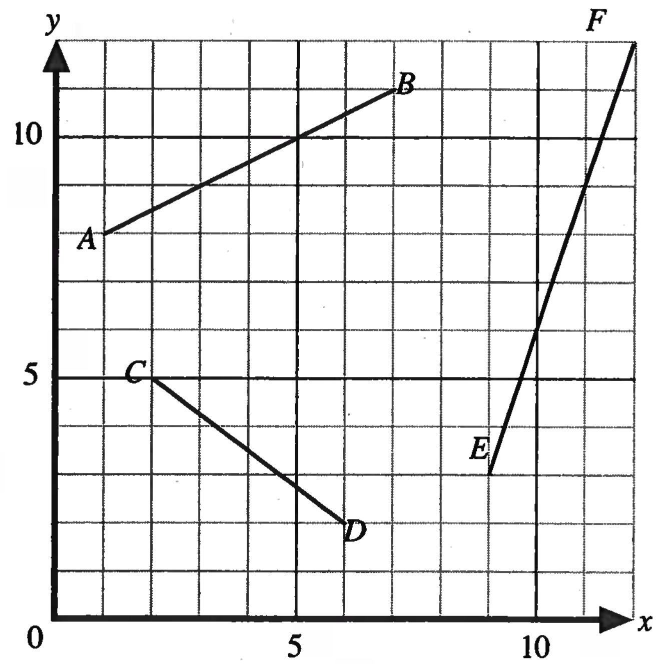
e) A horizontal line segment has a slope of 0.

f) A vertical line segment has an undefined slope.

g) The slopes of all line segments on a line are equal.

Developing the Slope Formula

a) Calculate the slope of line segment AB using slope $= \frac{\text{rise}}{\text{run}}$.



b) List the coordinates of the endpoints of line segment AB.

$A($, $)$ $B($, $)$

c) How can the rise of line segment AB be determined using y_B and y_A?

d) How can the run of line segment AB be determined using x_A and x_B?

e) Use your results from **c)** and **d)** to write a formula which describes how the slope of line segment AB can be calculated using its endpoints.

f) Calculate the slope of line segment AB using the formula in **e)**.

g) Calculate the slope of the line segments CD and EF using the method in **a)** and verify using the formula from **e)**.

The Slope Formula

In mathematics the letter "m" is used to represent slope. If the graph of a linear relation passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope of this line can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$$

Class Ex. #1

Find the slope of a line which passes through the points $G(-3, 8)$ and $H(7, -2)$.

$$m_{GH} = \frac{y_H - y_G}{x_H - x_G} = \frac{-2 - 8}{7 - (-3)} = \frac{-10}{10} = -1$$

(-3, 8)

(7, -2)

pt 2 pt 1

Class Ex. #2

Eleanor, Bonnie, and Carl are calculating the slope of a line segment with endpoints $E(15, 8)$ and $F(-10, 6)$. Their work is shown below.

	Eleanor	Bonnie	Carl
Step 1:	$m_{EF} = \frac{-10 - 15}{6 - 8}$	$m_{EF} = \frac{6 - 8}{15 - (-10)}$	$m_{EF} = \frac{8 - 6}{15 - 10}$
Step 2:	$= \frac{-25}{-2}$	$= \frac{-2}{25}$	$= \frac{2}{5}$
Step 3:	$m_{EF} = \frac{25}{2}$	$m_{EF} = -\frac{2}{25}$	$m_{EF} = \frac{2}{5}$

Since their answers are all different, at least two of the students have made errors in their calculations. Describe all the errors which have been made and determine the correct slope.

she put x-values on the top

she didn't line up the points

he missed subtracting a negative point

15 - -10

Chetan

$\frac{6-8}{-10-15} = \frac{-2}{-25} = \frac{2}{25}$

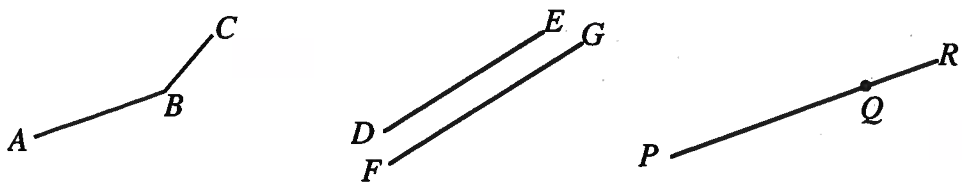
Jason

$\frac{8-6}{15-(-10)} = \frac{2}{25}$

Complete Assignment Questions #1 - #5

Collinear Points

Two lines in a plane can either be



at an angle — parallel and distinct — parallel and form a straight line.

Points that lie on the same straight line are said to be **collinear**, i.e. P, Q, and R are collinear.

If three points P, Q, and R are collinear then $m_{PQ} = m_{QR} = m_{PR}$.
Proving that any two of these three slopes are equal is sufficient for the third to be equal and for the points to be collinear.

Class Ex. #3

Consider points $A(5, -3)$, $B(2, 6)$, and $C(-7, 33)$.

a) Prove that the points A, B, and C are collinear.

$m_{AB} = \frac{6 - -3}{2 - 5} = \frac{9}{-3} = -3$

"slope of AB"

$m_{BC} = \frac{33-6}{-7-2} = \frac{27}{-9} = -3$

$m_{AC} = \frac{33 - -3}{-7-5} = \frac{36}{-12} = -3$

b) Find the value of y if the point $D(-4, y)$ lies on line segment AC.

$m_{AD} = -3$ since $m_{AC} = -3$ and the point D is on that line

$m_{AD} = \frac{y - -3}{-4-5} = -3$

$\frac{y+3}{-9} = -3$ (·−9) solve for y

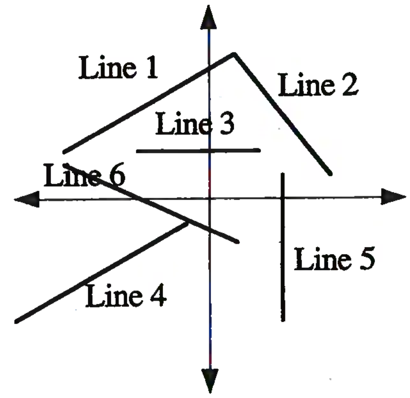
$y + 3 = 27$

$y = 24$

Complete Assignment Questions #6 - #12

Assignment

1. State whether the slope of each line is positive, negative, zero, or undefined.



2. Use the slope formula to calculate the slope of the line segment with the given endpoints.

a) $A(12,-2)$ and $B(0,3)$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} =$$

b) $C(-2,3)$ and $D(2,-2)$

c) $P(-15,-2)$ and $O(0,0)$

d) $S(36,-41)$ and $T(-20,-27)$

e) $U(-172,-56)$ and $V(-172,32)$

f) $K(8,-41)$ and $L(397,-41)$

3. Use the slope formula to calculate the slope of the line passing through the given points.

a) $(3,-6)$ and $(8,4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

b) $(-12,7)$ and $(0,-2)$

c) $(-3,-8)$ and $(1,5)$

d) $(21,1)$ and $(-4,-9)$

4. A coordinate grid is superimposed on a cross-section of a hill. The coordinates of the bottom and the top of a straight path up the hill are, respectively, $(3,2)$ and $(15,47)$, where the units are in metres.

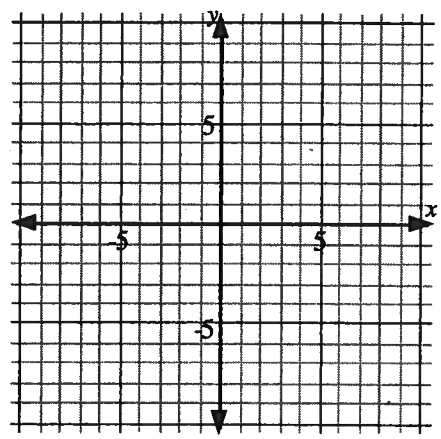
a) Calculate the slope of the hill.

b) Calculate the coordinates of the midpoint of the path up the hill.

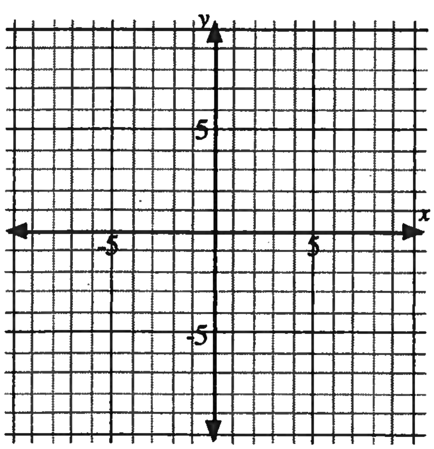
c) Calculate the length of the path to the nearest tenth of a metre.

5. The line segment joining each pair of points has the given slope. Determine each value of k and draw the line segment on the grid.

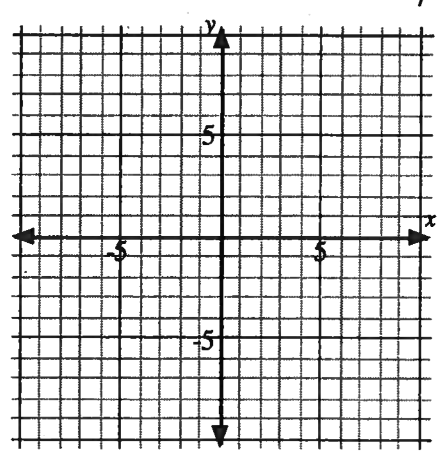
a) $S(4, 6)$ and $T(5, k)$ slope $= 3$



b) $L(k, -2)$ and $M(3, -7)$ slope $= -\frac{1}{2}$



c) $U(2, 5)$ and $V(k, 3)$ slope $= \frac{2}{7}$



6. Consider points $P(4,-9)$, $Q(-1,-7)$, and $R(-11,-3)$.

a) Use the slope formula to prove that the points P, Q, and R are collinear.

b) Use the distance formula to prove that the points P, Q, and R are collinear.

7. Consider points $A(8,-7)$, $B(-8,-3)$, and $C(-24,1)$.

a) Prove that the points A, B, and C are collinear.

b) Does the point $D(-2,-4)$ lie on line segment AC? Explain.

c) Find the value of k if the point $E(k,k)$ lies on line segment AC.

8. A private jet has crashed in the desert at the point $P(-10, 17)$. A search party sets out in an all terrain vehicle from A_1, passing in a straight line through A_2. A helicopter sets out from B_1 and flies in a straight line through B_2.
If the search parties continue in these directions, will either of them discover the crashed plane?

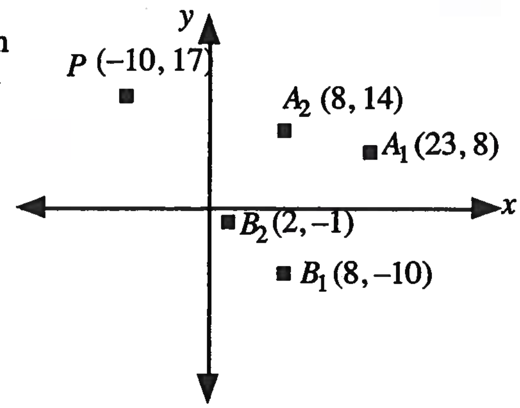


Diagram not to scale

Multiple Choice **9.** The slope of the line segment joining $E(5,-1)$ and $F(3,7)$, is

A. -3
B. -4
C. $-\frac{1}{3}$
D. $-\frac{1}{4}$

10. If the line segment joining $(2,3)$ and $(8,k)$ has slope $-\frac{2}{3}$, then $k =$

A. -1
B. -3
C. -6
D. 7

11. One endpoint of a line segment is $(1,6)$. The other endpoint is on the x-axis. If the slope of the line segment is -3, then the midpoint of the line segment is

A. $(4,6)$
B. $(2,3)$
C. $(-10,3)$
D. $\left(\frac{1}{2}, \frac{15}{2}\right)$

Numerical Response **12.** $P(3,6)$, $Q(8,-2)$, and $R(-6,0)$, are the vertices of a triangle. T is the midpoint of QR. The slope of the line PT, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

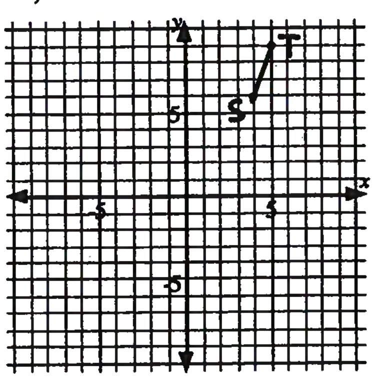
1. Line 1 - positive, Line 2 - negative, Line 3 - zero, Line 4 - positive, Line 5 - undefined, Line 6 - negative

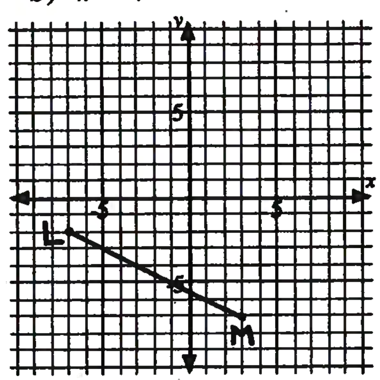
2. a) $-\frac{5}{12}$ b) $-\frac{5}{4}$ c) $\frac{2}{15}$ d) $-\frac{1}{4}$ e) undefined f) 0

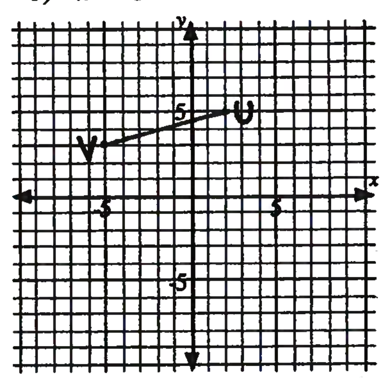
3. a) 2 b) $-\frac{3}{4}$ c) $\frac{13}{4}$ d) $\frac{2}{5}$

4. a) $\frac{15}{4}$ b) $\left(9, \frac{49}{2}\right)$ c) 46.6 m.

5. a) $k = 9$ b) $k = -7$ c) $k = -5$







6. a) $m_{PQ} = -\frac{2}{5}$, $m_{QR} = -\frac{2}{5}$. Since $m_{PQ} = m_{QR}$, the points P, Q and R are collinear.
b) $PQ = \sqrt{29}$, $QR = 2\sqrt{29}$, $PR = 3\sqrt{29}$. Since $PQ + QR = PR$, the points P, Q and R are collinear.

7. a) $m_{AB} = -\frac{1}{4}$, $m_{BC} = -\frac{1}{4}$. Since $m_{AB} = m_{BC}$, the points A, B and C are collinear.
b) $m_{AD} = -\frac{3}{10}$ Since $m_{AD} \neq m_{AB}$, the point D does not lie on line segment AC. c) $k = -4$

8. $m_{A_1A_2} = -\frac{2}{5}$, $m_{A_2P} = -\frac{1}{6}$. Since $m_{A_1A_2} \neq m_{A_2P}$, the search party in the all terrain vehicle will not discover the plane.
$m_{B_1B_2} = -\frac{3}{2}$, $m_{B_2P} = -\frac{3}{2}$. Since $m_{B_1B_2} = m_{B_2P}$, the search party in the helicopter will discover the plane.

9. B **10.** A **11.** B **12.**

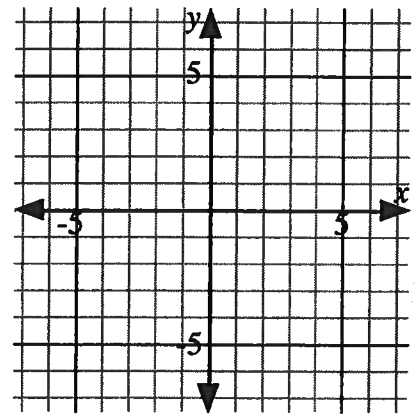
3	.	5	

Characteristics of Linear Relations Lesson #5: Parallel and Perpendicular Lines

Review of Transformations

In earlier mathematics courses we studied transformations: translations, reflections, and rotations. In order to investigate parallel and perpendicular line segments, we will review translations and rotations.

On the grid, show the image of the point $A(2, 5)$ after the following transformations. In each case write the coordinates of the image.



a) A translation 3 units right and 2 units up.

$A(2, 5) \rightarrow \quad B(\quad , \quad)$

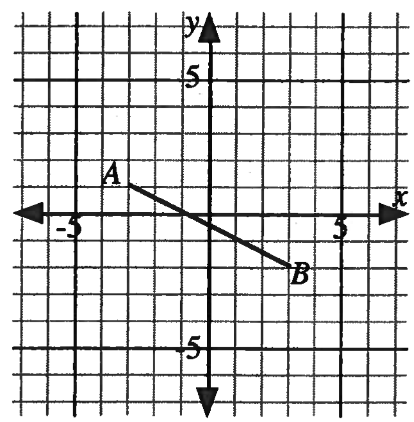
b) A 90° clockwise rotation about the origin.

$A(2, 5) \rightarrow \quad C(\quad , \quad)$

c) A 90° counterclockwise rotation about the origin.

$A(2, 5) \rightarrow \quad D(\quad , \quad)$

Investigating Parallel Line Segments



a) On the grid, show the image of line segment AB after the following transformations.

i) A translation 4 units right and 1 unit down to form line segment CD.

ii) A translation 3 units left and 6 units up to form line segment EF.

iii) A translation 3 units down to form line segment GH.

b) Calculate the slope of each of the line segments.

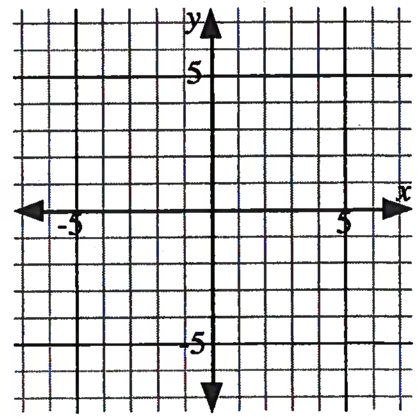
c) The four line segments are parallel. Make a conjecture about the slopes of parallel line segments.

Investigating Perpendicular Line Segments

a) i) On the grid, plot the point $A(5,2)$ and draw the line joining the point to the origin, O.

ii) Rotate the line through an angle of 90° clockwise about O and show the image on the grid.

iii) Find the slopes of the two perpendicular lines and multiply them together.



b) Repeat part **a)** for the point $B(-6, 1)$.

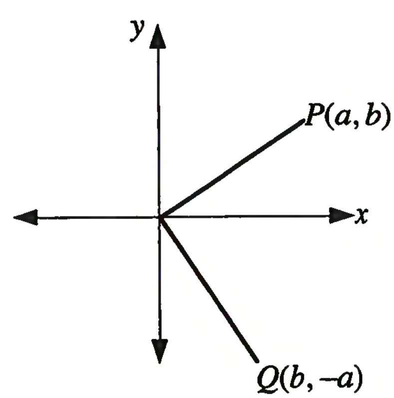
c) Make a conjecture about the slopes of perpendicular line segments.

d) Complete the following to prove the conjecture in **c)**.
Under a rotation of 90° clockwise about O, $P(a, b) \rightarrow Q(b, -a)$.

$$m_{OP} = \frac{y_P - y_O}{x_P - x_O} = \frac{b-0}{a-0} =$$

$$m_{OQ} = \frac{y_Q - y_O}{x_Q - x_O} = \frac{-a-0}{b-0} =$$

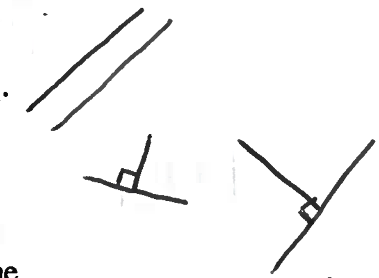
$$m_{OP} \times m_{OQ} =$$



Parallel Lines and Perpendicular Lines

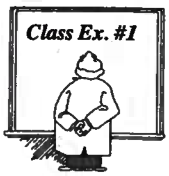
Recall that the slope of any line segment within a line represents the slope of the line.

Consider then two lines with slopes m_1 and m_2.



- The lines are **parallel** if they have the same slope, i.e. $m_1 = m_2$.
- The lines are **perpendicular** if the product of the slopes is -1, i.e. $m_1 \times m_2 = -1$ or $m_1 m_2 = -1$ or $m_1 = -\frac{1}{m_2}$
- For perpendicular lines, each slope is the negative reciprocal of the other provided neither slope is equal to zero.

$3 = \frac{-1}{3}$ $\quad \frac{-1}{4} = 4$



Class Ex. #1

Consider line segment AC with a slope of $\frac{3}{4}$.

$\frac{2}{3} = \frac{-3}{2}$

a) Write the slope of line segment GH which is parallel to AC.

$m_{GH} = \frac{3}{4}$ *parallel = same slope

b) Write the slope of line segment BF which is perpendicular to AC.

$m_{BF} = -\frac{4}{3}$ *perpendicular = flip the slope and change sign

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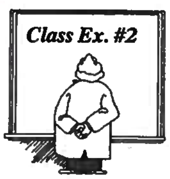


Class Ex. #2

The slopes of two lines are given.
Determine if the lines are parallel, perpendicular, or neither.

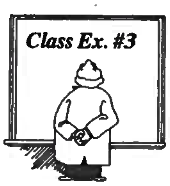
a) $m_1 = \frac{1}{4}$, $m_2 = \frac{3}{12} = \frac{1}{4}$ parallel

b) $m_1 = \frac{5}{7}$, $m_2 = \frac{14}{10} = \frac{7}{5}$ neither

*not parallel
*not perp cause not negative

$\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$

Complete Assignment Questions #1 - #6



Class Ex. #3

If P is the point $(4, 7)$ and Q is the point $(6, -2)$, find the slope of a line segment

a) parallel to line segment PQ **b)** perpendicular to line segment PQ

$m_{PQ} = \frac{-2-7}{6-4} = \frac{-9}{2}$ → $\frac{2}{9}$

parallel perp

Class Ex. #4

$\triangle LMN$ has coordinates $L(-4,2)$, $M(-2,7)$, and $N(1,0)$. Use slopes to show that the triangle is right-angled at L.

$$m_{LM} = \frac{7-2}{-2-4} = \frac{5}{2}$$

$$m_{LN} = \frac{0-2}{1--4} = \frac{-2}{5}$$

$$\left(\frac{5}{2}\right)\left(\frac{-2}{5}\right) = \frac{-10}{10} = -1$$

negative reciprocals so perpendicular

M (-2,7), L (-4,2), N (1,0)

disclaimer: terrible picture

to make a right-angle triangle LM & LN would have to be perp

Class Ex. #5

Two lines have slopes of $-\frac{3}{4}$ and $\frac{k}{5}$ respectively. Find the value of k if the lines are

a) parallel

$$\frac{-3}{4} = \frac{k}{5}$$

*solve for k

$$\frac{-15}{4} = k$$

b) perpendicular

$$\left(\frac{-3}{4}\right)\left(\frac{k}{5}\right) = -1$$

*two slopes multiplied must equal −1 to be perp

$$\frac{-3k}{20} = -1$$

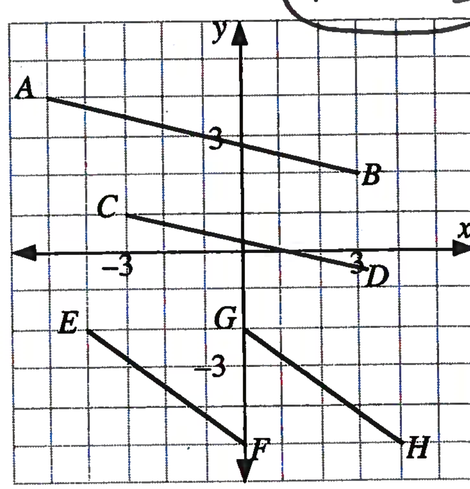
$$-3k = -20$$

$$k = \frac{20}{3}$$

Complete Assignment Questions #7 - #16

Assignment

1. AB is parallel to CD. EF is parallel to GH.



a) Determine the slopes of the following pairs of parallel line segments using $m = \frac{\text{rise}}{\text{run}}$.

Line Segment	Slope	Line Segment	Slope
AB		*EF*	
CD		*GH*	

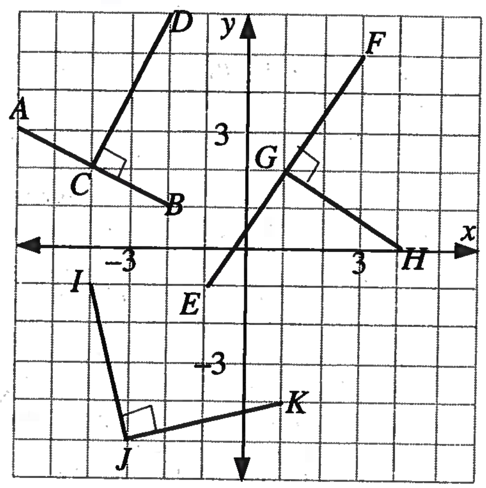
b) Observe the slopes of the pairs of parallel lines in a).
Write a rule in reference to the slopes of parallel lines.

2.a) Determine the slopes of the following pairs of perpendicular line segments using $m = \frac{\text{rise}}{\text{run}}$.

Line Segment	Slope
AB	
CD	

Line Segment	Slope
EF	
GH	

Line Segment	Slope
IJ	
JK	



b) Multiply the slopes of the pairs of perpendicular line segments.

$m_{AB} \times m_{CD}$

$-\frac{1}{2} \times 2 =$

$m_{EF} \times m_{GH}$

$m_{IJ} \times m_{JK}$

c) Write a rule in reference to the slope of two lines which are perpendicular to each other.

3. The slopes of two line segments are given. Determine if the lines are parallel, perpendicular, or neither.

a) $m_{AB} = \frac{8}{20}$, $m_{PQ} = \frac{2}{5}$ **b)** $m_{AB} = \frac{3}{2}$, $m_{PQ} = -\frac{2}{3}$ **c)** $m_{AB} = \frac{1}{6}$, $m_{PQ} = \frac{2}{12}$

d) $m_{AB} = \frac{7}{8}$, $m_{PQ} = \frac{8}{7}$ **e)** $m_{AB} = \frac{9}{3}$, $m_{PQ} = -\frac{1}{3}$ **f)** $m_{AB} = -5$, $m_{PQ} = \frac{1}{5}$

g) $m_{AB} = \frac{4}{8}$, $m_{PQ} = 2$ **h)** $m_{AB} = -\frac{12}{2}$, $m_{PQ} = -6$ **i)** $m_{AB} = -\frac{5}{2}$, $m_{PQ} = -\frac{2}{5}$

4. The slopes of some line segments are given.

$m_{AB} = 6$ $m_{CD} = \frac{1}{6}$ $m_{EF} = -6$ $m_{GH} = 6$ $m_{IJ} = -6$ $m_{KL} = \frac{1}{6}$

Which pairs of lines are parallel to each other?

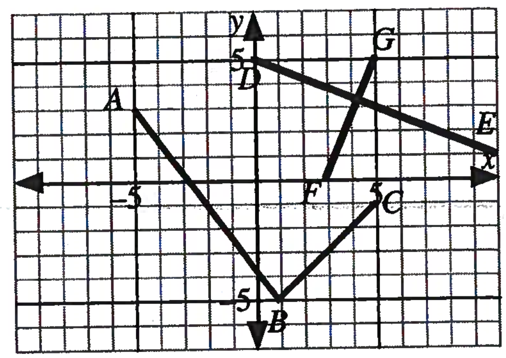
5. The slopes of some line segments are given.

$m_{RS} = -2$ $m_{UV} = \frac{1}{4}$ $m_{EF} = 0.5$ $m_{ZT} = 2$

$m_{PQ} = -4$ $m_{KL} = -\frac{1}{2}$ $m_{MN} = 4$ $m_{XY} = -\frac{1}{4}$

Which pairs of lines are perpendicular to each other?

6. The four line segments have endpoints with integer coordinates. In each case determine whether the two intersecting line segments are perpendicular.



7. A, B, and C are the points $(0, 4)$, $(-3, 1)$, and $(5, -2)$ respectively. Determine the slope of a line

a) parallel to line segment AB

b) perpendicular to line segment AB

c) parallel to line segment BC

d) perpendicular to line segment AC

8. ΔABC has vertices $A(3, 5)$, $B(-2, -5)$, $C(-5, 1)$.

a) Explain how we can determine if $\triangle ABC$ is a right triangle.

b) Determine if $\triangle ABC$ is a right triangle.

9. The vertices of two triangles are given.
Determine if either of the triangles is right-angled.

a) $\Delta PQR \rightarrow P(-3,3),\ Q(-1,1),\ R(-5,-1)$ **b)** $\Delta ABC \rightarrow A(-7,9),\ B(3,13),\ C(7,3)$

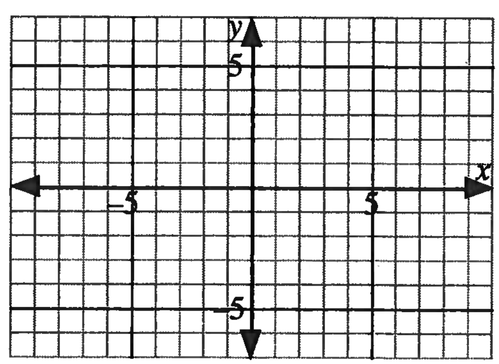
10. The slopes of parallel lines are given.
Determine the value of the variable.

a) $4, \frac{k}{3}$ **b)** $-2, \frac{2}{n}$ **c)** $\frac{5}{6}, 3m$ **d)** $\frac{3}{4}, -\frac{w}{6}$

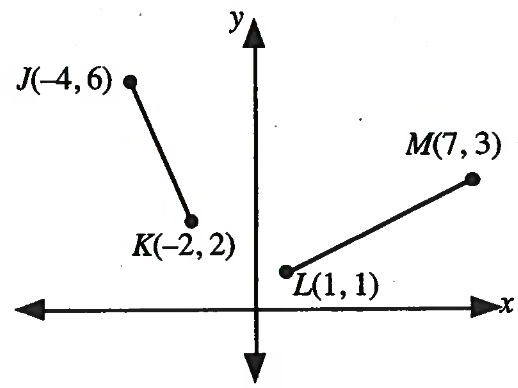
11. The slopes of perpendicular lines are given.
Determine the value of the variable.

a) $\frac{1}{3}, 3h$ **b)** $4, \frac{8}{p}$ **c)** $-5, \frac{s}{2}$ **d)** $-\frac{3}{4}, -\frac{q}{6}$

12. $P(-4,0)$ and $R(1,-3)$ are opposite vertices of a rhombus $PQRS$. Find the slope of diagonal QS.



13. a) Show that when line segments JK and ML are extended until they intersect, they will not meet at right angles.



b) If the y-coordinate of M is changed, the line segments, when extended, will meet at right angles. To what value should the y-coordinate of M be changed?

14. Given that $A, B,$ and C are the points $(-3, 3)$, $(0, 6)$, and $(5, 1)$ respectively, prove that triangle ABC is right angled by using

a) the slope formula

b) the distance formula

Multiple Choice

15. A and B are the points $(1, 2)$ and $(-2, 3)$ respectively. A line perpendicular to AB will have slope

A. -3

B. $-\frac{1}{3}$

C. 3

D. $\frac{1}{3}$

Numerical Response **16.** The line segment joining $U(-3, p)$ and $V(-6, 5)$ is perpendicular to the line segment joining $X(4, 2)$ and $Y(9, 0)$. The value of p, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) slope $AB = -\frac{1}{4}$ slope $CD = -\frac{1}{4}$ slope $EF = -\frac{3}{4}$ slope $GH = -\frac{3}{4}$

b) Lines which are parallel have the same slope

2. a) slope $AB = -\frac{1}{2}$ slope $EF = \frac{3}{2}$ slope $IJ = -4$

slope $CD = 2$ slope $GH = -\frac{2}{3}$ slope $JK = \frac{1}{4}$

b) All the products are -1. **c)** The product of the slopes is -1.

3. a) parallel **b)** perpendicular **c)** parallel

d) neither **e)** perpendicular **f)** perpendicular

g) neither **h)** parallel **i)** neither

4. *AB* and *GH*, *CD* and *KL*, *EF* and *IJ*.

5. *RS* and *EF*, *UV* and *PQ*, *ZT* and *KL*, *MN* and *XY*.

6. *AB* and *BC* are not perpendicular. *DE* and *FG* are perpendicular.

7. a) 1 **b)** -1 **c)** $-\frac{3}{8}$ **d)** $\frac{5}{6}$

8. a) Determine the slope of each side of the triangle. If two of the slopes are negative reciprocals of each, then the triangle is a right triangle.

b) $m_{BC} = -2$, $m_{AC} = \frac{1}{2}$. Since the slopes are negative reciprocals, the triangle is a right triangle.

9. a) ΔPQR is <u>not</u> a right triangle **b)** ΔABC is a right triangle

10. a) $k = 12$ **b)** $n = -1$ **c)** $m = \frac{5}{18}$ **d)** $w = -\frac{9}{2}$

11. a) $h = -1$ **b)** $p = -32$ **c)** $s = \frac{2}{5}$ **d)** $q = -8$

12. $m_{QS} = \frac{5}{3}$

13. a) $M_{JK} = -2$, $M_L = \frac{1}{3}$. The product of the slopes does not equal -1. **b)** $y_M = 4$

14. a) $m_{AB} = 1$, $m_{BC} = -1$ Since the product of the slopes $= -1$, AB and BC are perpendicular. Triangle *ABC* is right angled at *B*.

b) $AB = \sqrt{18}$, $BC = \sqrt{50}$, $AC = \sqrt{68}$. $AC^2 = 68$. $AB^2 + BC^2 = 68$.

$AC^2 = AB^2 + BC^2$ so the Pythagorean theorem is satisfied and the triangle is right angled at *B*.

15. C **16.**

1	2	.	5

Equations of Linear Relations Lesson #1:
The Equation of a Line in Slope y-intercept Form → $y = mx + b$

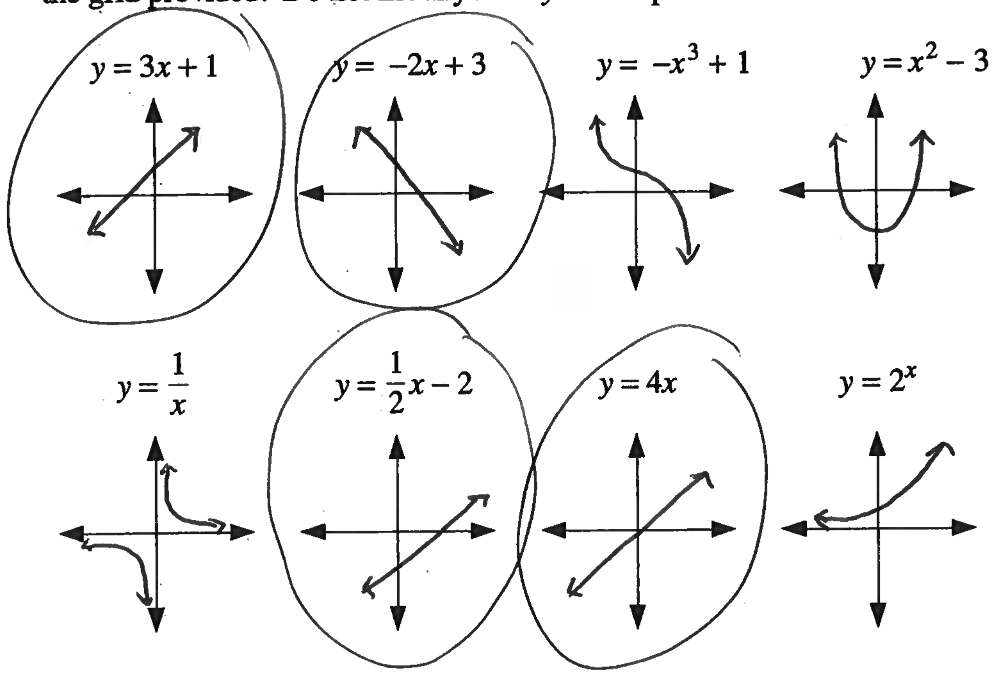
Overview of Unit

In this unit we express the equation of a linear relation in three different forms: slope *y*-intercept form, point-slope form, and general form. We relate linear relations expressed in these forms to their graphs.

We also determine the linear relation given: a graph, a table of data points, a point and the slope, two points, a point and the equation of a parallel or perpendicular line.

Investigating the Graphs of Linear and Non-Linear Relations

a) The equations of the graphs of some relations are given. In each case, use a graphing calculator to sketch the graph of the relation and make a rough sketch of the graph on the grid provided. Do not list any *x*- or *y*-intercepts.



b) List the equations of the graphs as linear or non-linear.

LINEAR: $y=3x+1$, $y=-2x+3$, $y=\frac{1}{2}x-2$, $y=4x$

NON-LINEAR: $y=-x^3+1$, $y=x^2-3$, $y=\frac{1}{x}$ x^{-1}, $y=2^x$

c) Compare the lists. Write a rule from the equation which can be used to determine whether the graph is a straight line or not.

-linear: no exponent on a variable

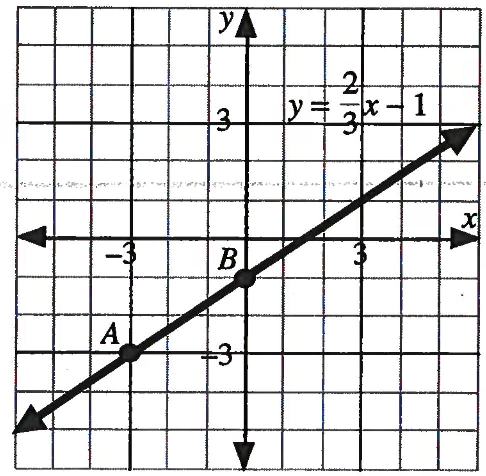
Linear Equation

A **linear equation** is an equation of the form $y = mx + b$, where $m, b \in R$.
The graph of a linear equation is a straight line.

Investigating m and b in the equation $y = mx + b$

Part One

Jenine used a graphing calculator to sketch the graph of the linear equation $y = \frac{2}{3}x - 1$.
Her sketch is shown on the grid.

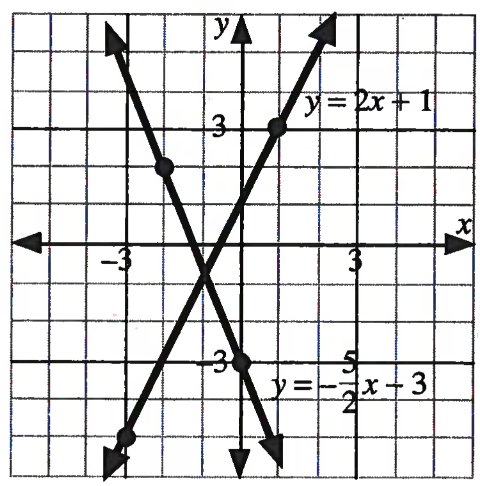


a) Use the sketch and points A and B to find the slope and y-intercept of the graph of $y = \frac{2}{3}x - 1$.

b) Compare the values found in **a)** with the coefficient of x and the constant term in the equation $y = \frac{2}{3}x - 1$.

c) Jenine sketched the graphs of two more linear equations. Use the grid to determine the slope and y-intercept of each graph.

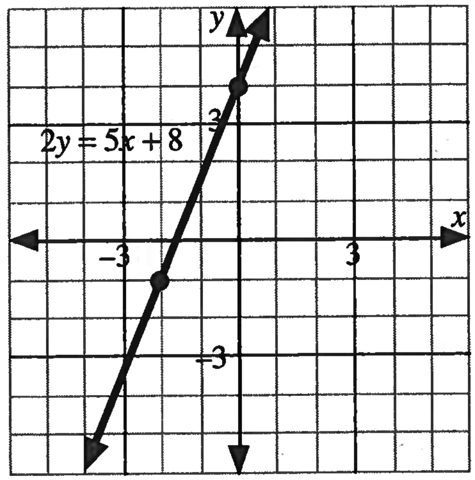
equation	slope	y-intercept
$y = 2x + 1$		
$y = -\frac{5}{2}x - 3$		



d) Make a conjecture about the slope and y-intercept of the graph of the linear equation $y = mx + b$.

Part Two

Hashib used a graphing calculator to graph the linear equation $2y = 5x + 8$.
The graph is shown on the grid.



a) Use the sketch to determine the slope and y-intercept of the graph of $2y = 5x + 8$.

b) Explain why, in this case, the slope is not 5 (the coefficient of x) and the y-intercept is not 8 (the constant term).

Slope y-intercept Form of the Equation of a Line → $y = mx + b$

The graph of an equation in the form $y = mx + b$ (or a function in the form $f(x) = mx + b$) is a straight line with **slope m** and **y-intercept b.**

The equation $y = mx + b$ is known as the **slope y-intercept form** of the equation of a line.

The graph of an equation in this form can be drawn without making a table of values.

Class Ex. #1

Determine the slope and y-intercept of the graph of each linear equation listed below:

$y=mx+b$

a) $y = 3x + 2$

m=slope
$=3$
b=y-int(x=0)
$=2$ $(0,2)$

b) $y = 7 - \frac{2}{3}x$

$m=\frac{-2}{3}$
$b=7$

c) $\frac{6y}{6} = \frac{8x}{6} + \frac{1}{6}$

$y=\frac{8x}{6}+\frac{1}{6}=\frac{4}{3}x+\frac{1}{6}$
$m=\frac{4}{3}$ $b=\frac{1}{6}$

Graphing an Equation of the Form $y = mx + b$

In this section, we will look at two ways of sketching the graph of a linear equation without using a graphing calculator or a table of values.

Class Ex. #2

Consider the equation $y = 2x - 5$.

a) State the slope and y-intercept.

m b

$m=2$ $b=-5$ $(0,-5)$

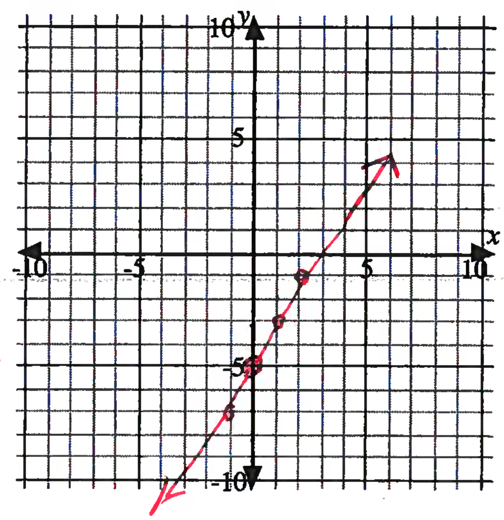
b) Mark the y-intercept on the grid.

y-int means $x=0$

c) Use the y-intercept and the formula slope $= \frac{\text{rise}}{\text{run}}$ to mark three other points on the grid. Join the points together, and extend the line.

↑2 →1

d) Verify the graph using a graphing calculator.



Class Ex. #3

Consider the equation $y = \frac{2}{3}x - 6$.

a) State the y-intercept.

-6 $(0,-6)$

b) Determine the x-intercept algebraically.

$y=0$ $0=\frac{2}{3}x-6$ *solve for x

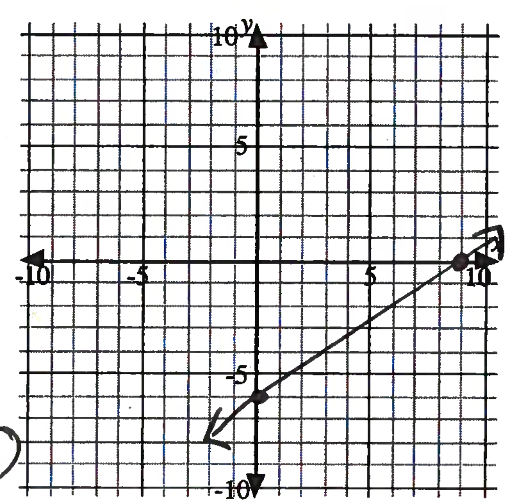
$+6$ $+6$

$6=\frac{2}{3}x$ $18=2x$

$(9,0)$ $\frac{18}{2}=\frac{2x}{2}$ $x=9$

c) Mark the x- and y-intercepts on the grid. Join the points together, and extend the line.

d) Verify the graph and the intercepts using a graphing calculator.



Complete Assignment Questions #1 - #14

Assignment

1. Each equation represents a relation.

a) $y = 6x + 1$ **b)** $y = x^2$ **c)** $y = 3x^4 + 5$ **d)** $y = -\frac{1}{4}x - 8$

e) $y = 1 - x$ **f)** $y = \frac{2}{1 - x}$ **g)** $y = 4x$ **h)** $y = 4^x$

Without sketching the graph of the relation, list the letters a) through h) as linear or non-linear

LINEAR:

NON-LINEAR:

2. State the slope and y-intercept of the graph of each linear equation.

a) $y = 7x - 2$ **b)** $y = \frac{4}{3}x + 3$ **c)** $y = 6 - \frac{1}{6}x$ **d)** $4y = 6x + 8$ **e)** $y = ax + b$

3. Write the equation of each line with the given slope and y-intercept.

a) slope $= 4$, y-intercept $= -9$

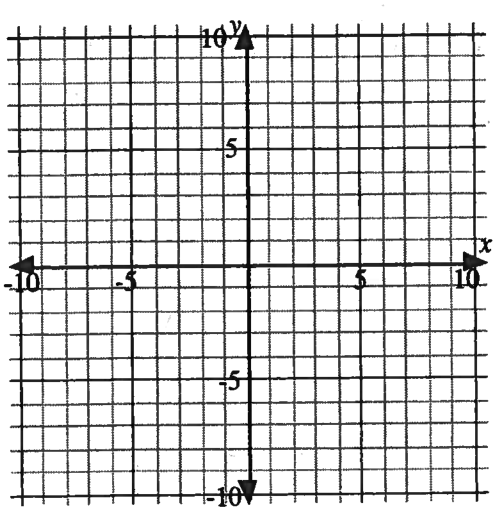
b) slope $= \frac{1}{5}$, y-intercept $= \frac{1}{2}$

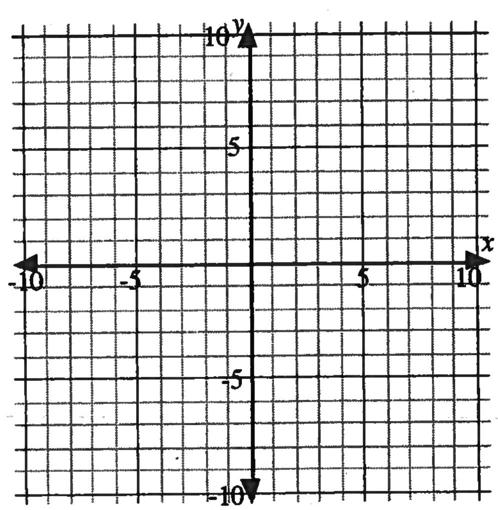
c) slope $= -3$, y-intercept $= 0$

d) slope $= m$, y-intercept $= b$

4. For each line, state the slope and the y-intercept. Graph the equation without using a graphing calculator.

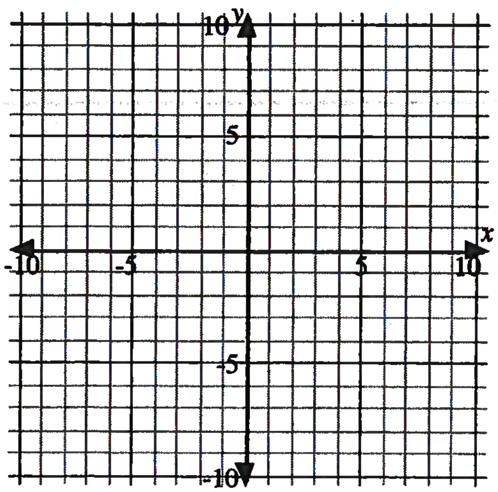
a) $y = \frac{1}{4}x + 2$ **b)** $y = -x - 1$ **c)** $y = -\frac{4}{3}x$ **d)** $y = 5$

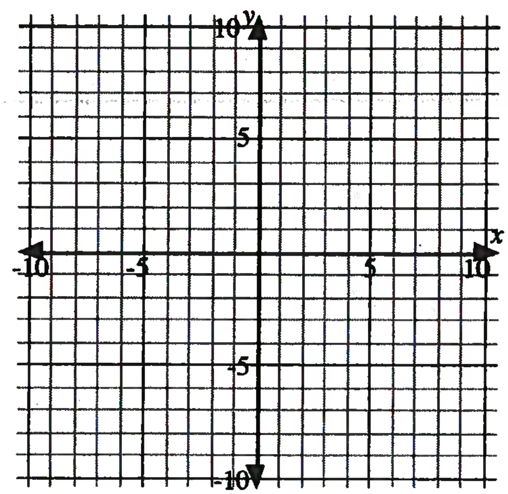




5. For each line, state the y-intercept. Determine the x-intercept algebraically, and graph the equation without using a graphing calculator.

a) $y = 2x + 6$ **b)** $y = -x - 4$ **c)** $y = \frac{6}{7}x - 6$ **d)** $y = -\frac{1}{2}x + 1$





6. Explain why the linear equation $y = 5x$ can be graphed using the method in question 4 but not by the method in question 5.

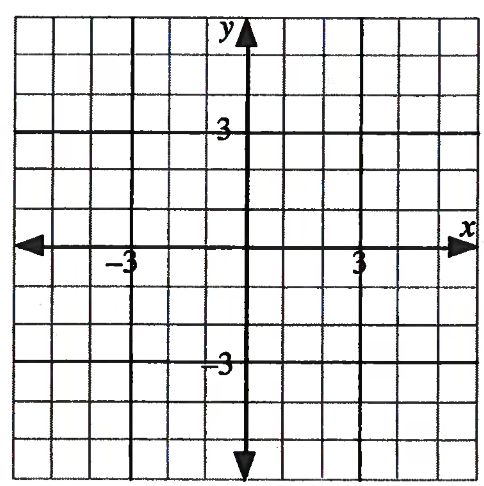
7. Consider the graph of the function with equation $y = x$.

a) State the values of m and b.

b) Determine the x- and y-intercepts.

c) Sketch the graph on the grid provided without using a graphing calculator.

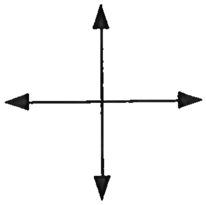
d) Determine the domain and range of the function.



e) Use a graphing calculator to graph the line $y = -x$, and sketch the graph on the grid.

8. Use a graphing calculator to sketch the graph of each of the following linear equations. Complete the table giving the x-intercept to the nearest hundredth.

i) $y = 7x - 8$

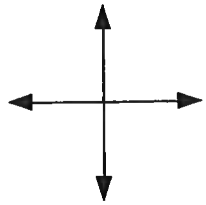


slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

ii) $y = -\frac{31}{2}x - 25$

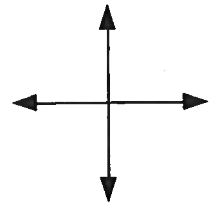


slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

iii) $y = 75 - \frac{5}{3}x$



slope	
x-intercept	
y-intercept	

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

Multiple Choice

9. Which of the following does not represent the equation of a straight line?

A. $y = 3x$
B. $y = 11 - 3x$
C. $y = \frac{x}{3}$
D. All of the above represent the equation of a straight line.

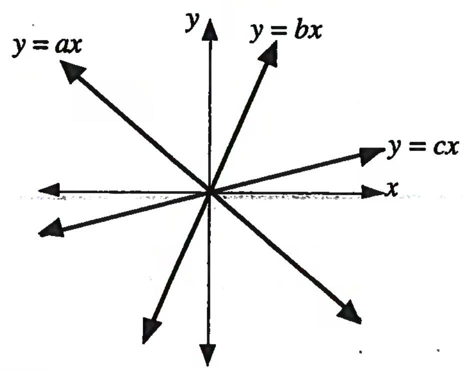
10. Which of the following statements is false for the line $y = -\frac{1}{2}x + 1$?

A. The graph of the line falls from left to right.
B. The x-intercept is 2.
C. The graph passes through the point $(8, -3)$.
D. The line is perpendicular to the line $y = -2x + 4$.

11. Which of the following statements is true for the line $2y = \frac{1}{4}x + 6$?

A. The x-intercept is 24.
B. The y-intercept is 6.
C. The slope is $\frac{1}{8}$.
D. The graph passes through the point $(-4, 5)$.

12. The lines $y = ax$, $y = bx$, and $y = cx$ are shown. Which of the following statements is true?



A. $a < b < c$
B. $a < c < b$
C. $c < a < b$
D. $c < b < a$

Use the following information to answer questions 13 and 14.

> Consider the line with equation $y = 3x + 5$. The line intersects the x-axis at P and the y-axis at Q. Triangle POQ is formed where O is the origin.

Numerical Response

13. The area of $\triangle POQ$, in square units, to the nearest tenth, is ______.

(Record your answer in the numerical response box from left to right)

14. To the nearest tenth, the perimeter of $\triangle POQ$ is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. LINEAR a), d), e), g). NON-LINEAR b), c), f), h).

2. **a)** slope = 7, y-int = –2 **b)** slope = $\frac{4}{3}$, y-int = 3

c) slope = $-\frac{1}{6}$, y-int = 6 **d)** slope = $\frac{3}{2}$, y-int = 2 **e)** slope = a, y-int = b

3. **a)** $y = 4x - 9$ **b)** $y = \frac{1}{5}x + \frac{1}{2}$ **c)** $y = -3x$ **d)** $y = mx + b$

4. **a)** slope = $\frac{1}{4}$, y-int = 2 **b)** slope = –1, y-int = –1

c) slope = $-\frac{4}{3}$, y-int = 0 **d)** slope = 0, y-int = 5

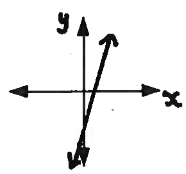
5. **a)** y-int = 6, x-int = –3 **b)** y-int = –4, x-int = –4 **c)** y-int = –6, x-int = 7 **d)** y-int = 1, x-int = 2

6. The method in #4 needs a point and a slope. We have point (0, 0) and slope = 5. The method in #5 needs two points to be joined. Since the x- and y-intercepts are the same point, the line cannot be drawn.

7. **a)** $m = 1$, $b = 0$ **b)** x-int = 0 and y-int = 0. **d)** $D = x \in R$ $R = y \in R$

8.

i) $y = 7x - 8$

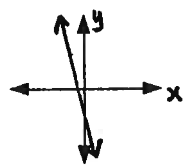


slope	7
x-intercept	1.14
y-intercept	–8

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -4
 Xmax= 4
 Xscl= 1
 Ymin= -10
 Ymax= 5
 Yscl= 2
 Xres=1
```

ii) $y = -\frac{31}{2}x - 25$

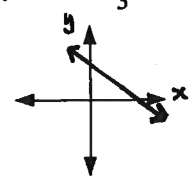


slope	–31/2
x-intercept	–1.61
y-intercept	–25

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -4
 Xmax= 4
 Xscl= 1
 Ymin= -40
 Ymax= 10
 Yscl= 10
 Xres=1
```

iii) $y = 75 - \frac{5}{3}x$



slope	–5/3
x-intercept	45
y-intercept	75

Graphing Window which includes both intercepts:

```
WINDOW
 Xmin= -10
 Xmax= 60
 Xscl= 10
 Ymin= -20
 Ymax= 100
 Yscl= 10
 Xres=1
```

9. D **10.** D **11.** C **12.** B

13.

4	.	2	

14.

1	1	.	9