Equations of Linear Relations Lesson #2:
Writing Equations Using $y = mx + b$

Review

We have learned that the graph of an equation in the form $y = mx + b$ is a straight line with slope m and y-intercept b.

Using the Form $y = mx + b$ to Write the Equation of a Line

The form $y = mx + b$ can be used to determine the equation of a line when the following information is given:

- the slope of the line
- the y-intercept of the line.

Class Ex. #1

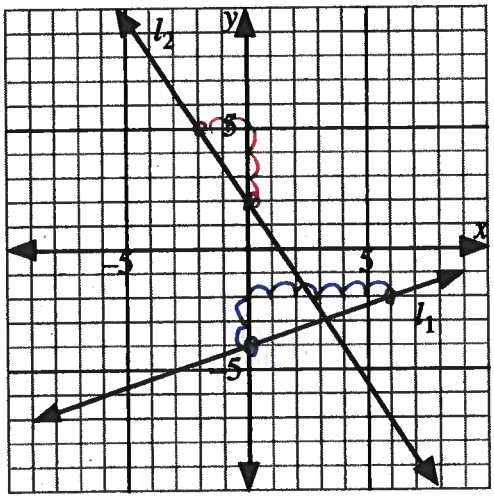
Write the equation of a line passing through the point $(0, 2)$ with slope $\frac{5}{2}$.

y-int

$y = \frac{5}{2}x + 2$

Class Ex. #2

Each line on the grid passes through points with integer coordinates. In each case, state the slope and y-intercept of the line, and determine the equation of the line.



l_1: $b = y\text{-int} = -4$

$m = \frac{rise}{run} = \frac{2}{6} = \frac{1}{3}$

$y = \frac{1}{3}x - 4$

l_2: $b = 2$

$m = \frac{3}{-2}$

$y = -\frac{3}{2}x + 2$

Class Ex. #3

Determine the equation of the following lines.

a) The line parallel to $y = \frac{1}{3}x + 4$, and with the same y-intercept as $y = 6x - 7$.

$y = \frac{1}{3}x - 7$

b) The line passing through $(0, 9)$ and perpendicular to the line joining $(2, -6)$ and $(-5, 0)$.

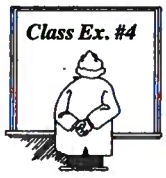
y-int

neg. reciprical

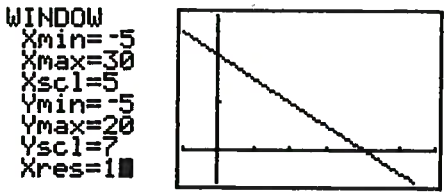
① calculate slope $\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{0 - -6}{-5 - 2} = \frac{6}{-7}$ → $\frac{7}{6}$ new slope

$y = \frac{7}{6}x + 9$



The diagram shows the display from a graphing calculator screen. The intercepts are integers. Determine the equation of the line shown.



Complete Assignment Questions #1 - #5

Horizontal and Vertical Lines

a) State the slope and y-intercept of the horizontal line L_1 shown on the grid.

b=4 m=0

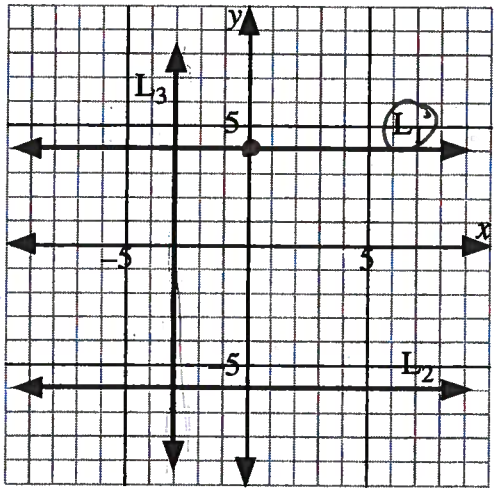
b) Use the form $y = mx + b$ to determine the equation of the horizontal line L_1.

y=0x+4

y=4

c) Predict the equation of the horizontal line L_2. Use a graphing calculator to verify.

y=-6 ✓



d) State the slope and y-intercept of the vertical line L_3 shown on the grid.

b=none m=undefined

e) Why can we not use the form $y = mx + b$ to determine the equation of the vertical line L_3?

x=-3

f) Predict the equation of the vertical line L_3.
Why can we not use a graphing calculator to verify?

The equation $y = k$ represents a horizontal line through $(0, k)$.
The equation $x = k$ represents a vertical line through $(k, 0)$.

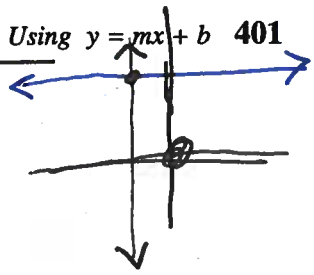
Determine the equation of the line through the point $(-2, 8)$ and

a) parallel to the y-axis

$x = -2$

b) parallel to the x-axis

$y = 8$



Complete Assignment Questions #6 - #15

Assignment

1. Write the equation of each line

a) with slope 4 and y-intercept -6

b) with a y-intercept of 3 and a slope of $-\frac{4}{3}$

c) passing through the origin with a slope of $-\frac{3}{5}$

$(0,0)$ $b=0$

$y = -\frac{3}{5}x$

d) with y-intercept -5 and parallel to $y = x$

e) with a y-intercept of -9 and perpendicular to $y = -\frac{2}{3}x + 7$

f) with the same y-intercept as $y = x + 2$ and parallel to $y = \frac{1}{4}x - 6$

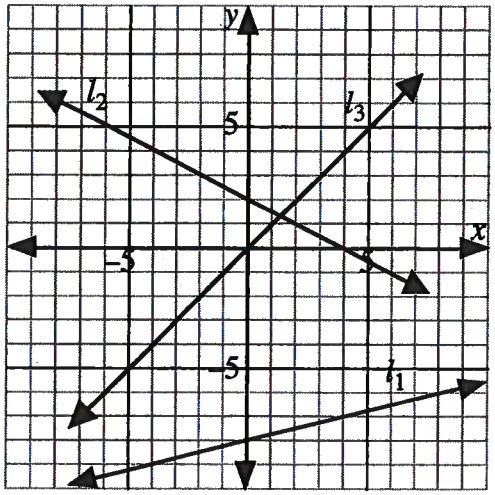
g) through the point $(0, 1)$ and perpendicular to $y = 4x - 2$

h) through the point $(0, 4)$ and parallel to $y = \frac{1}{10}x + 24$

i) with the same y-intercept as $y = 2x - 3$ and perpendicular to $y = \frac{7}{3}x - 2$

j) with the same y-intercept as $y = ax + b$ and perpendicular to $y = cx + d$

2. Every line on the grid passes through points with integer coordinates.
Determine the equation of each line.

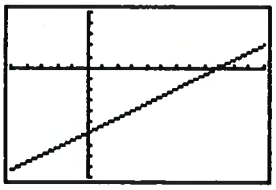


3. Each diagram represents the image from the display of a graphing calculator and the window setting used to graph a linear equation. The *x*- and *y*-intercepts of each graph are integers.

In each case complete the table.

a)

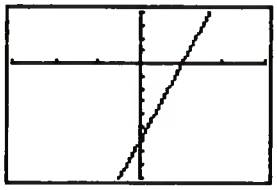
```
WINDOW
Xmin=-5
Xmax=12
Xscl=1
Ymin=-10
Ymax=5
Yscl=1
Xres=1
```



x-intercept	
y-intercept	
slope	
equation	

b)

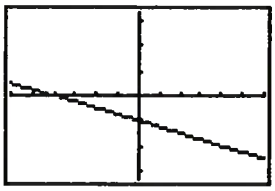
```
WINDOW
Xmin=-6
Xmax=6
Xscl=2
Ymin=-36
Ymax=16
Yscl=4
Xres=1
```



x-intercept	
y-intercept	
slope	
equation	

c)

```
WINDOW
Xmin=-6
Xmax=6
Xscl=1
Ymin=-10
Ymax=10
Yscl=3
Xres=1
```



x-intercept	
y-intercept	
slope	
equation	

4. Determine the equation of the line which passes through the point $(0, 16)$ and is parallel to the line which passes through $(1, 3)$ and $(4, -6)$.

5. Determine the equation of the line which passes through the point $(0, -1)$ and is perpendicular to the line which passes through $(7, -2)$ and $(12, -3)$.

6. State the equations of the following lines

a) through the point $(-5, 3)$ and parallel to the y-axis

b) through the point $(-5, 3)$ and parallel to the x-axis

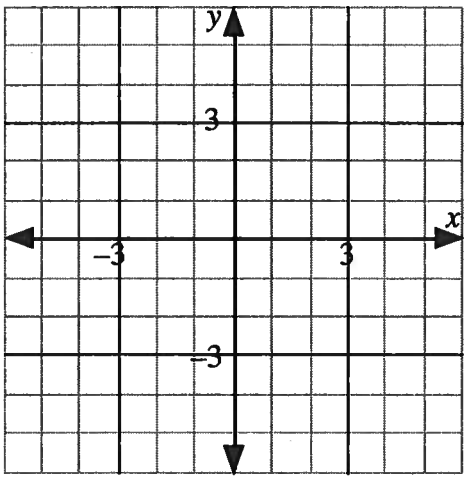
c) through the point $(1, -1)$ and parallel to the x-axis

d) through the point (a, b) and parallel to the y-axis

7. Consider the graph of the function with equation $y = 2$.

a) State the values of m and b.

b) Sketch the graph on the grid provided.



c) State the x- and y-intercepts of the graph.

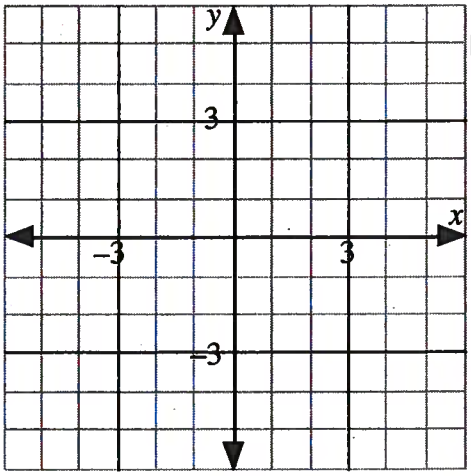
d) Determine the domain and range of the function.

e) On the same grid draw the line with equation $y = 2x - 4$ without using a graphing calculator.

f) State the coordinates of the point of intersection of the two lines.

g) On the grid, draw the line with equation $y = -5$.

8. Consider the graph of the relation with equation $x = -4$.



a) Sketch the graph on the grid provided.

b) State the x- and y-intercepts of the graph.

c) Explain why the relation is not a function.

d) Explain why the [Y=] editor key cannot be used to graph $x = -4$.

e) Determine the domain and range of the relation with equation $x = -4$.

f) On the grid draw the line with equation $x = 2$.

9. Write the equation of each line

a) parallel to the x-axis through $(3, -9)$

b) parallel to the y-axis through $(3, -9)$

c) perpendicular to the x-axis through $(1, 4)$

d) perpendicular to the y-axis through $(1, 4)$

e) the x-axis

f) the y-axis

Multiple Choice

10. A line is parallel to the y-axis and passes through the point $(2, -7)$. The equation of the line is

A. $x = 2$
B. $x = -7$
C. $y = 2$
D. $y = -7$

11. A line is parallel to the x-axis and passes through the point $(-6, 10)$. The equation of the line is

A. $x = 10$
B. $x = -6$
C. $y = 10$
D. $y = -6$

12. The line through the origin, perpendicular to the line with equation $y = \frac{2}{3}x$, has equation

A. $y = \frac{2}{3}x$

B. $y = \frac{3}{2}x$

C. $y = -\frac{2}{3}x$

D. $y = -\frac{3}{2}x$

13. The point $(2, -1)$ lies on a line with slope 3. The y- intercept of the line is

A. -7
B. -5
C. 5
D. 7

Numerical Response

14. Consider the line which is perpendicular to the line $y = \frac{1}{3}x + 4$ and has the same y-intercept as $y = 6x - 7$. If the equation of this line is written in the form $y = mx + b$, then the exact value of $m - b$ is _____.

(Record your answer in the numerical response box from left to right)

15. Two perpendicular lines intersect on the y-axis. One line has equation $y = 4x + 6$. If the equation of the other line is $y = mx + b$, then the exact value of $m + b$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y = 4x - 6$ **b)** $y = -\frac{4}{3}x + 3$ **c)** $y = -\frac{3}{5}x$ **d)** $y = x - 5$ **e)** $y = \frac{3}{2}x - 9$
f) $y = \frac{1}{4}x + 2$ **g)** $y = -\frac{1}{4}x + 1$ **h)** $y = \frac{1}{10}x + 4$ **i)** $y = -\frac{3}{7}x - 3$ **j)** $y = -\frac{1}{c}x + b$

2. l_1: $y = \frac{1}{4}x - 8$ l_2: $y = -\frac{1}{2}x + 2$ l_3: $y = x$

3. **a)**

x-intercept	9
y-intercept	−6
slope	$\frac{2}{3}$
equation	$y = \frac{2}{3}x - 6$

b)

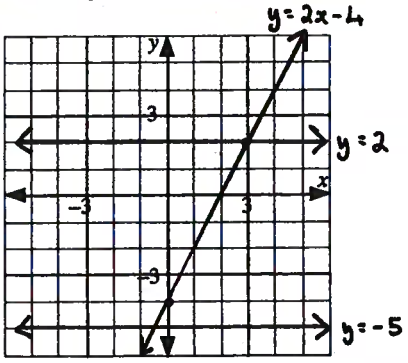
x-intercept	2
y-intercept	−24
slope	12
equation	$y = 12x - 24$

c)

x-intercept	−4
y-intercept	−3
slope	$-\frac{3}{4}$
equation	$y = -\frac{3}{4}x - 3$

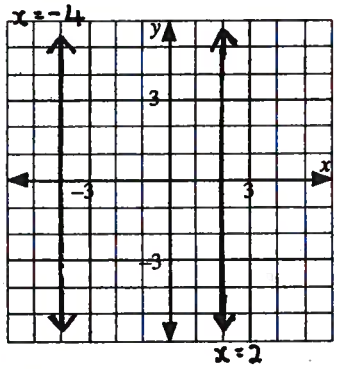
4. $y = -3x + 16$ **5.** $y = 5x - 1$ **6.** **a)** $x = -5$ **b)** $y = 3$ **c)** $y = -1$ **d)** $x = a$

7. **a)** $m = 0, b = 2$
b), e), g) see graph below



c) no *x*-intercept, *y*-intercept = 2
d) domain $x \in R$, range $y = 2$ **f)** (3, 2)

8. **a), f)** see graph below
b) *x*-intercept = −4, no *y*-intercept



c) When the input = −4, there are multiple values for the output. The graph of the relation does not pass the vertical line test.
d) The equation $x = -4$ cannot be written in the form y= . . . .
e) domain $x = -4$, range $y \in R$

9. **a)** $y = -9$ **b)** $x = 3$ **c)** $x = 1$ **d)** $y = 4$ **e)** $y = 0$ **f)** $x = 0$

10. A **11.** C **12.** D **13.** A

14.

4			

15.

5	.	7	5

Equations of Linear Relations Lesson #3: Writing Equations using $y - y_1 = m(x - x_1)$

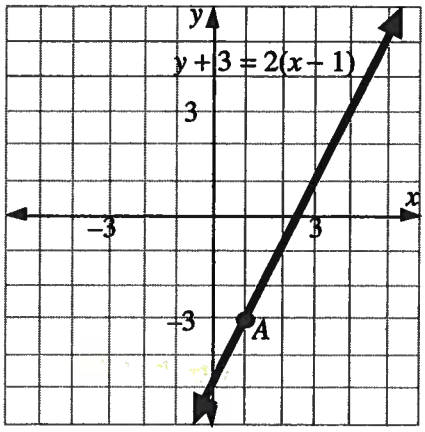
Review

In the last lesson we learned how to write the equation of a straight line using slope y-intercept form, namely $y = mx + b$ where m is the slope and b is the y-intercept.

There is another equally important method exists in writing the equation of straight lines. The investigation below explores this method.

Investigation *Point-Slope Form*

The graph of $y + 3 = 2(x - 1)$ is shown on the grid.

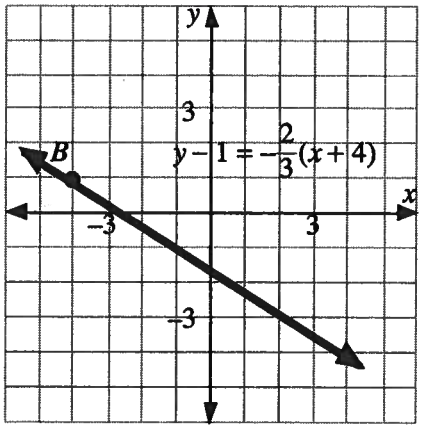


a) Determine the slope of the graph of $y + 3 = 2(x - 1)$.

b) List the coordinates of point A on the line.

c) Compare your answers in **a)** and **b)** with the numbers in the equation.

The graph of $y - 1 = -\frac{2}{3}(x + 4)$ is shown on the grid.



d) Determine the slope of the graph of $y - 1 = -\frac{2}{3}(x + 4)$.

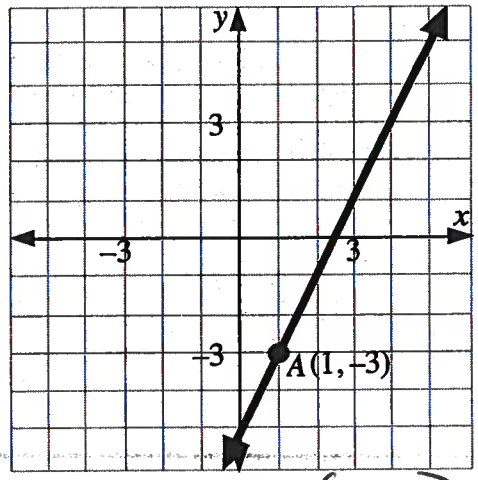
e) List the coordinates of point B on the line.

f) Compare your answers in **d)** and **e)** with the numbers in the equation.

g) Consider the graph of the linear equation $y - y_1 = m(x - x_1)$. Based on your observations in **c)** and **f)**, state the slope of the line, and write the coordinates of one point on the line.

Equation of a Line Given the Slope of the Line and a Point on the Line

Consider the line with slope 2 passing through the point $A(1, -3)$. The line is shown on the grid.



Our objective is to determine the equation of the line. In other words, to find a relation between x and y which is satisfied by every point (x, y) on the line.

Let $P(x, y)$ be any point on the line except A.

Using the slope formula we have

$$\frac{y_P - y_A}{x_P - x_A} = m_{AP} \qquad \frac{y-(-3)}{x-1} = 2$$

$y+3=2(x-1)$

Cross multiply and solve for y to determine the equation of the line in the form $y = mx + b$.

$y+3=2(x-1)$ $\quad y=2x-1-3$ $\quad y=2x-4$

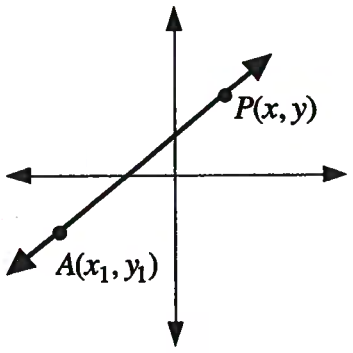
At this point in the exploration, the equation above is valid for all points on the line except A.

Note that the coordinates of A also satisfy the equation, and therefore it is the equation of all points on the line.

In the next section we will use the same procedure to develop a formula for the equation of any line, given the slope of the line and the point on the line.

The Equation of the Line with slope m through the point (x_1, y_1)

Consider the line with slope m passing through the point with coordinates (x_1, y_1).
We will use the same procedure as above to show that the equation of the line can be expressed in the form $y - y_1 = m(x - x_1)$.



Let $P(x, y)$ be any point on the line distinct from A.

Using the slope formula we have

$$m_{AP} = \frac{y_P - y_A}{x_P - x_A} \quad \text{so} \quad m = \frac{y -}{x -}$$

Point-Slope Equation of a Line → $y - y_1 = m(x - x_1)$

- The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$ where m is the slope of the line, and (x_1, y_1) represents a point on the line.
- To determine the equation of a line in future math courses, the point-slope equation, $y - y_1 = m(x - x_1)$, is used more frequently than the slope-y-intercept equation, $y = mx + b$.

- The point-slope equation is used when we have **the slope of a line and the coordinates of any point on that line.**
- It is customary to give the final equation in slope y-intercept form or in the general form, $Ax + By + C = 0$ (to be taught in the next lesson).

State the equation, in point-slope form, of the line through the given point and with the given slope.

a) (6, 5), 3 (point, slope)

$y - 5 = 3(x - 6)$

b) (1, –1), –4 (x_1, y_1, m)

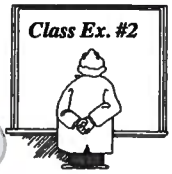
$y - (-1) = -4(x - 1)$

$y + 1 = -4(x - 1)$

c) (–9, –8), $\frac{1}{2}$ (x_1, y_1, m)

$y - -8 = \frac{1}{2}(x - -9)$

$y + 8 = \frac{1}{2}(x + 9)$



In each case the slope of a line and a point on the line are given. Determine the equation of the line in slope y-intercept form, $y = mx + b$.

a) $m = 5$, point (–5, 2)

$y - 2 = 5(x - -5)$

$y - 2 = 5(x + 5)$

$y - 2 = 5x + 25$ (+2)

$y = 5x + 27$

b) $m = -7$, point (–3, 4)

$y - 4 = -7(x + 3)$ (–3)

$y - 4 = -7x - 21$ (+4)

$y = -7x - 17$

John and Nicki were solving the following quiz question:

" Determine the equation, in slope y-intercept form, of a line with slope –2 passing through the point (3, –5)".

John could only remember the slope y-intercept form $y = mx + b$, but Nicki remembered the point-slope form $y - y_1 = m(x - x_1)$. Complete their work which is started below.

<u>John's work</u>

$y = mx + b$

$y = -2x + b$

$-5 = -2(3) + b$

$-5 = -6 + b$

$+6 \quad +6$

$1 = b \rightarrow y = -2x + 1$

<u>Nicki's work</u>

$y - y_1 = m(x - x_1)$

$y - y_1 = -2(x - x_1)$

$y - (-5) = -2(x - 3)$

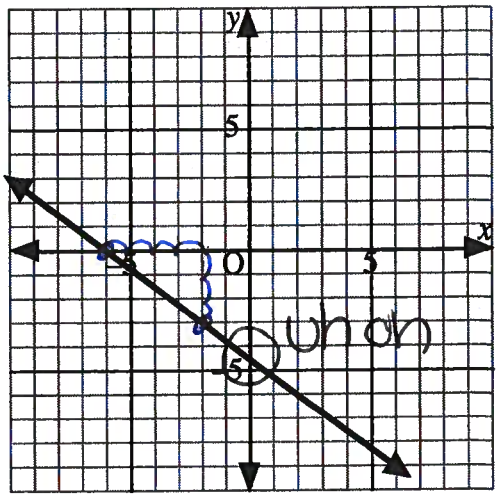
$y + 5 = -2x + 6$

$-5 \quad -5$

$y = -2x + 1$

Class Ex. #4

The line on the grid passes through at least two points with integer coordinates. Determine the equation of the line in slope y-intercept form.



(-2,-3) ↑3 ←-4

$y - y_1 = m(x - x_1)$ (-(-3)) (-(-2))

$y + 3 = \frac{-3}{4}(x + 2)$

Class Ex. #5

In each case, state the slope of the line and write the coordinates of a point on the line.

a) $y + 11 = \frac{1}{7}(x - 4)$ y-(-11) x-(4) slope: $\frac{1}{7}$ (4,-11)

b) $y - 9 = -\frac{5}{3}(x - 7)$ slope: $\frac{-5}{3}$ (7,9)

c) $y = -3(x - 6)$ slope: -3 (6,0)

d) $y = -3x - 6$ slope: -3 (0,-6)

Complete Assignment Questions #1 - #9

Vincent

$y = -3(x + 2)$ (-2,0)

Assignment

1. State the equation, in point-slope form, of the line through the given point and with the given slope.

a) (9,3), 4 **b)** (8,–2), –3 **c)** (–5,7), 1

d) (0,3), $\frac{1}{2}$ **e)** (–7,0), $\frac{1}{4}$ **f)** $(-\frac{1}{2}, -\frac{5}{4})$, $\frac{6}{5}$

2. Write the following equations in slope y-intercept form $y = mx + b$.

a) $y + 1 = 8(x - 2)$ **b)** $y - 3 = -2(x - 7)$ **c)** $y - 9 = -11(x + 3)$

3. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,4)$, 6 **b)** $(2,-1)$, 2 **c)** $(0,4)$, -2

d) $(-6,2)$, $\frac{1}{2}$ **e)** $(-7,-7)$, 1 **f)** $(0,b)$, m

4. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,-5)$, $\frac{1}{4}$ **b)** $(-4,2)$, $-\frac{1}{3}$ **c)** $(0,-8)$, $-\frac{3}{4}$

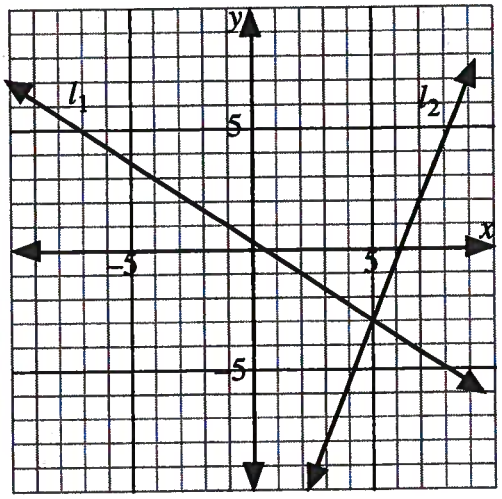
5. The point-slope equation of a line is given. State the slope and the coordinates of the point which were used to write the equation.

a) $y-9=-\frac{11}{3}(x+3)$ **b)** $y+3=\frac{1}{2}x$ **c)** $y-8=-2(x-6)$

d) $y=3(x+12)$ **e)** $y-9=-\frac{5}{3}x$ **f)** $y=\frac{2}{5}x$

6. Two lines have been drawn on the grid. Each line passes through at least two points with integer coordinates.

Determine the equation of each line.



Multiple Choice

7. The equation of the line passing through the point $(4, 2)$ with slope -3 is

A. $y = -3x - 14$
B. $y = -3x - 10$
C. $y + 2 = -3(x + 4)$
D. $y - 2 = -3(x - 4)$

8. Which of the following linear equations is equivalent to $y - 3 = -\frac{3}{4}(x + 7)$?

A. $y = -\frac{3}{4}x + \frac{9}{4}$

B. $y = -\frac{3}{4}x - \frac{9}{4}$

C. $y = -\frac{3}{4}x + 10$

D. $y = -\frac{3}{4}x - 10$

Numerical Response

9. The equation of the line with an x-intercept of -2 and slope 12 can be written in the form $y - A = C(x - B)$. The value of $A + B + C$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y - 3 = 4(x - 9)$ **b)** $y + 2 = -3(x - 8)$ **c)** $y - 7 = 1(x + 5)$
d) $y - 3 = \frac{1}{2}x$ **e)** $y = \frac{1}{4}(x + 7)$ **f)** $y + \frac{5}{4} = \frac{6}{5}\left(x + \frac{1}{2}\right)$

2. **a)** $y = 8x - 17$ **b)** $y = -2x + 17$ **c)** $y = -11x - 24$

3. **a)** $y = 6x - 8$ **b)** $y = 2x - 5$ **c)** $y = -2x + 4$ **d)** $y = \frac{1}{2}x + 5$ **e)** $y = x$ **f)** $y = mx + b$

4. **a)** $y = \frac{1}{4}x - \frac{11}{2}$ **b)** $y = -\frac{1}{3}x + \frac{2}{3}$ **c)** $y = -\frac{3}{4}x - 8$

5. **a)** $m = -\frac{11}{3}$, $P(-3, 9)$ **b)** $m = \frac{1}{2}$, $P(0, -3)$ **c)** $m = -2$, $P(6, 8)$ **d)** $m = 3$, $P(-12, 0)$
e) $m = -\frac{5}{3}$, $P(0, 9)$ **f)** $m = \frac{2}{5}$, $P(0, 0)$

6. $l_1 \Rightarrow 2x + 3y - 1 = 0$ or $y = -\frac{2}{3}x + \frac{1}{3}$ $l_2 \Rightarrow 5x - 2y - 31 = 0$ or $y = \frac{5}{2}x - \frac{31}{2}$

7. D **8.** B **9.**

1	0		

Equations of Linear Relations Lesson #4:
The General Form Equation $Ax + By + C = 0$

Review

In Lesson #2 and Lesson #3 of this unit we studied two forms of the equation of a straight line. The form of these equations are

- the slope y-intercept form which can also be writen as $y = mx + b$
- the point-slope form which can also be written as $y - y_1 = m(x - x_1)$

General Form of the Equation of a Line → $Ax + By + C = 0$

The **general form** of the equation of a line is an equation where all the terms are collected to the left side of the equation. The right side of the equation is zero.
It has the following characteristics:

- It is written as $Ax + By + C = 0$, where A, B, and C are expressed as **integers** if possible, and A is usually positive.
- It allows us to write equations for oblique lines, horizontal lines, and vertical lines.
- In some texts, the form $Ax + By + C = 0$ is referred to as **standard form**.

Convert the following equations from slope y-intercept form, $y = mx + b$, to general form, $Ax + By + C = 0$, where A, B, and C are integers.

a) $y + 2 = \frac{1}{2}(x - 3)$

$y + 2 = \frac{1}{2}x - \frac{3}{2}$

$0 = \frac{1}{2}x - \frac{3}{2} - y - 2$

$0 = x - 3 - 2y - 4$

$0 = x - 2y - 7$

b) $y = \frac{2}{3}x + 7$

$3y = 2x + 21$

$0 = 2x - 3y + 21$

c) $y = -\frac{1}{4}x + \frac{3}{5}$

$20y = -5x + 12$

$5x + 20y - 12 = 0$

$y = \frac{-20}{4}x + \frac{60}{5}$

Determine the equation, in general form, of the line with slope $-\frac{3}{7}$ passing through $(2, -1)$.

$y - y_1 = m(x - x_1)$

① $y + 1 = \frac{-3}{7}(x - 2)$

$7y + 7 = -3(x - 2)$

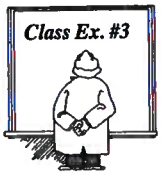
$7y + 7 = -3x + 6$

$7y + 7 + 3x - 6 = 0$

$3x + 7y + 1 = 0$

Determining the Slope and y-intercept from Ax + By + C = 0

Given the equation of a line in general form, $Ax + By + C = 0$, the slope and y-intercept can be found by converting the equation into slope y-intercept form, $y = mx + b$.



Determine the slope and y-intercept of the graph of the following lines.

a) $2x - 5y + 25 = 0$

$y=$ solve for y.

$-5y = -2x - 25$

$\frac{-5y}{-5} = \frac{-2x}{-5} - \frac{25}{-5}$

$y = \frac{2}{5}x + 5$

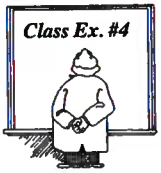
b) $6x + 2y - 15 = 0$

$2y = -6x + 15$

$y = -3x + \frac{15}{2}$

$y = \frac{5}{-3}x + 2$

Complete Assignment Questions #1 - #4



The lines ① $3x - 4y + 8 = 0$ and ② $5x - ky - 6 = 0$ have the same y-intercept. Determine the value of k.

① $3x - 4y + 8 = 0$

$-4y = -3x - 8$

$\frac{-4y}{-4} = \frac{-3x}{-4} - \frac{8}{-4}$

$y = \frac{3}{4}x + 2$ ← y-int

② $5x - ky - 6 = 0$

$-ky = -5x + 6$

$\frac{-ky}{-k} = \frac{-5x}{-k} + \frac{6}{-k}$

$y = \frac{5}{k}x - \frac{6}{k}$ ← y-int

$2 = -\frac{6}{k}$ *solve for k

$\frac{2k}{2} = \frac{-6}{2}$

$k = -3$

Which of the following lines is/are perpendicular to the line $4x - 2y + 9 = 0$?

i) $6x + 3y - 1 = 0$

$3y = -6x + 1$

$y = -2x + \frac{1}{3}$

slope: -2

ii) $x + 2y - 12 = 0$

$2y = -x + 12$

$y = -\frac{1}{2}x + 6$

slope: $-\frac{1}{2}$

iii) $5x + 10y = 0$

$10y = -5x$

$y = -\frac{1}{2}x$

slope: $-\frac{1}{2}$

parallel

Given line $4x - 2y + 9 = 0$

$-2y = -4x - 9$

$y = 2x + \frac{9}{2}$

slope: 2

perpendicular

$-\frac{1}{2}$ neg. recip. 2

Use the following information to answer Class Ex. #6.

A student made the following statements about the line with equation $2y = 5x + 12$.

Statement 1: The line has a slope of 5. FALSE

Statement 2: The line is parallel to $10x - 4y + 13 = 0$. TRUE $\frac{10}{4} = \frac{5}{2}$

Statement 3: The line passes through $(-2, 1)$. TRUE

Which of the above statement(s) is/are true?

A. 1 and 2 only

B. 1 and 3 only

C. 2 and 3 only

D. some other combination of statements 1, 2, and 3

② parallel → same slope

$10x - 4y + 13 = 0$

$-4y = -10x - 13$

$y = \frac{10}{4}x + \frac{13}{4}$

(testing a point)

① $2y = 5x + 12$

$y = \frac{5}{2}x + 6$

③ $2y = 5x + 12$

$2(1) = 5(-2) + 12$ is this true?

$2 = -10 + 12$

$2 = 2$ ✓

Complete Assignment Questions #5 - #16

Assignment

1. Convert the following equations to general form $(Ax + By + C = 0)$ where A, B, and C are integers.

a) $y - 4 = 7(x - 1)$ **b)** $y = -2x + 9$ **c)** $y = mx + b$

d) $y = -\frac{3}{4}x + 5$ **e)** $y + 8 = -\frac{3}{2}(x - 5)$ **f)** $y = \frac{5}{3}x - \frac{1}{4}$

2. Find the equation, in general form, of the line through the given point and with the given slope.

a) $(6,1)$, 3 **b)** $(-9,-2)$, $\frac{2}{5}$ **c)** $(0,0)$, $\frac{4}{3}$

3. Determine the slope and y-intercept of the graph of the following lines.

a) $x+y-11=0$ **b)** $3x-2y+30=0$ **c)** $3x+6y-7=0$

4. Determine the slope, y-intercept, and x-intercept of the graph of the following lines.

a) $2x+y-6=0$ **b)** $5x-2y+20=0$ **c)** $4x-5y-3=0$

5. Consider the lines $x-2y+1=0$ and $4x+ky-8=0$.

a) Determine the value of k if the lines have the same slope.

b) Determine the value of k if the lines have the same y-intercept.

6. Consider the lines $3x-5y-15=0$ and $ax+2y-6=0$.

a) Determine the value of a if the lines have the same slope.

b) Determine the value of a if the lines have the same x-intercept.

Multiple Choice **7.** The equation of the line passing through the orgin with slope $-\frac{1}{2}$ is

A. $x+2y=0$

B. $x-2y=0$

C. $2x+y=0$

D. $2x-y=0$

8. Match each equation on the left with the correct characteristic of the graph of the equation on the right. Each characteristic may be used once, more than once, or not at all.

Equation	Characteristic
i) $6x - 2y + 5 = 0$	**A.** Slope $= -\frac{1}{3}$
ii) $2x - 5y = 0$	**B.** y-intercept $= -\frac{5}{2}$
iii) $x + 3y + 6 = 0$	**C.** Passes through $(-10, -4)$
iv) $x - 4y + 10 = 0$	**D.** Slope $= 0$
v) $2x - y - 5 = 0$	**E.** y-intercept $= \frac{5}{2}$
	F. Perpendicular to $y = \frac{5}{2}x - 3$
	G. x-intercept $= \frac{5}{2}$

9. The slope of the line with equation $6x + 5y - 1 = 0$ is

A. $-\frac{6}{5}$

B. $-\frac{5}{6}$

C. $\frac{6}{5}$

D. $\frac{1}{5}$

10. Which line has a y-intercept of 1?

A. $x + 5y + 1 = 0$

B. $x + 3y + 3 = 0$

C. $x - 2y + 2 = 0$

D. $2y = 3x + 1$

11. The slope of a line perpendicular to the line $x+3y+8=0$ is

A. -8

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

12. The line $2y+3x+6=0$ intersects the y-axis at P.
The slope of the line joining P to $Q(6,-2)$ is

A. $-\frac{5}{6}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $-\frac{2}{3}$

13. The lines with equations $ay=4x+9$ and $y=5x-7$ are perpendicular.
The value of a is

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{5}{4}$

D. -20

Use the following information to answer the next question.

Consider the following statements about all the lines in the form $kx+4y-8=0$, where $k \in R$.

Statement 1: The lines have the same slope.
Statement 2: The lines have the same y-intercept.
Statement 3: . The lines have the same x-intercept.

14. Which of the above statement(s) is/are true?

A. 1, 2, and 3
B. 1 only
C. 2 only
D. 3 only

15. Line L has equation $5x-3y+21=0$. A is the point $(-6,-3)$, B is $(3,-2)$, and C is $(-3,2)$. Which of these points lie on line L?

A. A only
B. A and B only
C. A and C only
D. B and C only

Numerical Response

16. Given that the line joining the points $(2,3)$ and $(8,-q)$, where $q \in W$, is perpendicular to the line $3x-2y-5=0$, then the value of q is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $7x-y-3=0$ b) $2x+y-9=0$ c) $mx-y+b=0$
d) $3x+4y-20=0$ e) $3x+2y+1=0$ f) $20x-12y-3=0$

2. a) $3x-y-17=0$ b) $2x-5y+8=0$ c) $4x-3y=0$

3. a) slope $=-1$, y-int $=11$ b) slope $=\frac{3}{2}$, y-int $=15$ c) slope $=-\frac{1}{2}$, y-int $=\frac{7}{6}$

4. a) slope $=-2$, y-int $=6$, x-int $=3$
b) slope $=\frac{5}{2}$, y-int $=10$, x-int $=-4$
c) slope $=\frac{4}{5}$, y-int $=-\frac{3}{5}$, x-int $=\frac{3}{4}$

5. a) -8 b) 16 **6.** a) $-\frac{6}{5}$ b) $\frac{6}{5}$

7. A
8. i) E ii) C iii) A iv) E v) G
9. A **10.** C **11.** D **12.** B
13. D **14.** C **15.** C **16.** 1

Equations of Linear Relations Lesson #5:
Further Practice with Linear Equations

Writing Linear Equations

*great practice #1-6

Linear equations can be written in different forms:

$Ax + By + C = 0$	→	General form of a linear equation.
$y = mx + b$	→	Slope y-intercept form of a linear equation.
$y - y_1 = m(x - x_1)$	→	Point-slope form of a linear equation.

The slope y-intercept form is used when we are given the slope of a line and the y-intercept.
The point-slope form is used when we are given the slope of a line and any point on the line.
In many cases, either the point or the slope of the line must be determined from the information given before the equation can be used.

Given $P(3,-1)$ and $Q(-2,-6)$, determine the equation, in general form, of a line passing through the two points.

① Find slope $\frac{-1--6}{3--2} = \frac{5}{5} = 1$

② use point/slope form

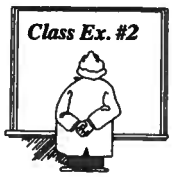
$y - y_1 = m(x - x_1)$

$y + 1 = 1(x - 3)$

$y + 1 = x - 3$ or $y + 6 = x + 2$

---

x-int



Determine the equation, in general form, of a line through the point $(5,0)$ and perpendicular to the line with equation $3x - 5y + 17 = 0$.

① Rearrange general form to $y = mx + b$

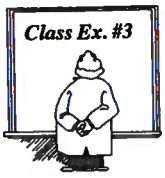
$-5y = -3x - 17$

$y = \frac{3}{5}x + \frac{17}{5}$

② slope $\frac{3}{5} \rightarrow -\frac{5}{3}$ perpendicular

③ pant/slope form $y - 0 = -\frac{5}{3}(x - 5)$

$y = -\frac{5}{3}(x - 5)$



Find the equation, in general form, of the line perpendicular to the line $5x - 7y - 10 = 0$ and with the same x-intercept as $x - 2y - 12 = 0$.

① Find slope $5x-7y-10=0$

$-7y = -5x+10$

$y = \frac{5}{7}x - \frac{10}{7}$

$\frac{5}{7} \xrightarrow{\text{perp}} \frac{-7}{5}$

② x-int, $y=0$

$x-12=0$

$x=12$

$(12,0)$

③ put into point/slope form

$y = \frac{-7}{5}(x-12)$

Complete Assignment Questions #1 - #14

Assignment

1. Find the equation, in general form, of the line through each pair of points.

a) (7, 5) and (6, 1) **b)** (3, –7) and (–5, 9) **c)** (–3, 4) and (11, 25)

d) (10, –15) and (–2, –12) **e)** (4, –7) and (3, –7) **f)** (–5, –8) and (–4, –10)

2. Identify the lines in **1.** which are
i) parallel **ii)** perpendicular

3. Write the equation of each line in general form

a) with slope $\frac{2}{7}$ and an x-intercept of -6 **b)** with a y-intercept of $-\frac{8}{3}$ and a slope of 7

c) through the point $(2,0)$ and perpendicular to $3x-5y+19=0$

d) through the point $(3,-6)$ and parallel to $5x+3y+9=0$

4. Write the equation of each line in general form

a) perpendicular to $y=x$ and with the same x-intercept as $y=2x+10$

b) parallel to $2x-3y+7=0$ and with the same y-intercept as $5x-3y-12=0$

5. Write the equation of each line in general form

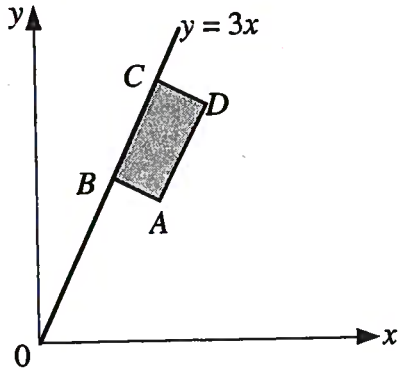
a) perpendicular to $6x - 2y + 5 = 0$ and with the same y-intercept as $x - y + 8 = 0$

b) with the same x-intercept as $9x - 2y + 18 = 0$ and through the point $(4, -5)$

6. Line l contains the point $A(7, 9)$ and is parallel to a line which contains the points $B(-4, 5)$ and $C(8, -1)$. Determine the equation of line l in the form $y = mx + b$.

7. A Cartesian plane is placed on the plan of a farm. The farmhouse is at the origin, and $ABCD$ represents a rectangular field of wheat. A farm road, with equation $y = 3x$, runs from the farmhouse along one side of the field.

a) If the point A has coordinates $(2, 4)$, determine the equation of AD.



b) Determine the equation of AB.

8. A child with a fixed amount of money can buy 2 bags of chips and 5 cans of pop, or 3 bags of chips and 2 cans of pop. A linear relationship exists between the number of bags of chips, x, and the number of cans of pop, y, which can be bought.

a) Write the coordinates of two points which lie on the graph of this linear relationship.

b) Determine the equation of the linear relationship.

Multiple Choice

9. The equation of the line through the point $(7, -4)$ and perpendicular to the line with equation $5x - 4y + 13 = 0$, can be written in the form

A. $y + 4 = \frac{5}{4}(x - 7)$

B. $y = -\frac{4}{5}(x + 7)$

C. $y + 4 = -\frac{4}{5}(x - 7)$

D. $y + 4 = \frac{4}{5}(x - 7)$

10. A line passing through the point $(0, 3)$ is perpendicular to the line $x - 2y - 5 = 0$. The equation of the line is

A. $2x + y - 3 = 0$

B. $2x + y + 3 = 0$

C. $x - 2y + 6 = 0$

D. $2x - y + 3 = 0$

11. Which of the following linear relations is not equivalent to the other three?

A. $y - 4 = -\frac{1}{3}(x + 6)$

B. $x + 3y + 2 = 0$

C. the line passing through $(0, 2)$ and $(6, 0)$

D. $y = -\frac{1}{3}x + 2$

12. A line passing through the point $(0,3)$ is parallel to the line $x-2y-5=0$. The equation of the line is

A. $2x+y-3=0$
B. $2x+y+3=0$
C. $x-2y+6=0$
D. $2x-y+3=0$

13. The image of $y=2x+7$ after a counterclockwise rotation of $90°$ about the origin is

A. $y=-\frac{1}{2}x+\frac{7}{2}$
B. $y=\frac{1}{2}x-\frac{7}{2}$
C. $y=-\frac{1}{2}x-\frac{7}{2}$
D. $y=-2x-7$

Numerical Response

14. The line through the points $(-3,4)$ and $(-1,-2)$ has equation $y+ax+b=0$, where a and b are integers. The value of $a+b$ is _____.
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $4x-y-23=0$ **b)** $2x+y+1=0$ **c)** $3x-2y+17=0$
d) $x+4y+50=0$ **e)** $y+7=0$ **f)** $2x+y+18=0$
2. **i)** b and f **ii)** a and d
3. **a)** $2x-7y+12=0$ **b)** $21x-3y-8=0$ **c)** $5x+3y-10=0$ **d)** $5x+3y+3=0$
4. **a)** $x+y+5=0$ **b)** $2x-3y-12=0$
5. **a)** $x+3y-24=0$ **b)** $5x+6y+10=0$ **6.** $y=-\frac{1}{2}x+\frac{25}{2}$
7. **a)** $y=3x-2$ or $3x-y-2=0$ **b)** $y=-\frac{1}{3}x+\frac{14}{3}$ or $x+3y-14=0$
8. **a)** $(2,5)$ and $(3,2)$ **b)** $3x+y-11=0$
9. C **10.** A **11.** B **12.** C **13.** C **14.**

8			

Equations of Linear Relations Lesson #6:
Graphing Linear Equations

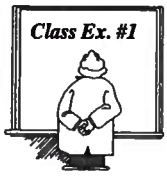
Graphing Linear Equations Without Technology

slope: $\frac{y_2 - y_1}{x_2 - x_1}$

Linear equations can be written in different forms:

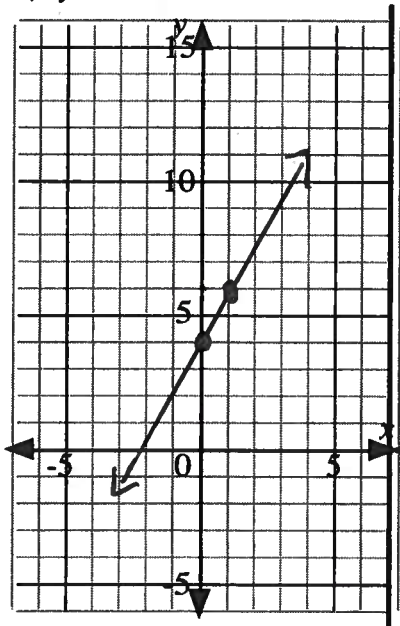
$Ax + By + C = 0$	→	General form of a linear equation.
$y = mx + b$	→	Slope y-intercept form of a linear equation.
$y - y_1 = m(x - x_1)$	→	Point-slope form of a linear equation.

The method used to graph a linear relation without technology depends on the form in which the linear equation is written.



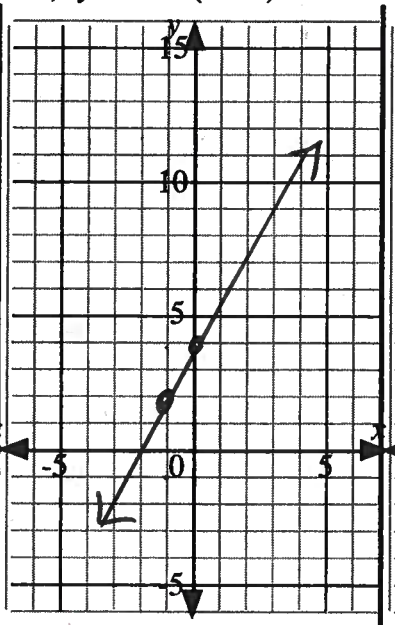
Without altering the form in which the linear equation is written, explain the different strategies used to graph (without technology) each of the following linear relations. Draw the graph of each linear relation on the grid provided.

a) $y = 2x + 4$



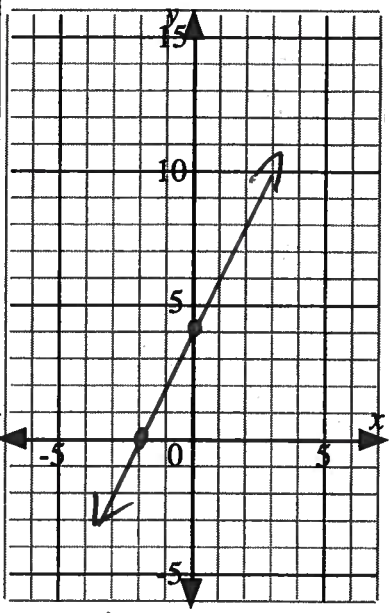
y-int: 4
plot (0,4)
• use slope to find next point
• connect into line

b) $y - 2 = 2(x + 1)$

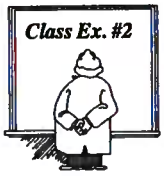


• plot point (-1, 2)
• use slope to find next point
• connect into line

c) $2x - y + 4 = 0$



• x-int, y=0
$2x + 4 = 0$
$2x = -4$
$x = -2$
• y-int, x=0
$-y + 4 = 0$
$y = 4$
• plot the intercepts
• connect into line



Match each linear relation to its graph.

Equation 1: $2x - y + 3 = 0$ D

Equation 2: $y = -2x + 3$ A

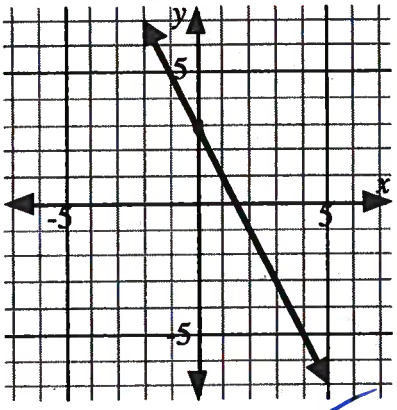
check ← intercept

then slope

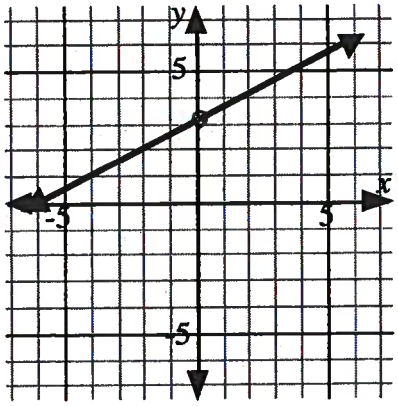
Equation 3: $y + 1 = -\frac{1}{2}(x + 4)$ C

Equation 4: $2y - x - 6 = 0$ B

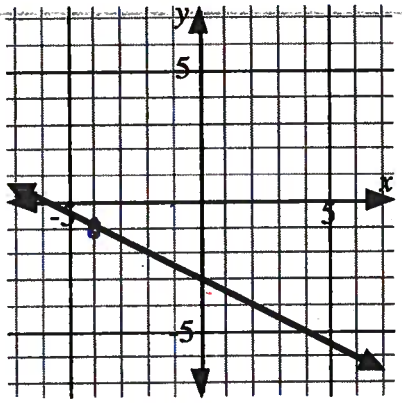
Graph A



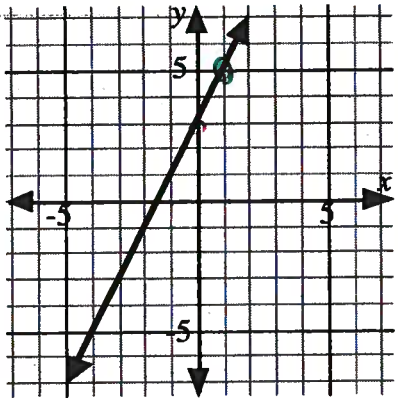
Graph B



Graph C



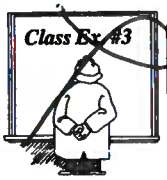
Graph D



check point B (0,3) (1,5) (-4,-1)

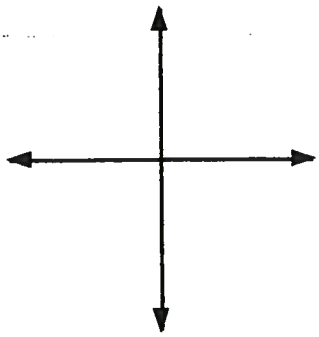
① $2(1) - 5 + 3 \stackrel{?}{=} 0$

$2 - 5 + 3 \stackrel{?}{=} 0$ ✓



Graphing Linear Equations With Technology

- Explain the strategy used to graph (with technology) the linear relations.
  $y + 8 = -5(x - 2)$ and $4x - y + 9 = 0$

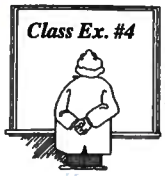


- State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the same grid.

- Determine the x and y-intercepts of $4x - y + 9 = 0$.

Complete Assignment Questions #1 - #6

Determining Linear Relationships from Tables of Data



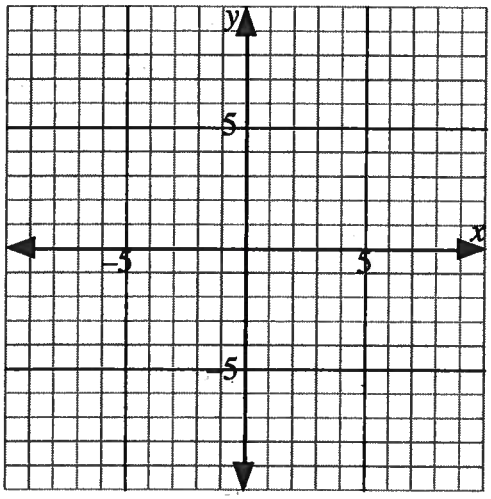
Consider the following data points expressed in a table of values

x	2	4	6	8	10
y	–1	0	1	2	3

a) Plot the data points on the grid to verify that there is a linear relationship.

b) Join the points together and determine the slope of the line.

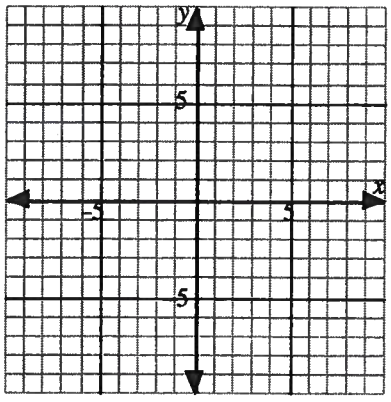
c) Determine the equation of the linear relationship in the form $y = mx + b$.



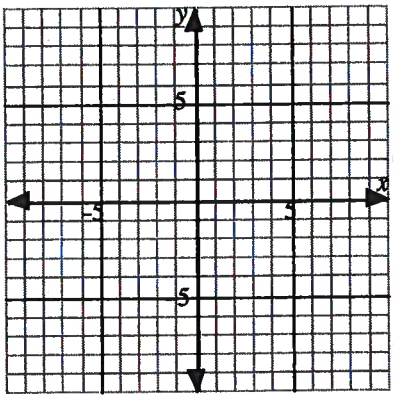
Complete Assignment Questions #7 - #10

Assignment

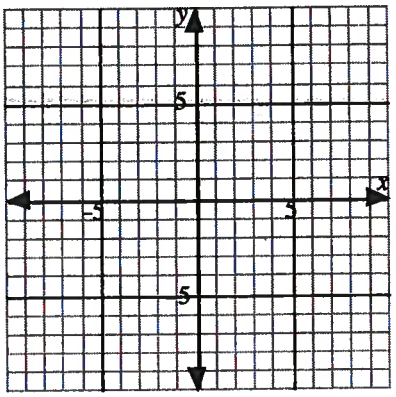
1. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y = -3x - 6$ on a grid.
Draw the graph on the grid provided.



2. Without using technology or without altering the form in which the linear equation is written, explain how to graph $2x - 5y + 20 = 0$ on a grid.
Draw the graph on the grid provided.

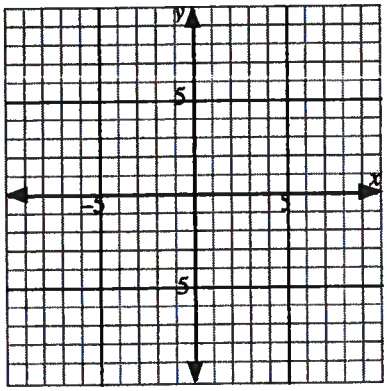


3. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y + 4 = \frac{1}{2}(x - 2)$ on a grid.
Draw the graph on the grid provided.

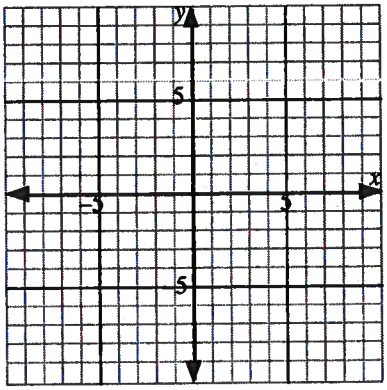


4. Without altering the form in which the linear equation is written, draw the graph (without technology) of each of the following linear relations on the grid provided.

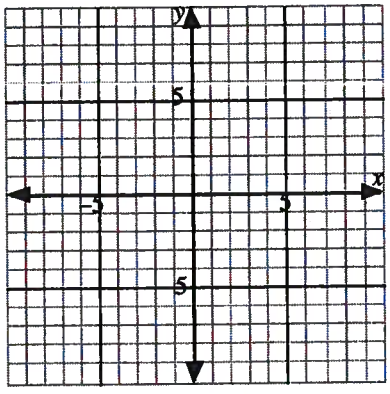
a) $y = -\frac{1}{3}x + 3$



b) $y - 5 = -2(x + 6)$



c) $x - 4y - 8 = 0$



5. Match each linear relation to its graph.

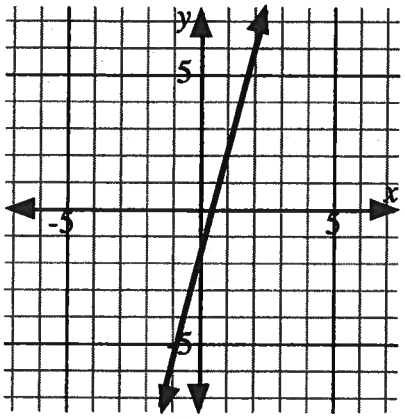
Equation 1:
$y - 6 = -4(x + 1)$

Equation 2:
$x - 4y - 8 = 0$

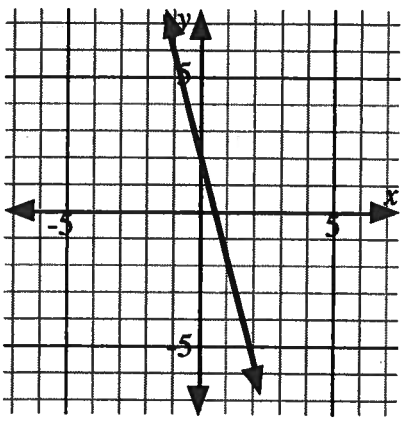
Equation 3:
$4x - y - 2 = 0$

Equation 4:
$y = -\frac{1}{4}x + 2$

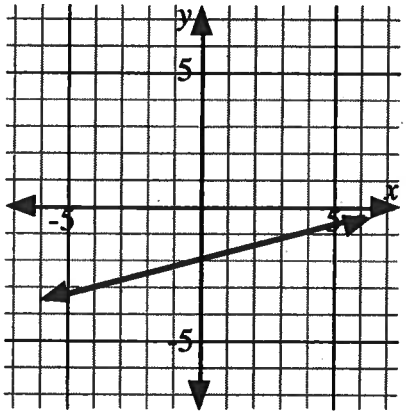
Graph A



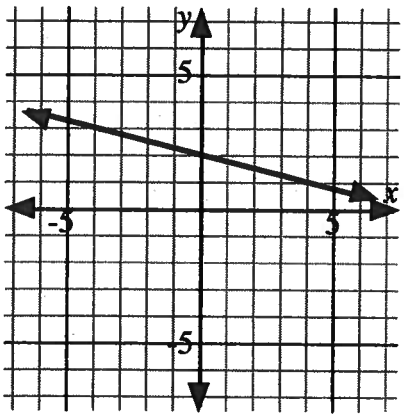
Graph B



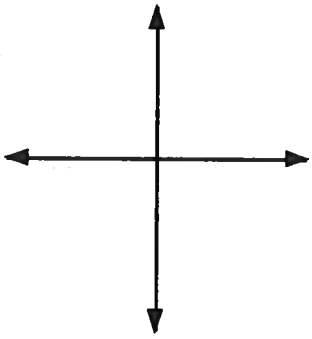
Graph C



Graph D



6. a) Explain the strategy used to graph (with technology) the linear relations $x + 5y + 10 = 0$ and $y - 3 = \frac{1}{3}(x + 6)$.



b) State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the grid.

c) Determine the x- and y-intercepts of each graph.

7. Consider the following data points expressed in a table of values

a)

x	–1	0	1	2	3
y	–3	–1	1	3	5

b)

x	–6	–3	0	3	6
y	6	5	4	3	2

c)

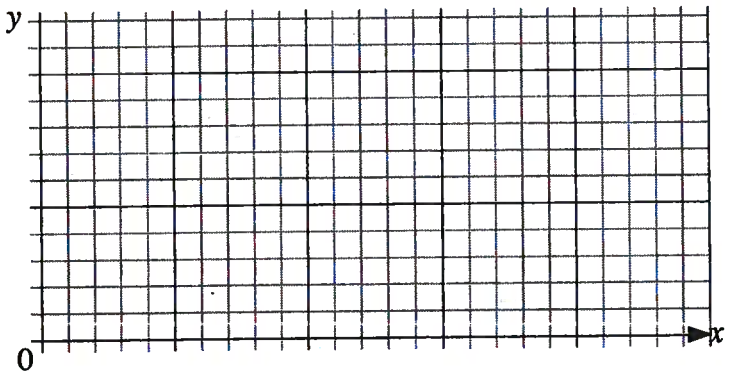
x	0	1	2	3	4
y	1	4	9	16	25

In each case, determine the equation of the linear relationship if it exists.

8. The following data is taken from a continuous linear relationship involving two quantities x and y.

x	32	41	50	59	68	77	86	95
y	0	5	10	15	20	25	30	35

a) Plot the data on the grid and obtain, in general form, the equation of the linear relation which is represented by the data.



b) Rewrite the equation of the linear relation

i) in terms of x (i.e. $y = ...$) **ii)** in terms of y (i.e. $x = ...$)

c) The formulas in **b)** are well known in the scientific field. Can you suggest what scientific variables are represented by x and y?

Use the following information to answer the questions #9 and #10.

The following data is taken from a continous linear relationship involving two quantities, x and y.

x	-10	–5	5	10	20
y	-4	–3	–1	0	2

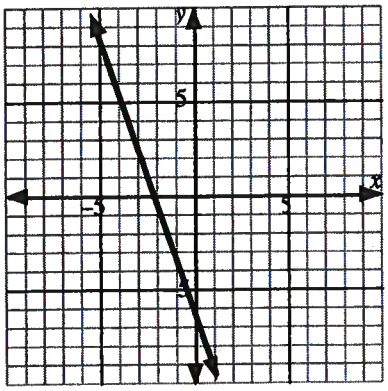
Multiple Choice **9.** When the data is graphed on a grid, the slope of the line is

A. 5
B. –5
C. $\frac{1}{5}$
D. $-\frac{1}{5}$

Numerical Response **10.** The equation of the linear relationship can be written in the form $Ax - 5y + C = 0$. The value of $A - C$ is _____ .

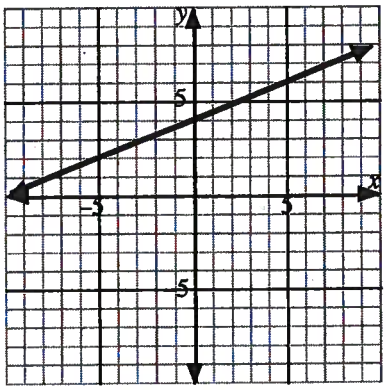
Answer Key

1.



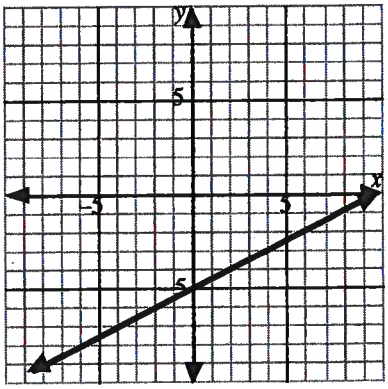
Plot the y-intercept $(0, -6)$. Since the slope, -3, equals rise over run, move 3 up and 1 left and plot another point. Repeat for two more points and draw a line through the points.

2.



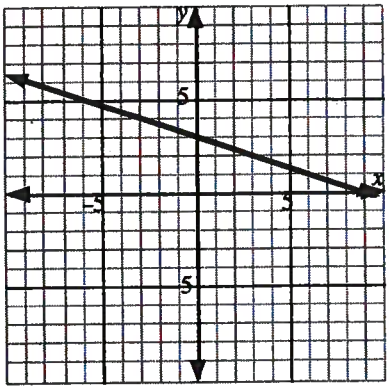
Plot the x-intercept $(-10, 0)$ and the y-intercept $(0, 4)$. Draw a line through these two points.

3.

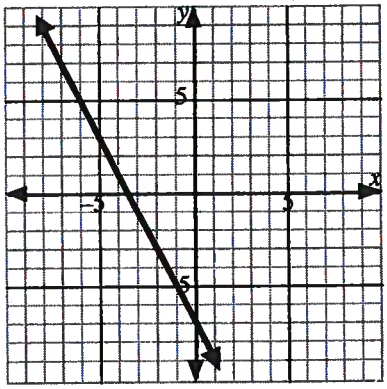


Plot the point $(2, -4)$. Since the slope equals 1/2, move 1 up and 2 right and plot another point. Repeat for two more points and draw a line through the points.

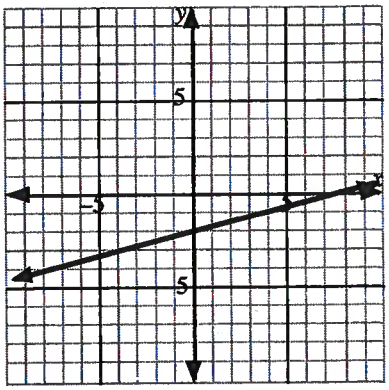
4. a)



b)



c)



5. 1B, 2C, 3A, 4D.

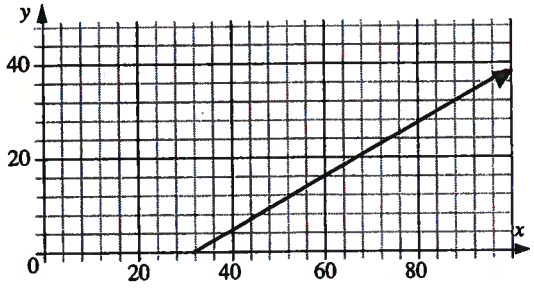
6. a) Solve each equation for y. Then input Y_1 and Y_2 into the equation editor of the graphing calculator. Press Graph.

b) x:[–20, 10, 5] y:[–6, 10, 2], answers may vary.

c) For $x + 5y + 10 = 0$, x-int = -10, y-int = -2, and for $y - 3 = \frac{1}{3}(x + 6)$, x-int = -15, and y-int = 5.

7. a) $y = 2x - 1$ **b)** $y = -\frac{1}{3}x + 4$ **c)** not linear

8. a) $5x - 9y - 160 = 0$



b) i) $y = \frac{5}{9}x - \frac{160}{9}$, or $y = \frac{5}{9}(x - 32)$

ii) $x = \frac{9}{5}y + 32$

c) x is temperature in °F, and y is temperature in °C.

9. C **10.**

1	1		