

Characteristics of Linear Relations Lesson #1: Line Segments on a Cartesian Plane



Lesson 1 and Lesson 2 of this unit are not required for this curriculum, but are included because

1. they are important characteristics of linear relations not covered elsewhere
2. this information is required in higher level math courses such as Calculus

Unit Overview

The graph of a **linear relation** is represented by a **straight line**. The line can be infinite or finite depending on the domain and range of the linear relation. In some cases we are only interested in a portion of a line. This portion is called a **line segment**.

We have already studied some of the characteristics of the graph of a linear relation: intercepts, domain, and range. In this unit we study some characteristics of line segments: namely, **length**, **midpoint**, **distance**, and **slope**. We demonstrate an understanding of slope with respect to rise and run, the slope formula, and rate of change. We then discuss the slopes of parallel and perpendicular lines.

Line Segment

A line segment is the portion of a line between two points on the line.

If the endpoints of a line segment are A and B , we refer to it as line segment AB .

NOTE: Line segment AB may also be written as \overline{AB} .



Length of a Horizontal Line Segment

Consider the line segments shown on the grid.

- a) Find the length of each line segment by counting.

- length of AB is 6 units.
- length of CD is 10 units.
- length of EF is 7 units.

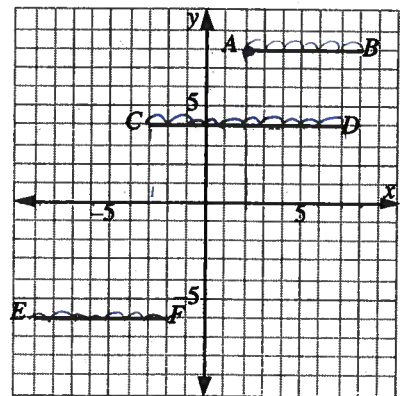
- b) Determine the coordinates of the endpoints of each line segment.

- $AB \rightarrow A(2, 8) \quad B(8, 8)$
- $CD \rightarrow C(-3, 4) \quad D(7, 4)$
- $EF \rightarrow E(-9, -6) \quad F(2, -6)$

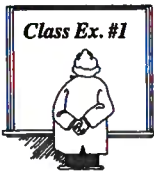
- c) Complete the following.

- The difference in the x -coordinates, $x_B - x_A$, is 6.
- The difference in the x -coordinates, $x_D - x_C$, is 10.
- The difference in the x -coordinates, $x_F - x_E$, is 7.

- d) How can the coordinates of the end points of a horizontal line segment be used to find the length of the line segment?



Handwritten notes:
 \leftarrow x -coordinate of B
 \leftarrow x -coordinate of A
 $8 - 2 = 6$



a) Line segment AB has endpoints $A(2, 8)$ to $B(-5, 8)$. Determine the length of \overline{AB} .

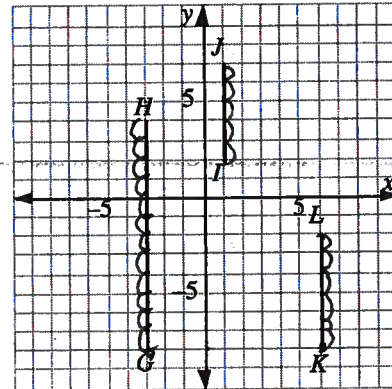
right left right-left

$$x_B - x_A \quad -5 - 2 = -7 \rightarrow 7 \text{ units}$$

b) Determine the length of the line segment from $P(a-2, b)$ to $Q(a+4, b)$.

Length of a Vertical Line Segment

Consider the line segments shown on the grid.



a) Find the lengths of each line segment by counting.

- length of GH is 12 units.
- length of IJ is 5 units.
- length of KL is 6 units.

b) Determine the coordinates of the endpoints of each line segment.

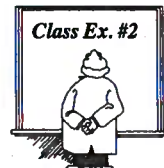
- $GH \rightarrow G(-3, -8) \quad H(-3, 4)$
- $IJ \rightarrow I(1, 2) \quad J(1, 7)$
- $KL \rightarrow K(6, -8) \quad L(6, -2)$

c) Complete the following.

- The difference in the y-coordinates, ^{top-bottom} $y_H - y_G$, is 12. $4 - -8$
- The difference in the y-coordinates, $y_J - y_I$, is 5. $7 - 2$
- The difference in the y-coordinates, $y_L - y_K$, is 6. $-2 - -8$

d) How can the coordinates of the end points of a vertical line segment be used to find the length of the line segment?

top y coordinate subtract bottom y-coordinate



a) Line segment RS has endpoints $R(1, -4)$ to $S(1, -9)$. Determine the length of \overline{RS} .

$$-4 - -9 = \textcircled{5} \text{ units.}$$

b) Determine the length of the line segment from $P(a, b)$ to $Q(a, b+10)$.

$$b+10 - b = 10 \text{ units.}$$

Complete Assignment Questions #1 - #5

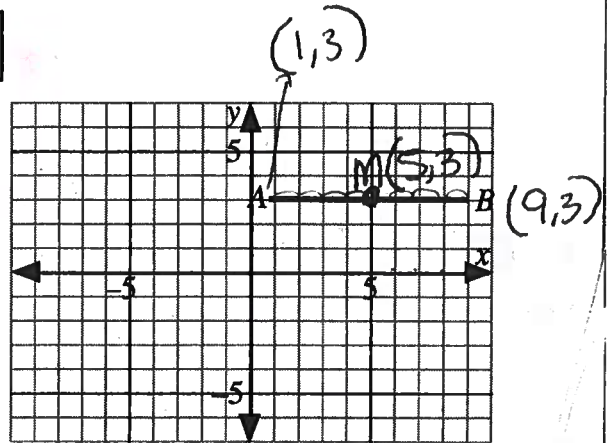
Midpoint

The **midpoint**, M , of a line segment on the graph of a linear relation is the point at the centre of the line segment.

Midpoint of a Horizontal Line Segment

Consider the line segment AB shown on the grid.

- a) Determine the coordinates of the midpoint by counting. Label the midpoint, M , on the grid and list the coordinates beside it.
- b) List the coordinates of point A and point B on the grid. How can the x -coordinates of points A and B be used to find the coordinates of the midpoint of a horizontal line?

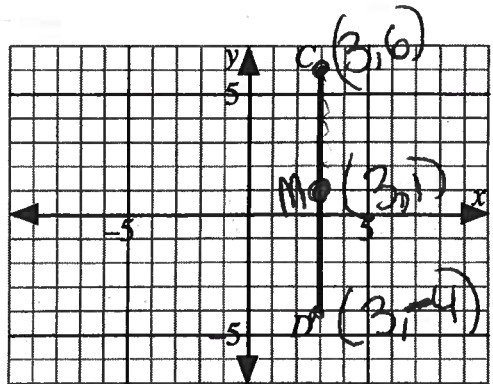


$9 - 1 = 8 \text{ units} \div 2 = 4 \text{ units}$ $\frac{9 + 1}{2} = 5 \text{ coordinate}$

Midpoint of a Vertical Line Segment

Consider the line segment CD shown on the grid.

- a) Determine the coordinates of the midpoint by counting. Label the midpoint, M , on the grid and list the coordinates beside it.
- b) List the coordinates of point C and point D on the grid. How can the y -coordinates of points C and D be used to find the coordinates of the midpoint?

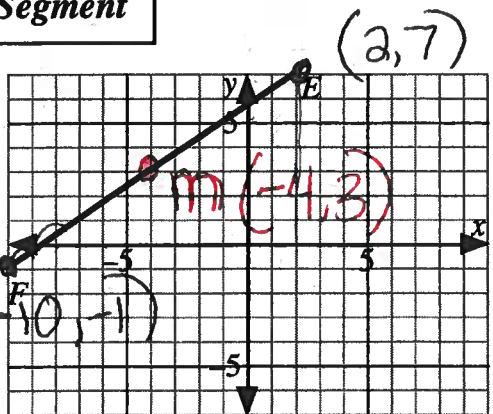


Average of y -coord. $\frac{6 + 4}{2} = \frac{10}{2} = 5$

Midpoint of an Oblique (Diagonal) Line Segment

Consider the line segment EF shown on the grid.

- a) Use the results from above to determine the midpoint of EF .
- b) Express in words how to find the midpoint, M , of the line segment joining the points (x_1, y_1) and (x_2, y_2) . Average for x -coord and average for y -coord



- c) Complete the formula to express the relationship in b). $x_M = \frac{x_1 + x_2}{2}$ $y_M = \frac{y_1 + y_2}{2}$

$\frac{2 + (-10)}{2} = -4$ $\frac{7 + (-1)}{2} = 3$

Midpoint of a Line Segment

Consider line segment PQ with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$.

The midpoint, M , of the line segment has coordinates.

$$M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Line segment PQ can also be written as \overline{PQ} .



Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

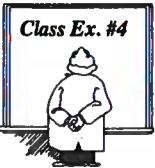
- a) $P(4, 7), Q(12, 3)$ b) $E(-5, 7), F(-11, -2)$ c) $A(w+3, 2w), C(5w-1, 7w+1)$

Handwritten solutions for Class Ex. #3:

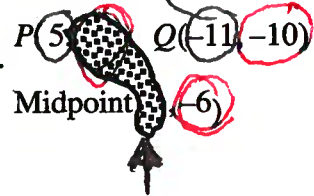
a) $M \left(\frac{4+12}{2}, \frac{7+3}{2} \right) = (8, 5)$

b) $M \left(\frac{-5+(-11)}{2}, \frac{7+(-2)}{2} \right) = \left(\frac{-16}{2}, \frac{5}{2} \right) = (-8, \frac{5}{2})$

c) $M \left(\frac{(w+3)+(5w-1)}{2}, \frac{2w+(7w+1)}{2} \right) = \left(\frac{6w+2}{2}, \frac{9w+1}{2} \right) = \left(3w+1, \frac{9w+1}{2} \right)$



Ruby was doing a question in her coordinate geometry homework, and her little brother Max wrote over part of the question as a prank.



Calculate the missing coordinates.

Handwritten work for Class Ex. #4:

$$\frac{x_p + x_q}{2} = \frac{5 + (-11)}{2} = -3$$

$$\frac{? + (-10)}{2} = -6 \implies ? - 10 = -12 \implies ? = -2$$

The final answer is $y_p = -2$.

Complete Assignment Questions #6 - #17

Assignment

1. Determine the length of each line segment.

a) $A(2, 7)$ to $B(5, 7)$

b) $C(-5, 3)$ to $D(-5, 12)$

c) $I(-3, -8)$ to $J(-3, -3)$

d) $K(7, -10)$ to $L(-35, -10)$

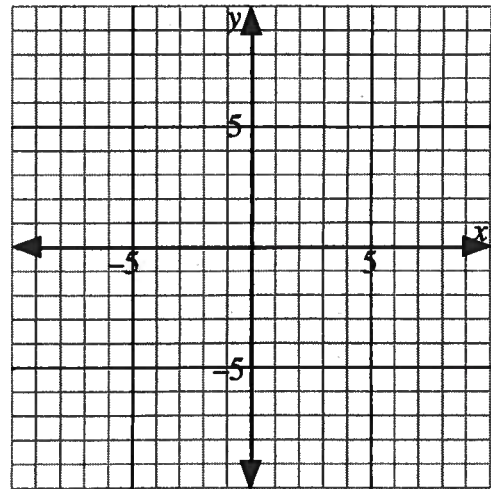
2. Determine whether each line segment is horizontal or vertical, and write an expression for its length.

a) $A(p, q)$ to $B(p - 4, q)$

b) $C(m - 3, n + 5)$ to $D(m - 3, n + 12)$

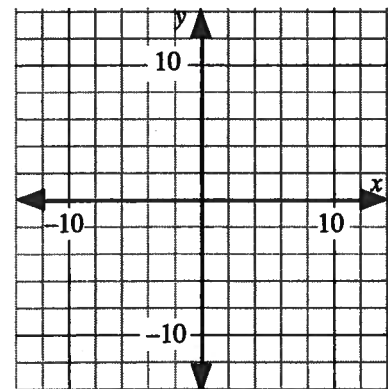
3. A triangle has vertices $P(-4, -3)$, $Q(9, -3)$, and $R(1, 5)$.

- Sketch the triangle on the grid.
- Calculate the area of the triangle.



4. On the grid, plot the points $P(-6, 6)$, $Q(-6, -10)$, and $R(6, -10)$.

- a) Determine the distance from P to R using the Pythagorean Theorem.



- b) Calculate the area and perimeter of $\triangle PQR$.

5. Rebecca uses quadrant I in a Cartesian plane to describe the location of the bases in a game of high school softball. The four bases form a square. The origin is at home plate. First base is at $(18, 0)$, and the distance between each base is 18 m. The pitcher's mound is located between home plate and second base.

- a) State the coordinates of second base.
- b) The pitcher stands on the mound 12 m from home plate. If she has to throw a ball to second base, what distance to the nearest tenth of a metre, would she throw the ball?

6. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

- a) $A(2, 6), C(4, 16)$ b) $X(-3, -8), Y(-11, 0)$ c) $K(15, -17), L(-11, 3)$

7. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

- a) $C(3x, 8y), D(7x, -4y)$ b) $S(a + b, a + 7b), T(a + b, a - 3b)$

8. Otto was given two points: $A(-6, 4)$ and $B(12, -8)$. He was asked to divide \overline{AB} into four equal parts. State the coordinates of the points which will divide \overline{AB} into four equal parts.

9. In each case M is the midpoint of \overline{AB} . Determine the value of x .

a) $A(2, 6)$, $B(6, x)$, $M(4, -1)$

b) $A(3, 6)$, $B(x, 0)$, $M(0, 3)$

Multiple
Choice

10. $ABCD$ is a square with vertices $(\sqrt{5}, 0)$, $(0, \sqrt{5})$, $(-\sqrt{5}, 0)$, and $(0, -\sqrt{5})$ respectively. The area of the square, in unit^2 , is

- A. 5
- B. 10
- C. 20
- D. 100

11. $P(4, -8)$ and $Q(-2, 10)$ are the endpoints of a diameter of a circle. The coordinates of the centre of the circle are

- A. $(-3, 9)$
- B. $(2, 2)$
- C. $(3, -9)$
- D. $(1, 1)$

12. AB is a diameter of a circle; the centre is C . If $A(8, -6)$ and $C(5, -2)$ then B is the point

- A. $(2, 2)$
- B. $(6.5, -4)$
- C. $(11, -10)$
- D. $(13, -8)$

13. Which statement is always true?

- A. Two line segments of equal length have the same midpoint.
- B. Two line segments with the same midpoint are of equal length.
- C. A point equidistant from the endpoints of a line segment is the midpoint.
- D. None of the above statements is always true.

Numerical Response

14. To the nearest tenth, the perimeter of $\triangle PQR$ with vertices, $P(3, 8)$, $Q(3, 0)$, and $R(-1, 8)$ is _____.

(Record your answer in the numerical response box from left to right)

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15. The midpoint of line segment ST is $M\left(\frac{1}{2}, -4\right)$. If the coordinates of T are $(-3, 3)$, and the coordinates of S are (x, y) , the value of x is _____.

(Record your answer in the numerical response box from left to right)

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16. The point $M(a, 6)$ is the midpoint of \overline{GH} with $G(22, b)$ and $H(6, -8)$. The value of $a + b$ is _____.

(Record your answer in the numerical response box from left to right)

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17. The midpoint of line segment AB lies on the y -axis. A lies on the x -axis, and B has coordinates $(-4, 5)$. The length of AB , to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

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Answer Key

1. a) 3 b) 9 c) 5 d) 42 2. a) horizontal, 4 b) vertical, 7
 3. 52 units² 4. a) 20 units b) area = 96 units², perimeter = 48 units
 5. a) (18,18) b) 13.5 m 6. a) (3, 11) b) (-7, -4) c) (2, -7)
 7. a) (5x, 2y) b) (a + b, a + 2b) 8. $\left(-\frac{3}{2}, 1\right)$, (3, -2), $\left(\frac{15}{2}, -5\right)$
 9. a) -8 b) -3 10. B 11. D 12. A 13. D

14.

2	0	.	9
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15.

4			
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16.

3	4		
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17.

9	.	4	
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Characteristics of Linear Relations Lesson #3: Slope of a Line Segment

A trucker driving up a hill with a heavy load may be concerned with the steepness of the hill. When building a roof, a builder may be concerned with the steepness (or pitch) of the roof. A skier going down a hill may be concerned with the steepness of the ski hill.

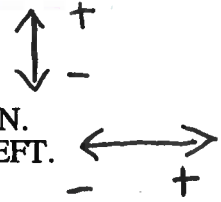
In mathematics, the term **slope** is used to describe the steepness of a line segment.

Slope of a Line Segment

The slope of a line segment is a measure of the steepness of the line segment.

It is the ratio of **rise** (the change in vertical height between the endpoints) over **run** (the change in horizontal length between the endpoints).

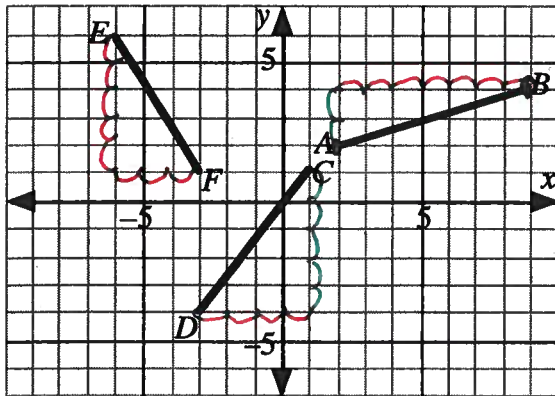
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$



- the **rise** is **POSITIVE** if we count **UP**, and **NEGATIVE** if we count **DOWN**.
- the **run** is **POSITIVE** if we count **RIGHT**, and **NEGATIVE** if we count **LEFT**.



Each line segment on the grid has endpoints with integer coordinates. Complete the table below.



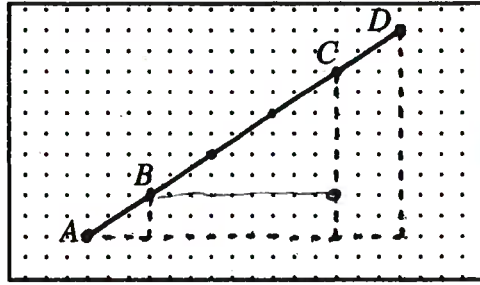
Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	7	$\frac{2}{7}$
CD	-5	-4	$\frac{-5}{-4} = \frac{5}{4}$
EF	-5	3	$\frac{-5}{3}$

$$\frac{-5}{3} = \frac{5}{-3} = -\frac{5}{3}$$

Investigation #1 Investigating the Slope of Line Segments

a) Complete the chart. Write the slopes in simplest form.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	2	3	$\frac{2}{3}$
AC	8	12	$\frac{8}{12} = \frac{2}{3}$
AD	10	15	$\frac{10}{15} = \frac{2}{3}$
BC	6	9	$\frac{6}{9} = \frac{2}{3}$



b) How are the slopes of the line segments related?

the slope is consistent

any two parts on a segment will give the same slope.

Slope of a Line

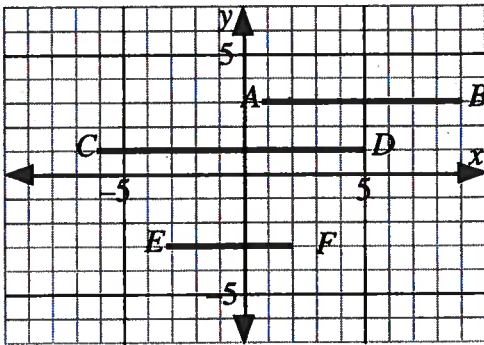
The slopes of all line segments on a line are equal.
The slope of a line representing the graph of a linear relation can be found using

$$\text{slope} = \frac{\text{rise}}{\text{run}} \text{ for any two points on the line.}$$

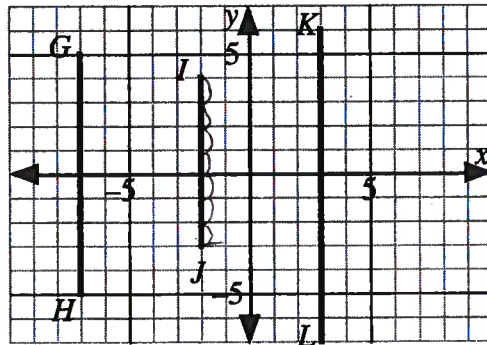
Investigation #2 Slopes of Horizontal and Vertical Line Segments

Consider the line segments in Grid 1 and Grid 2 below.

Grid 1



Grid 2



a) Determine the slopes of all the line segments in Grid 1.

$$\frac{0}{8} = 0$$

b) Determine the slopes of all the line segments in Grid 2.

$\frac{7}{0}$ not possible

c) Complete the following statements.

- Horizontal line segments have a slope of 0.
- Vertical line segments have an undefined slope.

Investigation #3

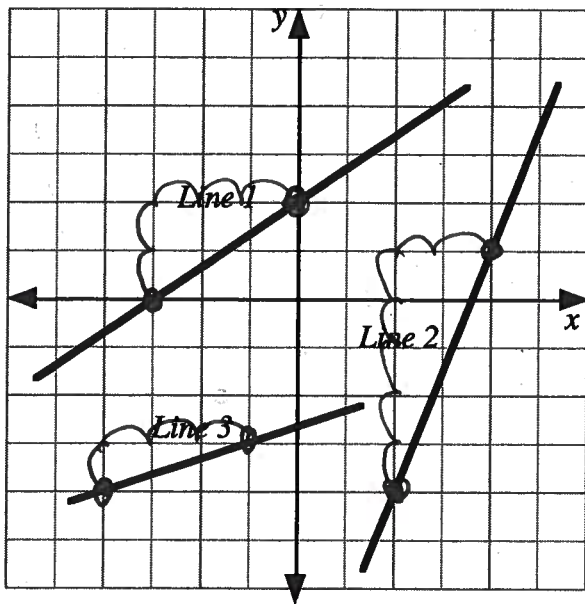
Positive and Negative Slopes

- a) Each line on the grids passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

Remember on a Cartesian Plane

- the rise is POSITIVE if we count UP, and NEGATIVE if we count DOWN
- the run is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT

Grid 1



Grid 2

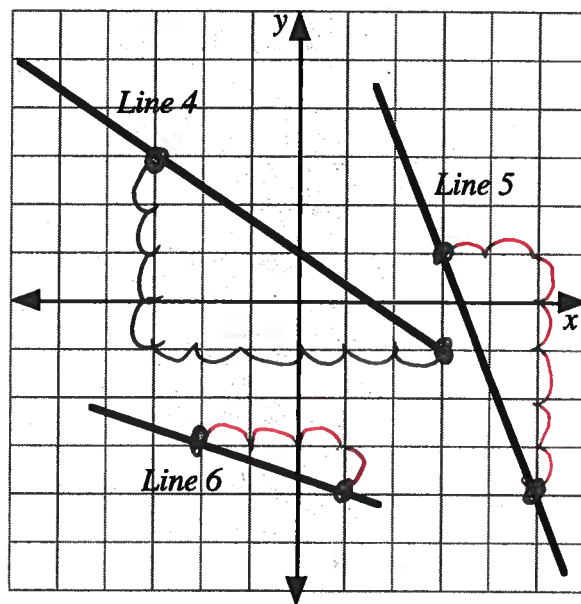


Table For Grid 1

Line	Slope
1	$\frac{2}{3}$
2	$\frac{5}{2}$
3	$\frac{1}{3}$

Table For Grid 2

Line	Slope
4	$\frac{-4}{6} = -\frac{2}{3}$
5	$-\frac{5}{2}$
6	$-\frac{1}{3}$

- b) Compare the slopes of:

- Line 1 and Line 4
- Line 2 and Line 5
- Line 3 and Line 6

Slopes are opposite signs, same steepness

- c) Complete the following statements.

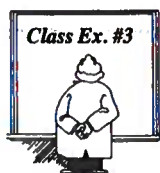
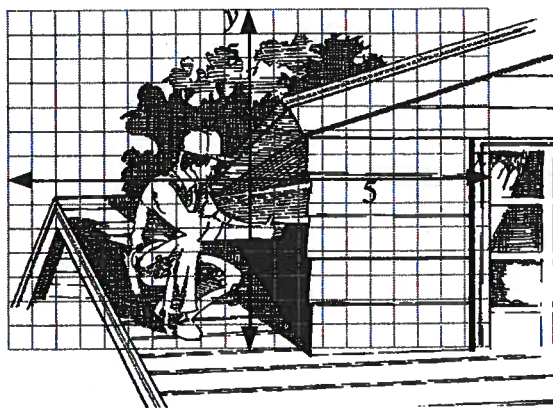
- A line which rises ⁺ from left to right has a positive slope.
- A line which falls ⁻ from left to right has a negative slope.

Complete Assignment Questions #1 and #2



A grid has been superimposed on the sketch.

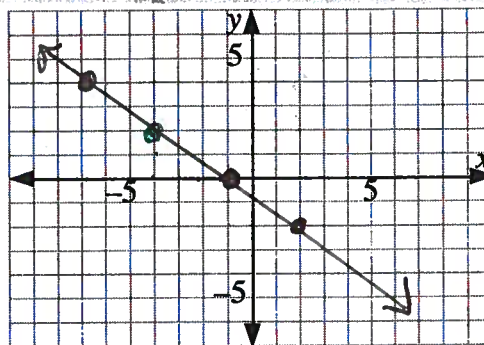
- a) Estimate the pitch (slope) of the roof to the right of the worker's head.
- b) Could the grid be used to estimate the pitch of the roof the worker is standing on? Explain.



Draw a line segment on the grid which passes through the point $(-4, 2)$ and has a slope of $-\frac{2}{3}$. The line segment must be long enough to cross both the x -axis and the y -axis.

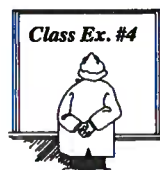
Write the coordinates of three other points on the line segment which have integer coordinates.

$$\begin{array}{ccc} -2 & \downarrow 2 & \uparrow 2 \\ \hline 3 & \rightarrow 3 & \leftarrow 3 \end{array}$$



rise
run

$$(-7, 4), (-1, 0), (2, -2)$$



A line segment has a slope of $-\frac{5}{7}$ and a rise of 12. Calculate the run as an exact value.

$$-\frac{5}{7} \text{ or } \frac{5 \text{ rise}}{-7 \text{ run}}$$

$$\frac{-5}{7} = \frac{12}{x}$$

Solve for x

$$x = \frac{12 \cdot 7}{-5}$$

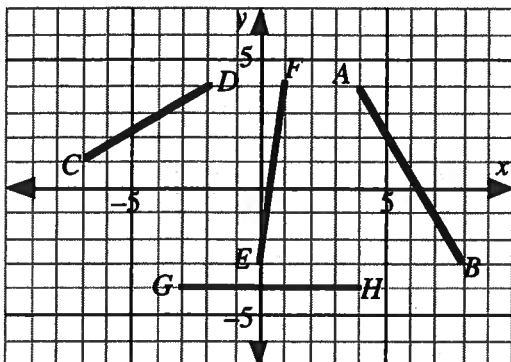
$$= \frac{84}{-5} \text{ run}$$

$$\frac{7}{-5} = \frac{x}{12}$$

Complete Assignment Questions #3 - #13

Assignment

1. Each line segment on the grid has endpoints with integer coordinates. Complete the table.



Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB			
CD			
EF			
GH			

2. Every line on the grid passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

slope of Line 1 :

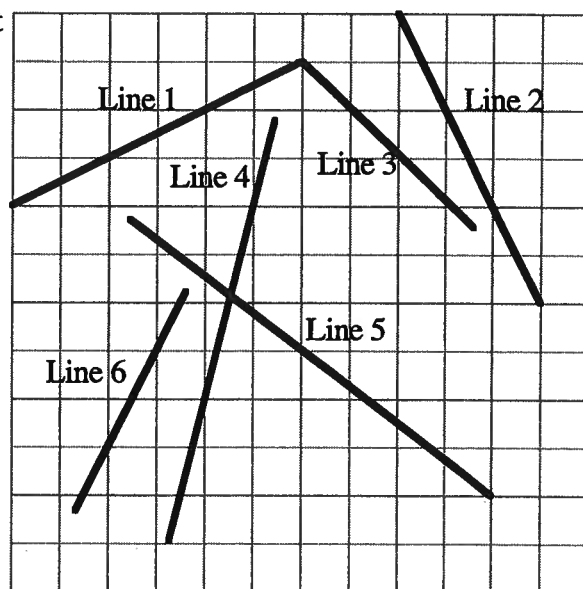
slope of Line 2:

slope of Line 3:

slope of Line 4:

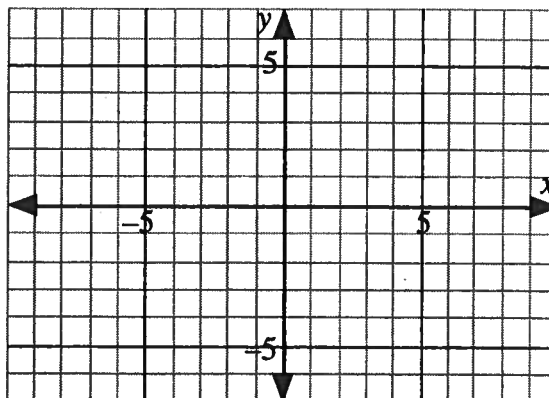
slope of Line 5:

slope of Line 6:



3. Draw a line segment on the grid which passes through the point $(-5, -2)$ and has a slope of $\frac{2}{3}$. The line segment must be long enough to cross both the x -axis and the y -axis.

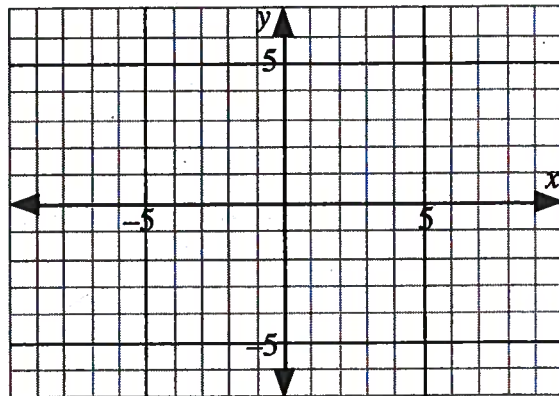
Write the coordinates of three other points on the line segment which have integer coordinates.



4. Repeat question #3 for line segments with the given slope passing through the given point.

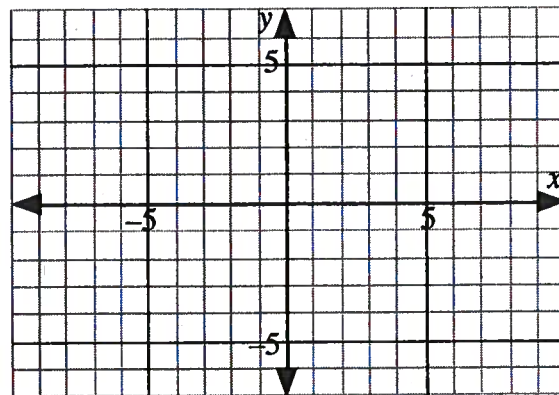
a) slope = $\frac{2}{5}$, (2, 1)

b) slope = $-\frac{1}{3}$, (6, -3)



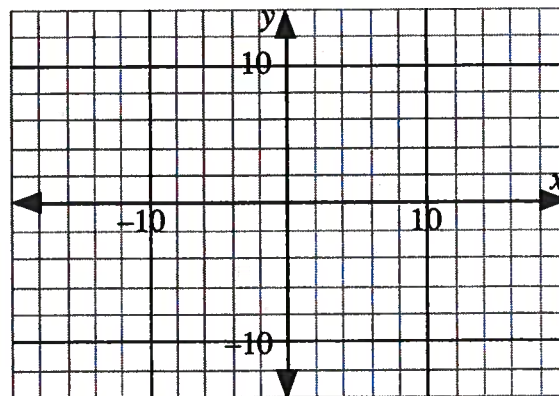
c) slope = $-\frac{4}{3}$, (-9, 6)

d) slope = 4, (0, -7)



e) slope = -2, (4, -12)

f) slope = 0, (0, 6)



5. P has coordinates $(-1, 2)$. Find two positions for point Q so that the slope of PQ is

a) 2

b) -3

c) $\frac{1}{3}$

d) $-\frac{2}{5}$

e) 0

f) undefined

6. Two of three measures are given for rise, run, and slope.

Calculate the value of the third measure in each of the following.

a) slope = $\frac{5}{7}$ and run = 49

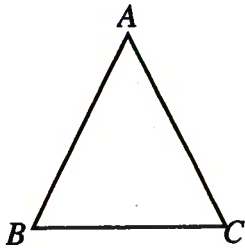
b) slope = $-\frac{3}{8}$ and rise = 15

c) slope = $-\frac{6}{11}$ and run = 33

d) slope = $\frac{3}{4}$ and rise = 15

7. A ramp which has been set up by skateboarders has a slope of $\frac{2}{3}$. Calculate the height of the ramp if it has a base length of 1.5 metres.

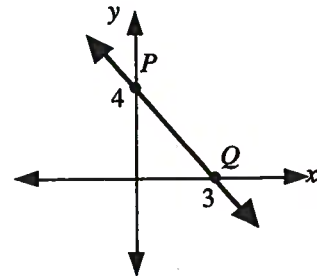
8. Triangle ABC is isosceles with $AB = AC$ and $BC = 6.8$ cm. Calculate the area of the triangle if the slope of $AC = -\frac{5}{4}$.



Multiple
Choice

9. The slope of \overline{PQ} is

- A. $\frac{3}{4}$
 B. $-\frac{3}{4}$
 C. $\frac{4}{3}$
 D. $-\frac{4}{3}$

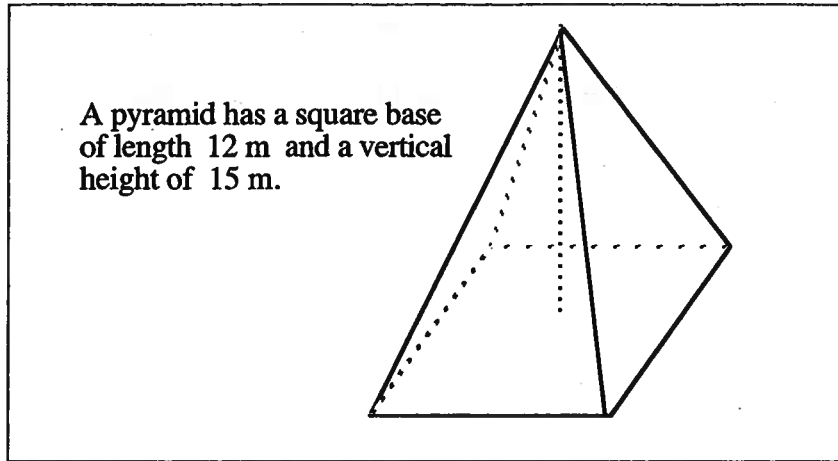


10. The point $(-4, 0)$ is on a line which has a slope of $-\frac{2}{5}$. The next point with integer coordinates on the line to the right of $(-4, 0)$ is
- A. $(-9, -2)$
 B. $(-9, 2)$
 C. $(1, -2)$
 D. $(-2, -5)$
11. P is a point in quadrant I, Q is a point in quadrant II, R is a point in quadrant III, and S is a point in quadrant IV.

Which one of the following statements must be true?

- A. Line segment PQ has a positive slope.
 B. Line segment QR has a positive slope.
 C. Line segment PR has a positive slope.
 D. Line segment QS has a positive slope.

Use the following information to answer questions #12 and #13.



Numerical
Response

12. A beetle starts to climb the pyramid starting from the midpoint of one of the faces. To the nearest tenth, the slope of the beetle's climb is _____.

(Record your answer in the numerical response box from left to right)

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13. A fly starts to climb the pyramid along one of the edges. To the nearest tenth, the slope of the fly's climb is _____.

(Record your answer in the numerical response box from left to right)

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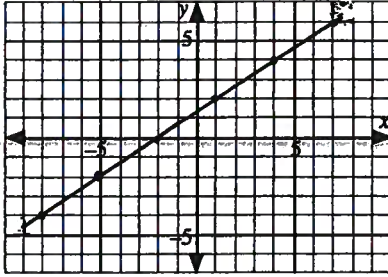
Answer Key

1. 2. slope of line 1 = $\frac{1}{2}$, slope of line 2 = -2, slope of line 3 = -1

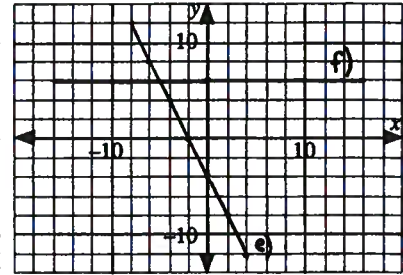
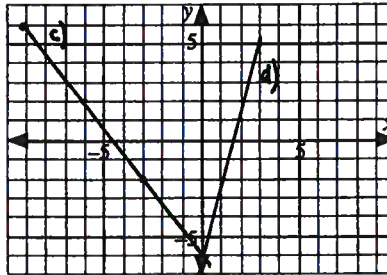
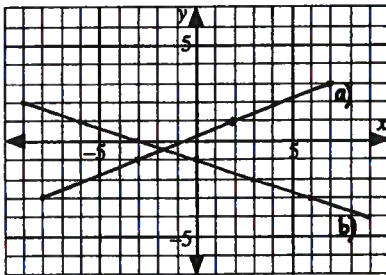
Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	-7	4	$-\frac{7}{4}$
CD	3	5	$\frac{3}{5}$
EF	7	1	$\frac{7}{1} = 7$
GH	0	7	$\frac{0}{7} = 0$

- slope of line 4 = 4, slope of line 5 = $-\frac{3}{4}$, slope of line 6 = 2

3. Any three of (-8, -4), (-2, 0), (1, 2), (4, 4)



- 4.



- a) (-8, -3), (-3, -1), (7, 3) c) (-6, 2), (-3, -2), (0, -6) e) Many possible answers including (2, -8), (0, -4), (-2, 0)
 b) Any 3 of (-9, 2), (-6, 1), (-3, 0), (0, -1), (3, -2), (9, -4) d) (1, -3), (2, 1), (3, 5) f) Many possible answers including (1, 6), (2, 6), (3, 6)

5. Many possible answers, including any two from:

- a) (-3, -2), (-2, 0), (0, 4), (1, 6) b) (-3, 8), (-2, 5), (0, -1), (1, -4),
 c) (2, 3), (5, 4), (-4, 1), (-7, 0) d) (-11, 6), (-6, 4), (4, 0), (9, -2)
 e) (-3, 2), (-2, 2), (0, 2), (1, 2) f) (-1, 1), (-1, 0), (-1, -1), (-1, 3)

6. a) rise = 35 b) run = -40 c) rise = -18 d) run = 20

7. 1 metre 8. 14.45 cm² 9. D 10. C 11. C

12.

2	.	5	
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13.

1	.	8	
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