Relations Lesson #3:
x- and y-intercepts and Interpreting Relations

Review

a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ordered pairs.

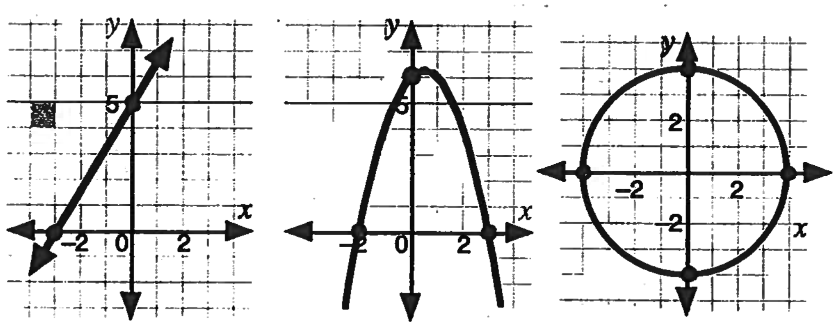
b) The first component of a set of ordered pairs is the x coordinate, also known as the input. Values of the input are values of the independent variable.

c) The second component of a set of ordered pairs is the y coordinate, also known as the output. Values of the output are values of the dependent variable.

Exploring x- and y-intercepts

Consider the following graphs.

Graph 1 **Graph 2** **Graph 3**



a) List the coordinates of the point(s) where each graph crosses the x-axis.

- Graph 1 crosses the x-axis at (−3, 0).
- Graph 2 crosses the x-axis at (−2, 0) and (3, 0).
- Graph 3 crosses the x-axis at (−4, 0) and (4, 0).

b) What do all the points in a) have in common?

the y-coordinate is zero

c) List the coordinates of the point(s) where each graph crosses the y-axis.

- Graph 1 crosses the y-axis at (0, 5).
- Graph 2 crosses the y-axis at (0, 6).
- Graph 3 crosses the y-axis at (0, −4) and (0, 4).

d) What do all the points in c) have in common?

the x-coordinate is zero

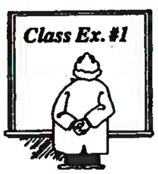
x- and y- intercepts of a Graph

The **x-intercept** of a graph is the x-coordinate of the ordered pair where the graph intersects the x-axis. An x-intercept occurs at a point on the graph where the y-coordinate is zero. The x-intercept can be given as a value or as an ordered pair.

The **y-intercept** of a graph is the y-coordinate of the ordered pair where the graph intersects the y-axis. A y-intercept occurs at a point on the graph where the x-coordinate is zero. The y-intercept can be given as a value or as an ordered pair.

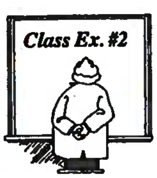
1. Given the equation of the graph of a relation:
   - to determine the x-intercept, set $y = 0$ and solve for x.
   - to determine the y-intercept, set $x = 0$ and solve for y.

2. The equation of a graph can be written in different forms, all of which are equivalent. The equation of Graph 1 on the previous page is $y = \frac{5}{3}x + 5$, which can be written as $3y = 5x + 15$ or $5x - 3y + 15 = 0$. Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the x- and y-intercepts of the graph of an equation given in any form.



The equation of Graph 1 on the previous page is $3y = 5x + 15$.
Algebraically determine the values of the x-intercept and the y-intercept of Graph 1.

x-intercept Let $y = 0$	y-intercept Let $x = 0$
$3(0) = 5x + 15$	$3y = 5(0) + 15$
$0 = 5x + 15$	$3y = 0 + 15$
$-15 \quad -15$ ← subtract 15 from both sides	$3y = 15$
$\frac{-15}{5} = \frac{5x}{5}$ ← divide both sides by 5.	$\frac{3y}{3} = \frac{15}{3}$ ← divide both sides by 3.
$x = -3$ xint $= -3$	$y = 5$ yint $= 5$



The equation of Graph 3 on the previous page is $x^2 + y^2 = 16$.
Calculate the x-intercept and the y-intercept of the graph of $x^2 + y^2 = 16$. Give the answers as ordered pairs.

x-intercept Let $y = 0$	y-intercept Let $x = 0$
$x^2 + (0)^2 = 16$ ordered pairs	$(0)^2 + y^2 = 16$
$x^2 + 0 = 16$ $(-4,0)$ and $(4,0)$	$0 + y^2 = 16$
$x^2 = 16$	$y^2 = 16$
$x = \sqrt{16}$ ← take the square root of both sides.	$y = \sqrt{16}$ ← tak the square root of both sides
$x = \pm 4$	$y = \pm 4$

Complete Assignment Questions #1 - #3

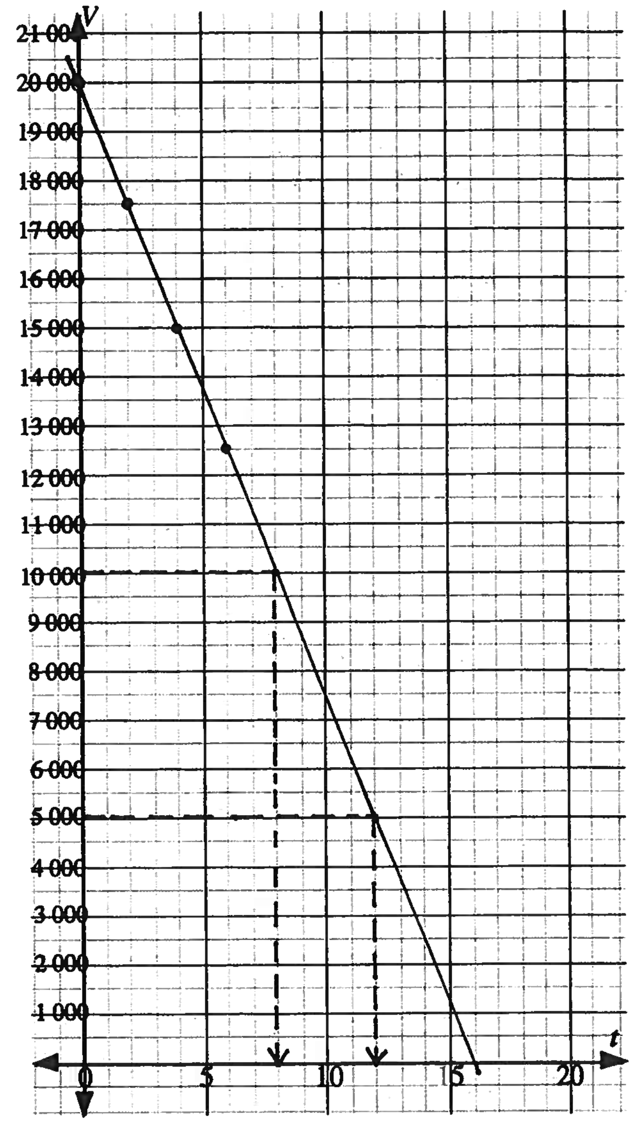
ordered pairs $(0,-4)$ and $(0,4)$

Class Ex. #3

Lisa purchases a new car for \$20 000. The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars, and t is the age of the car in years.

a) Complete the table of values and plot the ordered pairs on the grid.

Input (t)	Output (V)	Ordered pair (t, V)
0	20 000	(0, 20000)
2	17500	(2, 17500)
4	15 000	(4, 15000)
6	12500	(6, 12500)



Connect the points with a straight line, and extend the line.

b) What does the ordered pair (0, 20 000) represent?

The value of the new car.

y-intercept

c) Use the graph to estimate the t-intercept. What does the t-intercept represent?

$t_{int} = 16$

d) Use the graph to estimate the value of the car after
i) 3 years ii) 10 years iii) 14 years.

\$16 250 \$ 7500 \$ 2500

Part i – interpolation, as we are making conclusions inside given data from the above table
Part ii and iii – extrapolation, as we are making conclusions extra/outside given data from the above table.

e) Use the formula to verify d) ii).

$V = 20000 - 1250(10)$
$= 7500$

f) Use the graph to estimate when the car will be worth
i) \$5 000 ii) half of the purchase price.

12 years

(\$20 000 ÷ 2 = \$10 000)
8 years

g) Use the formula to verify f) ii).

$(10\,000) = 20000 - 1250t$
$1250t = 10\,000$
$t = \frac{10\,000}{1250} = 8$ years.

h) Complete the following statement to describe the relation:

The original value of the car is \$20000. It depreciates in value by \$1250 per year and has no value after sixteen years.

x-intercept.

3. Determine the *x*- and *y*-intercepts of each equation. Answer as ordered pairs.

a) $y = 4x + 7$ **b)** $y = 15 - 6x$ **c)** $4x - 2y + 16 = 0$

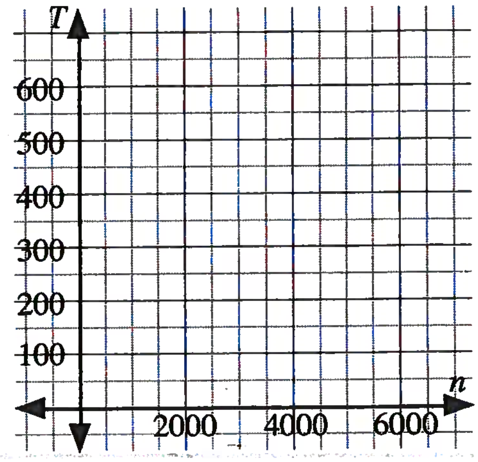
d) $y = \frac{x^2}{2} - 18$ **e)** $x^2 + y^2 = 25$ **f)** $y = 3x$

g) $y = x^2 + 4$ **h)** $9x^2 + y^2 = 81$ **i)** $9x^2 - y^2 = 81$

4. Triple A Car Rental charges $100 per rental plus 10¢ per km. The total cost, T, in dollars of renting the car can be represented by the formula, $T = 100 + 0.10n$, where n is the number of km travelled.

a) Complete the table of values, and plot the ordered pairs on the grid provided.

Number of km (n)	Total Rental Cost (T) dollars
0	
1000	
3500	
5000	



Connect the points with a straight line, and extend the line in both directions.

b) What does the ordered pair (0, 100) represent?

c) Determine the n-intercept of the graph. Explain why it is not applicable to this problem.

d) Interpolate from the graph to estimate the cost for a journey of:
i) 2000 km **ii)** 4500 km

e) Use the formula to verify the answers in **d)**.

f) If the total cost of rental is $650, use the graph to estimate the number of km travelled.

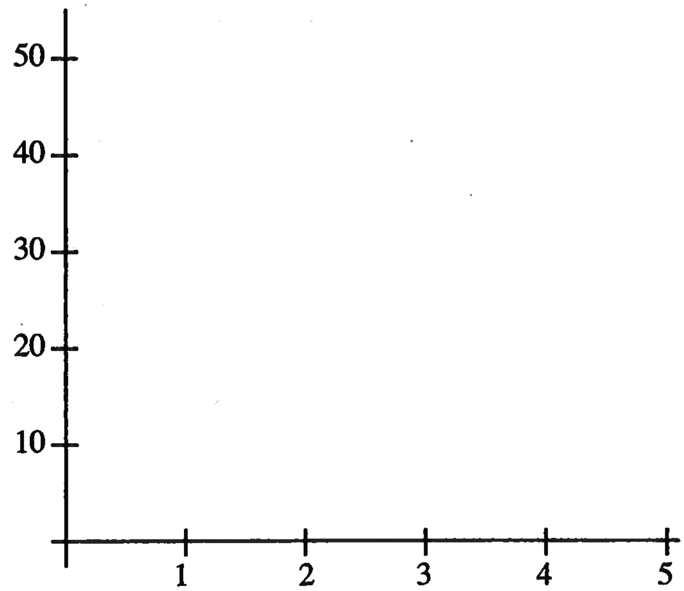
g) Verify the answer in **f)** using the formula.

5. An arrow is shot vertically into the air using a bow. The height, h metres, above the ground after t seconds, where $t \geq 0$ is approximated by the equation $h = -5t^2 + 20t + 25$.

a) The maximum height of the arrow is reached after 2 seconds.
Calculate the maximum height.

b) Complete the table of values, and plot the points on the grid.
Join the points with a smooth curve, and label the graph.

time (seconds)	height (metres)
0	
1	
2	
3	
4	
5	



c) Is this a linear or nonlinear relation?

d) For how many seconds is the arrow in the air?

e) What does the h-intercept represent in the context of the question?

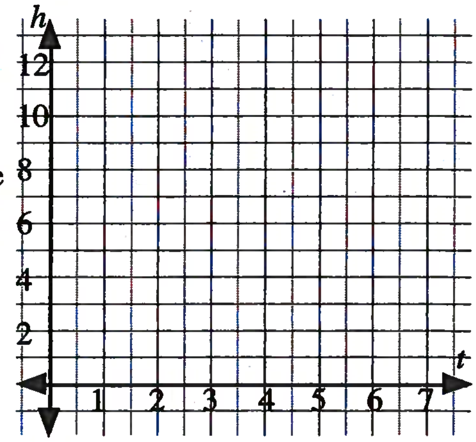
f) What does the t-intercept represent in the context of the question?

g) i) Use the graph to estimate the height of the arrow after 1.5 seconds.

ii) Use the equation to calculate the exact height of the arrow after 1.5 seconds.

h) Does it make sense to extend the graph of the relation $h = -5t^2 + 20t + 25$ further in a downward direction to the left or right? Explain.

6. A candle manufacturer determined that its "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours.



a) Make a table of values and use this to construct the graph of $h = -2t + 12$.

t				
h				

Use your graph to answer ***b – e***.

b) How tall is the candle before it begins to melt?

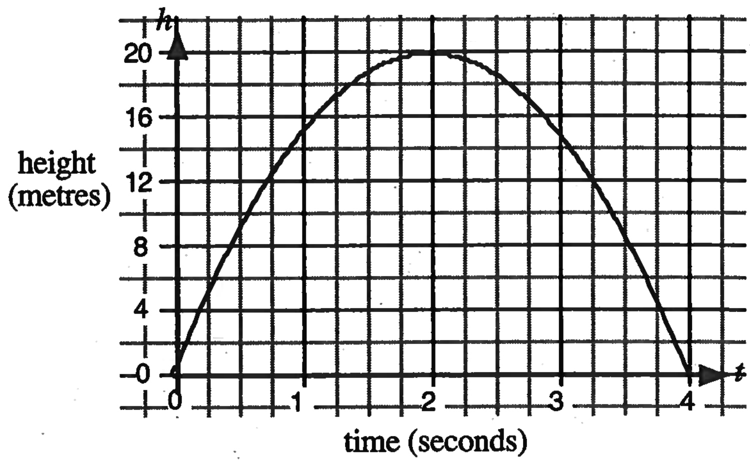
c) How many hours will the candle last before it will completely burn out?

d) How tall will the candle be after burning for 5 hours?

e) How long will it take for the candle to burn down to a height of 7 cm?

f) Verify the answers from **b)** - **e)** using the formula.

7. A football is kicked by a student. The graph of the relation between the height of the football above the ground and time is shown. The formula that represents the relation is given by $h = -4.9t^2 + 19.4t + 0.6$, where h is the height in metres above the ground and t is the time in seconds the football is in the air.



*Use the graph to answer **a** – **c**:*

a) Estimate, to the nearest metre, the maximum height of the football above the ground.

b) Estimate how long it takes for the football to reach the ground.

c) Estimate the height, to the nearest metre, of the football when it is in the air for 3 seconds.

d) Use the formula to calculate the exact answer to **c)**.

e) Calculate the h-intercept, and describe what it represents in the context of the question.

Multiple Choice **8.** In which of the following relations does the graph of the relation have x- and y-intercepts with equal values?

A. $y = x + 8$
B. $2x + 2y = 7$
C. $2x - 3y + 4 = 0$
D. none of the above

Numerical Response **9.** The graph of the relation $4x^2 + 9y^2 - 36 = 0$ has x-intercepts a and b, and y-intercepts c and d. The value of the product $abcd$ is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) −5 b) −15 c) 6 d) $\frac{1}{3}$ e) −30 f) 12.44

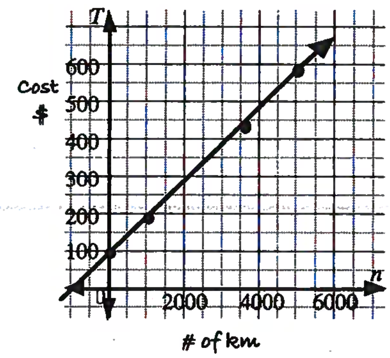
2. a) 2 b) 4 c) 6 d) $-\frac{5}{6}$ e) ±3 f) 4

3. a) x-int $= \left(-\frac{7}{4}, 0\right)$, y-int = (0. 7) b) x-int $= \left(\frac{5}{2}, 0\right)$ y-int = (0, 15) c) x-int = (−4, 0) y-int = (0, 8)

d) x-int = (6, 0) and (−6, 0), y-int = (0, −18)
e) x-int = (5, 0) and (−5, 0) y-int = (0, 5) and (0, −5)
f) x-int = (0, 0), y-int = (0, 0)
g) no x-int, y-int = (0, 4)
h) x-int = (3, 0) and (−3, 0) y-int = (0, 9) and (0, −9)
i) x-int = (3,0) and (−3,0) no y-int

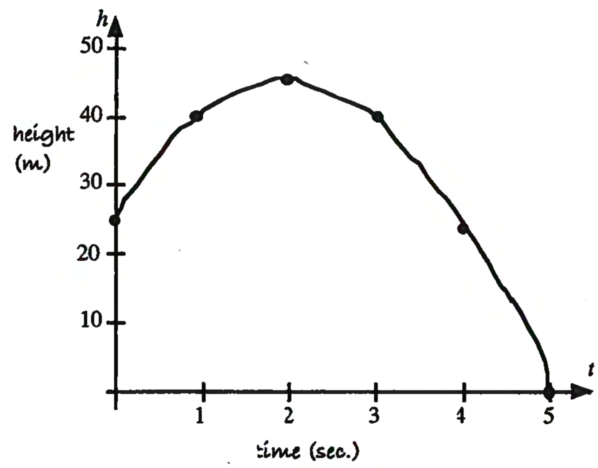
4. a) see table and graph
b) Triple A Car Rental charges a fixed rate of $100 before any distance is travelled
c) n–int = −1000, distance in this scenario cannot be represented by a negative value
d) i) $300 ii) $550
f) 5500 km

Number of km (n)	Total Rental Cost (T)dollars
0	100
1000	200
3500	450
5000	600



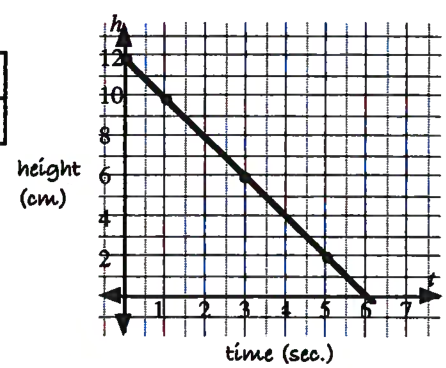
5. a) 45 m
b) see table and graph
c) non-linear
d) 5
e) The arrow was fired from a height of 25 m above the ground
f) The number of seconds it takes to strike the ground.
g) i) approximately 44 m
ii) 43.75
h) No to the left because time cannot be negative.
No to the right because the ground stops the arrow from going further.

time (seconds)	height (metres)
0	25
1	40
2	45
3	40
4	25
5	0



6. a) see table and graph
answers may vary
b) 12 cm
c) 6 hours
d) 2 cm
e) 2.5 hours

t	0	1	3	5
h	12	10	6	2



7. a) approx 20 m
b) approx 4 seconds
c) approx 15 m
d) 14.7 m
e) h–int is 0.6 m.
The football was punted 0.6 m above the ground.

8. B

9.

3	6		

Relations Lesson #4:
Domain and Range

Number Systems Review

Number systems were studied in previous math courses. Recall the symbols and letters used by completing the following.

a) Each letter below is represents a number system. Write the name of the number system. $\overline{Q}$ has been completed.

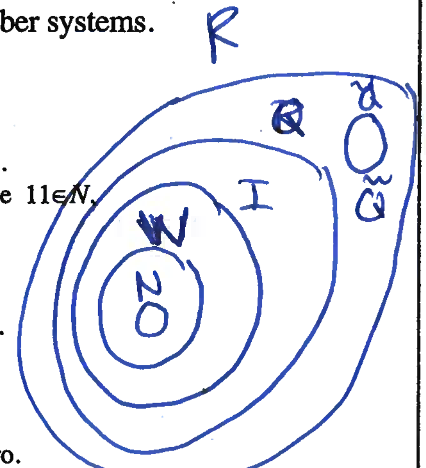
N	W	I	Q	$\overline{Q}$	R
natural	whole	integer	rational	Irrational Numbers	Real

b) The symbol $\in$ represents the phrase " includes" "belongs to"

c) The brackets { } represents the phrase " set of all"

d) Recall the following by reading following notes regarding number systems.

Natural Numbers no zero
- The natural numbers (counting numbers) are 1, 2, 3, 4, etc.
- There are an infinite number of natural numbers.
- The set of all natural numbers can be written as $N = \{1, 2, 3, 4, \dots\}$.
- The number 11 belongs to the set of natural numbers and we can write $11 \in N$.



Whole Numbers
- The whole numbers are the natural numbers together with 0.
- The set of all whole numbers can be written as $W = \{0, 1, 2, 3, \dots\}$.
- $6 \in N$ but $\frac{1}{2} \notin W$.

Integers
- The integers consist of positive and negative natural numbers, and zero.
- The set of all integers can be written as $I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- In some texts Z or J is used instead of I.

Rational Numbers
- Rational numbers are those numbers which can be expressed as a ratio between two integers, e.g. the fractions $\frac{1}{4}$ and $-\frac{2}{3}$ are both rational numbers. All the integers are included in the rational numbers since any integer can be written as $\frac{x}{1}$ e.g. $5 = \frac{5}{1}$.
- The set of rational numbers can be written as $Q = \left\{\frac{a}{b},\ \text{where } a, b \in I,\ b \neq 0\right\}$.

Irrational Numbers
- The set of irrational numbers can be written as $\overline{Q} = \{\text{non-terminating and non-repeating decimals}\}$ e.g. $\pi \in \overline{Q}$, $\sqrt{2} \in \overline{Q}$. π

Real Numbers
- The real numbers consists of all rational numbers and irrational numbers.
- The set of all real numbers can be written as $R = \{Q \text{ and } \overline{Q}\}$.
- In some texts $\Re$ or Re are used instead of R.

Complete Assignment Question #1 and #2

Domain and Range

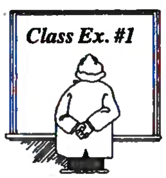
The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable** (x). ⟷

The **range** of a relation is the set of all possible values of the **output** of the **dependent variable** (y). ↕

In lesson 2 on page 235 we described the relation in each of the following forms

- in words
- a mapping (or arrow) diagram
- a table of values
- an equation
- a set of ordered pairs
- a graph

In this lesson we will study the domain and range given in any of these forms.



Class Ex. #1

List the domain and range of the following set of ordered pairs.

a) (1,2), (0,5), (3,8), (5,9), (−3,2)

D: {−3, 0, 1, 3, 5} *x-values

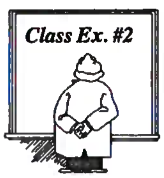
R: {2, 5, 8, 9} *y-value

↑ only write once

b) (3,3), (0,3), (−3,3), (2,9), (−8,3)

D: {−8, −3, 0, 2, 3}

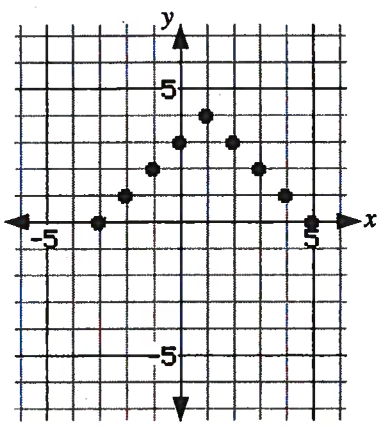
R: {3, 9}



Class Ex. #2

In each case, state the domain and range of the relation represented by the graph.

a)



D: {−3, −2, −1, 0, 1, 2, 3, 4, 5}

or {$-3 \le x < 5$, $x \in I$}

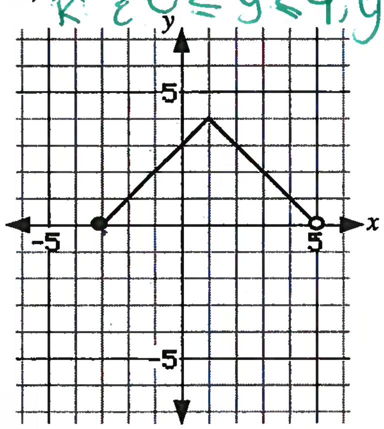
R: {0, 1, 2, 3, 4}

or {$0 \le y \le 4$, $y \in W$}

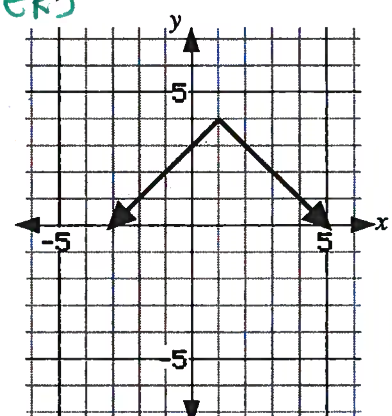
b)

D: {$-3 \le x < 5$, $x \in R$}

R: {$0 \le y \le 4$, $y \in R$}



c)



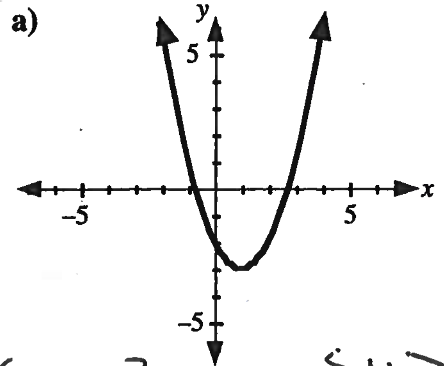
D: {$x \in R$}

R: {$y \le 4$, $y \in R$}

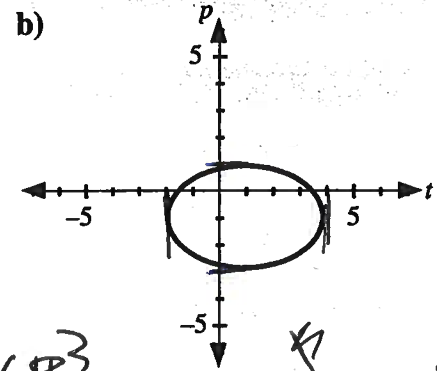
Class Ex. #3

State the domain and range of the following relations.

a)



b)



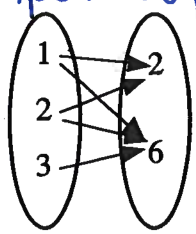
D: $\{x \in R\}$ R: $\{y \geq -3, y \in R\}$

$\{-2 \leq x \leq 4, x \in R\}$

$\{-3 \leq y \leq 1, y \in R\}$

c)

input (x) output (y)



D: $\{1, 2, 3\}$

R: $\{2, 6\}$

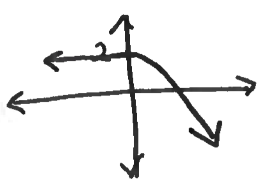
d) A circle with centre $(-2, 3)$ and a radius of 3.

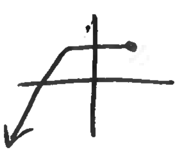
Class Ex. #4

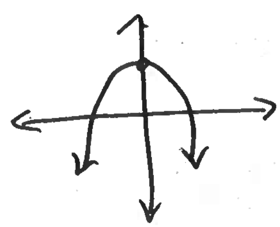
a) Draw the graph of a relation which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and

i) only one x-intercept

ii) two x-intercepts







b) Explain why it is not possible to draw a graph which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and no x-intercepts.

Class Ex. #5

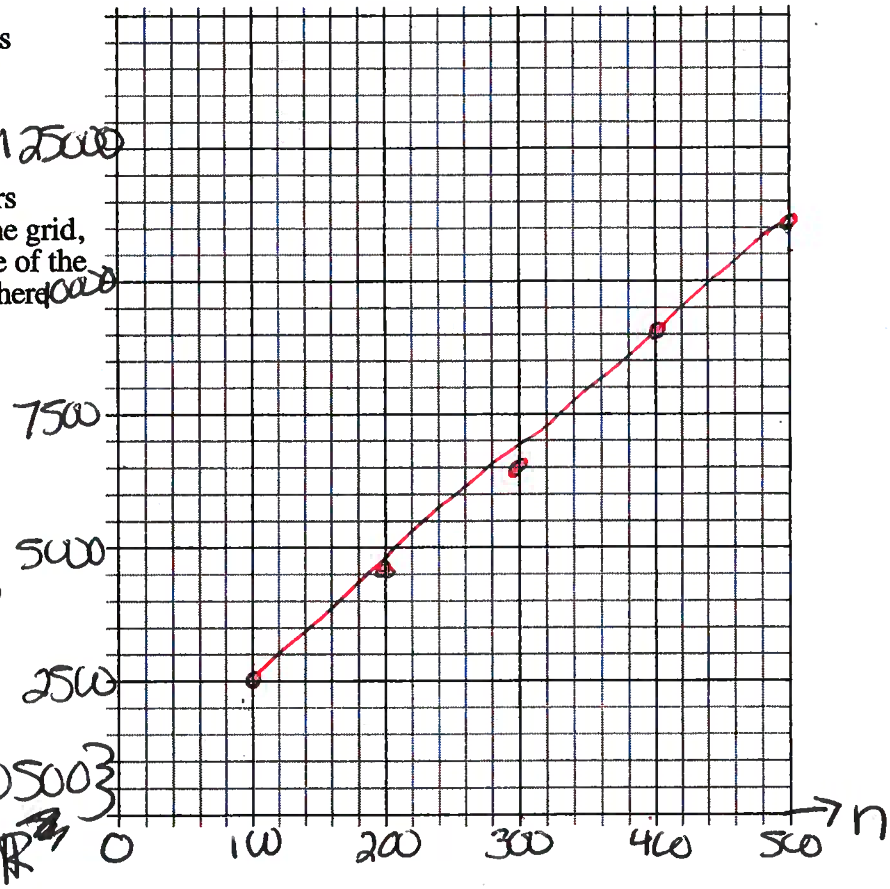
A high school football team is hosting a banquet to celebrate winning the championship. The caterer charges a set up fee of \$500 plus \$20 per person.
The equation $C = 500 + 20n$ represents the cost of hosting the banquet for n people.

a) Make a table of values with 8 entries for a minimum of 100 and a maximum of 500 people.

n	C
100	2500
150	3500
200	4500
250	5500
300	6500
350	7500
400	8500
500	10500

b) Plot the eight ordered pairs from **a)** on the grid.

c) If all possible ordered pairs from **b)** were plotted on the grid, state the domain and range of the relation and explain why there are restrictions on both.



D: {x ≥ 100

D: {x∈N st

D: {100 ≤ x ≤ 500}
x ∈ W

R: {2500 ≤ y ≤ 10500}
y ∈ R

Complete Assignment Questions #3 - #12

Assignment #1, 3acf, 4, 7, 8, 9

1. Use check marks to indicate all the sets to which each number belongs.

	N	*W*	*I*	*Q*	$\bar{Q}$	*R*
$1/3$				✓		✓
123 983	✓	✓	✓	✓		✓
-2						
$7.53\bar{4}$			✓	✓		✓
9.5						
$\sqrt{75}$						
$-\pi$						
$-355/113$						
$-\sqrt{49}$						
0.000005						
2.232425...					✓	✓

2. The addition of ANY two **natural** numbers is still a natural number.

Does this property hold true for the operations of subtraction, multiplication, or division (except for zero)?

Investigate for whole numbers, integers, rational numbers, and real numbers and write your conclusions.

3. State the domain and range of each relation.

a) (2, 3), (0, 2), (4, 8), (–1, 8), (–3, 1)

b) (–3, 3), (0, –5), (–3, 3), (5, –2), (–8, 1)

c)

Input (x)	Output (y)
0	3
2	4
4	5
6	3

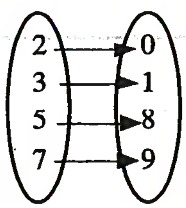
d)

Input (x)	Output (y)
2	3
0	4
–3	5
2	6

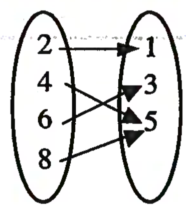
e)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

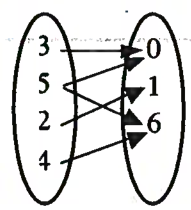
f)



g)

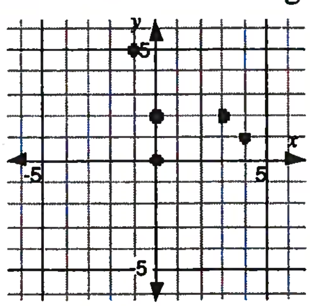


h)

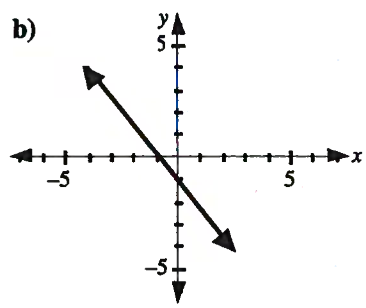


4. State the domain and range for each relation.

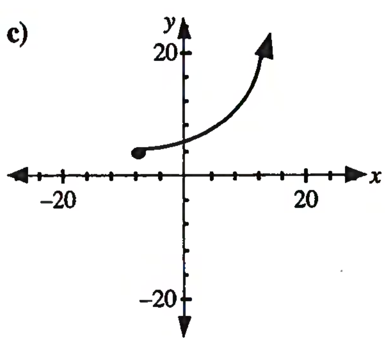
a)



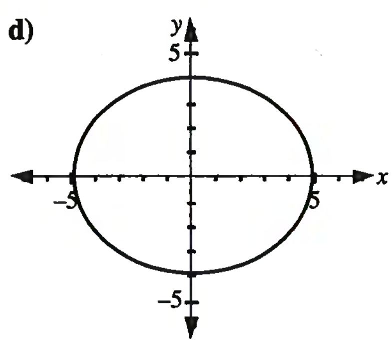
b)



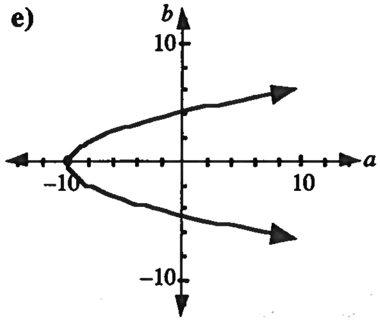
c)



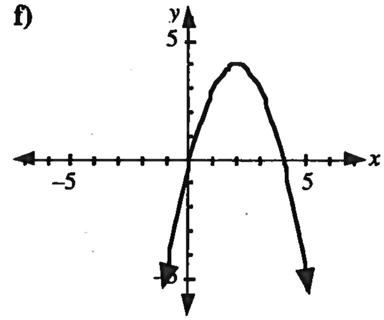
d)



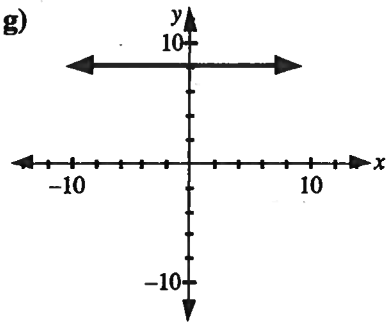
e)



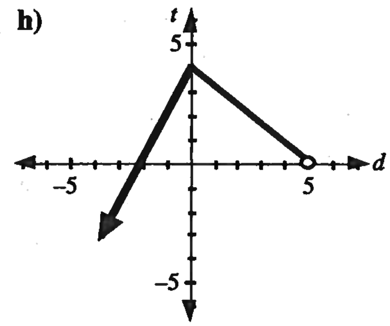
f)



g)



h)



5. In each case a relation is graphed on a grid. State the domain and range of the relation if the graph is

a) a circle whose centre is located at $(-1, 12)$ and with a radius of 5 units.

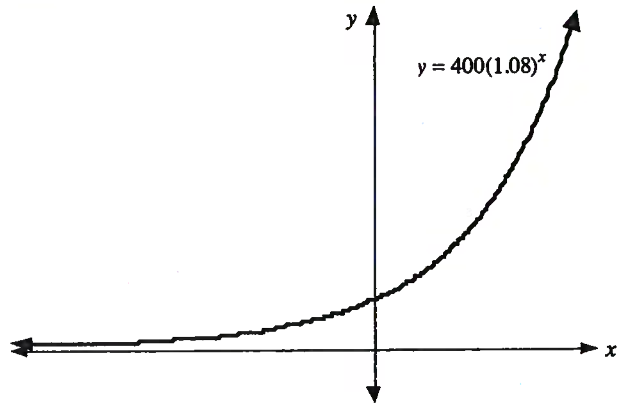
b) a circle with centre $(-3, -5)$ and diameter 40 units.

c) a rectangle with vertices $A(-8, 10)$, $B(-8, -2)$, $C(7, -2)$, and $D(7, 10)$.

d) a triangle with vertices $T(-50, -75)$, $U(-35, -25)$, and $V(-65, -25)$.

6. The graph of the relation $y = 400(1.08)^x$ is shown on the grid.

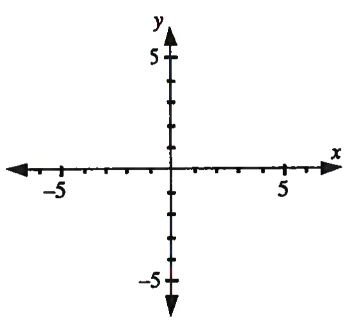
a) State the domain, range, and y-intercept of the relation.

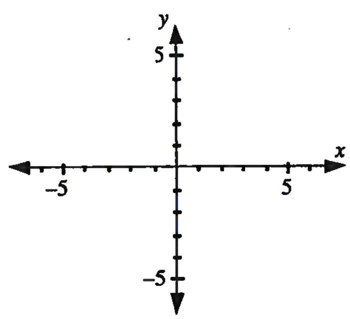


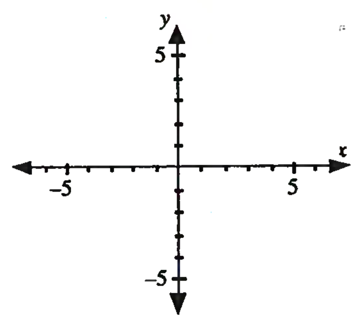
b) The relation $A = 400(1.08)^t$ represents the amount of money when an original investment of \$400 is compounded annually at 8% for a period of t years. State the domain and range of this relation, and explain why the answer is different from **a)**.

7. In each case draw a graph on the domain of real numbers which could represent a linear relation with

a) one x-intercept **b)** no x-intercept **c)** an infinite number of x-intercepts.



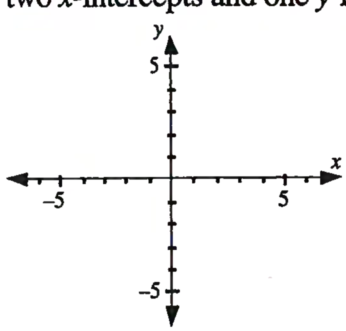


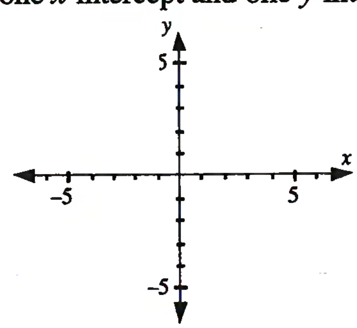


8. In each case draw a graph of a non-linear relation with

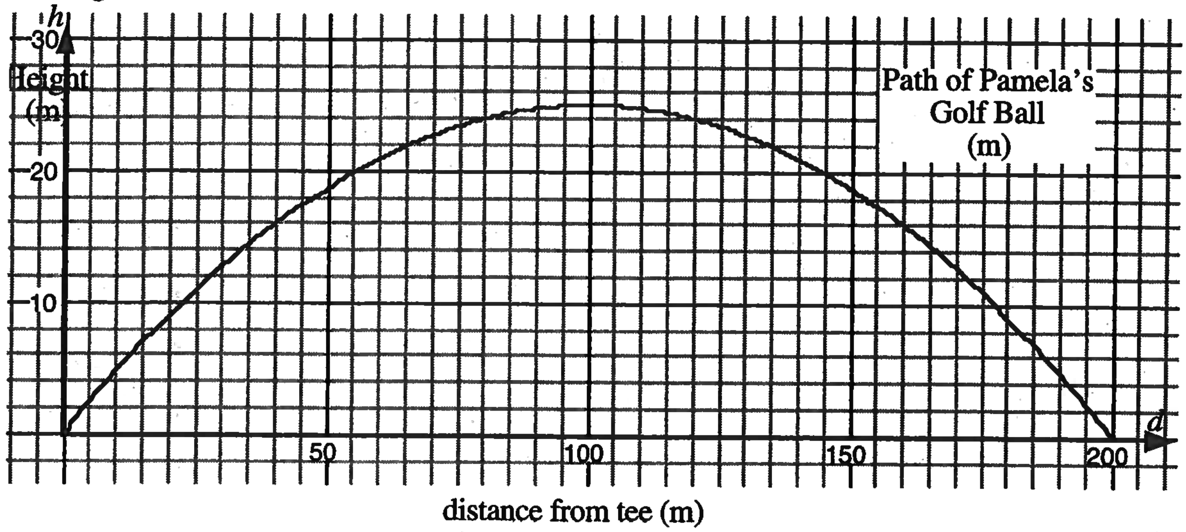
a) domain $x \in R$, range $y \geq -3, y \in R$ two x-intercepts and one y-intercept

b) domain $x \in R$, range $y \geq -3, y \in R$ one x-intercept and one y-intercept.





9. The graph shows the flight of Pamela's golf ball from the tee to a sand trap at the edge of the green.



a) State the h-intercept and the d-intercepts of the graph, and explain their significance in relation to the question.

b) State the maximum height of the golf ball, and explain its relevance to the domain or range of the relation.

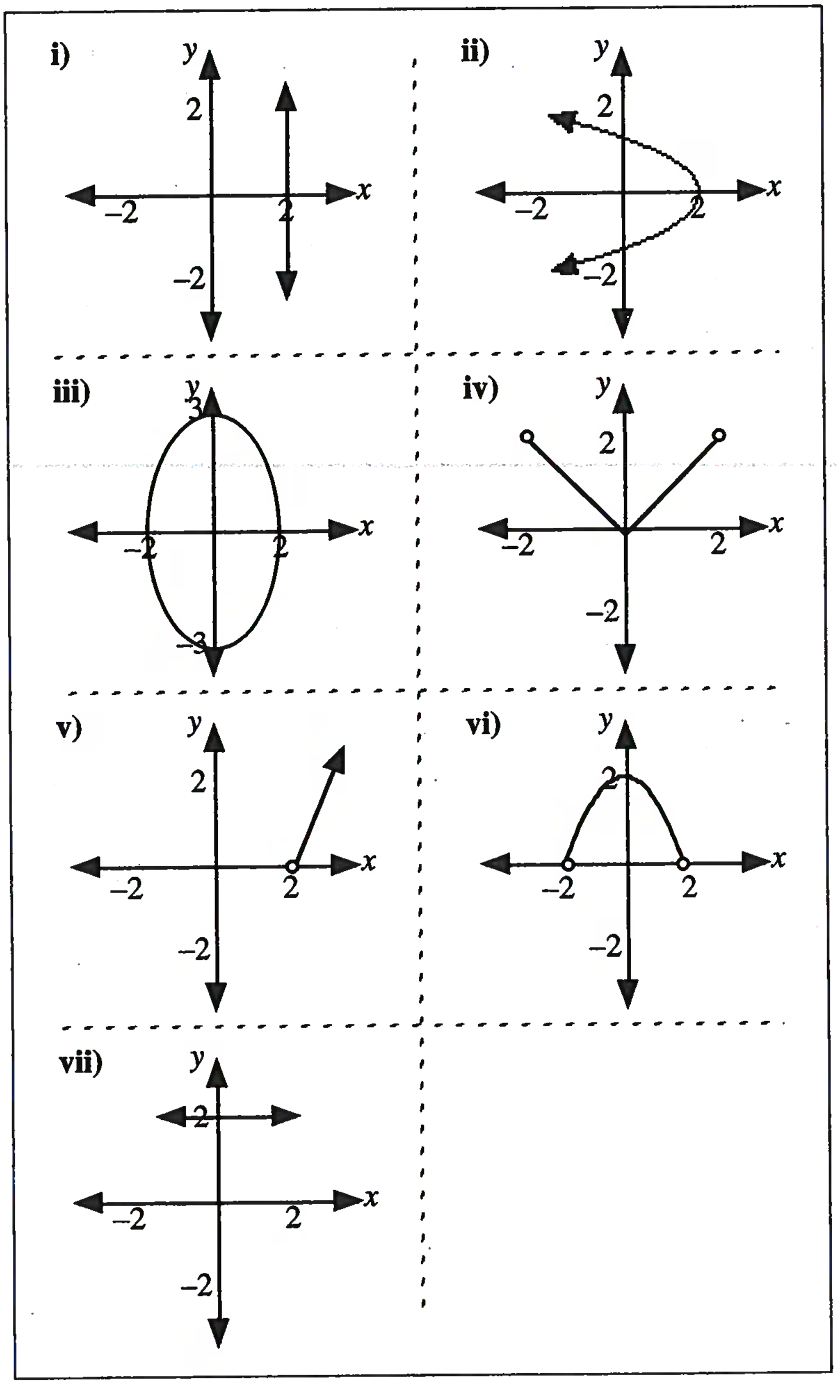
c) State the domain and range of the relation.

d) From the graph, estimate the horizontal distance the ball has traveled when it is 20 m in the air. Explain why there are two answers.

e) Using the graph, estimate the height of the golf ball when the horizontal distance from the tee is 80 m.

f) Give a brief description of the relationship between the height of the golf ball and the horizontal distance from the tee.

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.



A. $x = 2$

B. $x < 2$

C. $x \leq 2$

D. $x > 2$

E. $x \geq 2$

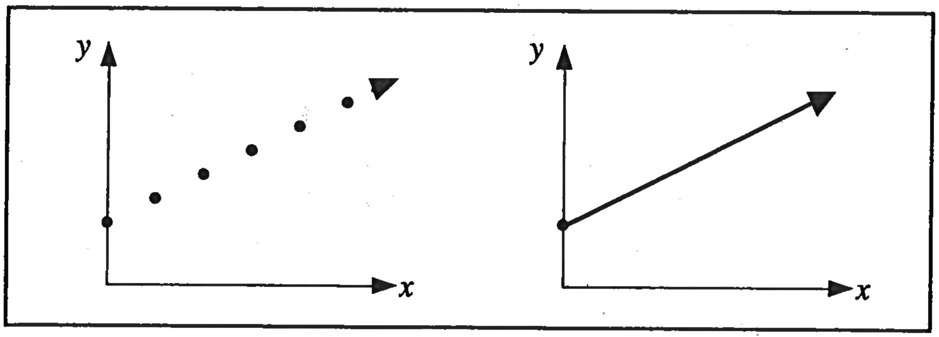
F. $-2 < x < 2$

G. $-2 \leq x \leq 2$

H. $2 < x < -2$

I. $x \in R$

Multiple Choice **11.** The graphs of two relations are shown. Which of the following statements is true?



A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **12.** The relation between the distance traveled, d km, and the cost, C dollars, of renting a truck is given by the formula $C = 60 + 0.27d$. The domain of the relation can be expressed in the form $d \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes and the value of x in the last two boxes.

(Record your answer in the numerical response box from left to right)

Answer Key

1. See chart below

	N	W	I	Q	$\overline{Q}$	R
$\frac{1}{3}$				√		√
123 983	√	√	√	√		√
−2			√	√		√
$7.5\overline{34}$				√		√
9.5				√		√
$\sqrt{75}$					√	√
$-\pi$					√	√
$-\frac{355}{113}$				√		√
$-\sqrt{49}$			√	√		√
0.000005				√		√
2.232425...					√	√

2. See below.

- For natural numbers and whole numbers, the property is true for addition and multiplication only.
- For integers, the property is true for addition, subtraction, and multiplication.
- For rational numbers and real numbers, the property is true for addition, subtraction, multiplication, and division.

3. a) $D=\{2,0,4,-1,-3\}$ $R=\{3,2,8,1\}$
b) $D=\{-3,0,5,-8\}$ $R=\{3,-5,-2,1\}$
c) $D=\{0,2,4,6\}$ $R=\{3,4,5\}$
d) $D=\{2,0,-3\}$ $R=\{3,4,5,6\}$
e) $D=\{1,-1,3,7\}$ $R=\{5\}$
f) $D=\{2,3,5,7\}$ $R=\{0,1,8,9\}$
g) $D=\{2,4,6,8\}$ $R=\{1,3,5\}$
h) $D=\{3,5,2,4\}$ $R=\{0,1,6\}$

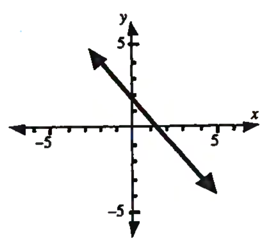
4. a) $D=\{-1,0,3,4\}$ $R=\{5,0,2,1\}$
b) $D=\{x\in R\}$ $R=\{y\in R\}$
c) $D=\{x\geq -8, x\in R\}$ $R=\{y\geq 4, y\in R\}$
d) $D=\{-5\leq x\leq 5, x\in R\}$ $R=\{-4\leq y\leq 4, y\in R\}$
e) $D=\{a\geq -10, a\in R\}$ $R=\{b\in R\}$
f) $D=\{x\in R\}$ $R=\{y\leq 4, y\in R\}$
g) $D=\{x\in R\}$ $R=\{8\}$
h) $D=\{d<5, d\in R\}$ $R=\{t\leq 4, t\in R\}$

5. a) $D=\{-6\leq x\leq 4, x\in R\}$ $R=\{7\leq y\leq 17, y\in R\}$
b) $D=\{-23\leq x\leq 17, x\in R\}$ $R=\{-25\leq y\leq 15, y\in R\}$
c) $D=\{-8\leq x\leq 7, x\in R\}$ $R=\{-2\leq y\leq 10, y\in R\}$
d) $D=\{-65\leq x\leq -35, x\in R\}$ $R=\{-75\leq y\leq -25, y\in R\}$

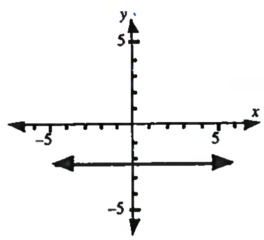
6. a) $D=\{x\in R\}$ $R=\{y>0, y\in R\}$, y-int is 400
b) $D=\{t\geq 0, t\in R\}$ different from a) because time is never a negative value.
$R=\{A\geq 400, A\in R\}$ different from a) because the amount of money can never be less than \$400.

7. Answers may vary.

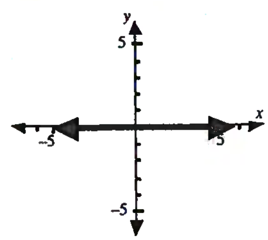
a)



b)

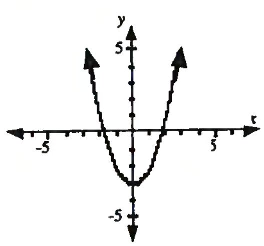


c)

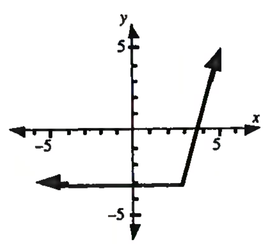


8. Answers may vary.

a)



b)



9. a) h–int = 0, d–int = 0 and 200. On the tee the ball is on the ground. It returns to ground level 200 m from the tee.
b) max height = 25 m. The maximum height is the upper limit of the range.
c) $D=\{0\leq d\leq 200, d\in R\}$ $R=\{0\leq h\leq 25, h\in R\}$
d) 55 m from the tee when the ball is rising and 145m from the tee when the ball is descending.
e) 24 m
f) Starting from a height of 0 m at the tee, the golf ball increases in height to a maximum height of 25m, 100 m from the tee. Then the golf ball starts decreasing in height until it hits the ground 200 m from the tee.

10. i) A ii) C iii) G iv) F v) D vi) F vii) I

11. D

12.

6	0	0	0

Functions Lesson #1:
Functions

Review

We have considered six ways in which the relationship between two quantities can be represented.

- in words
- a mapping (or arrow) diagram
- function notation (this unit)
- a table of values
- an equation
- a set of ordered pairs
- a graph

In a **relation** each element of the **domain** (the **input**) is related to an element or elements of the **range** (the **output**).

In this lesson we will study a special type of relation called a **function**.

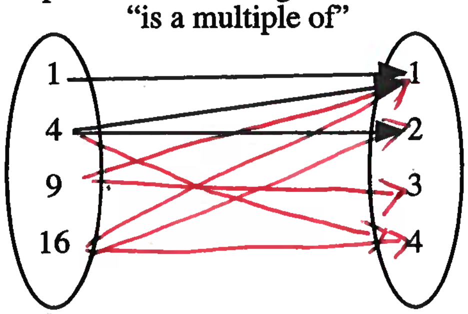
Exploration

To illustrate the concept of function, we will look at two relations described in words with domain $D = \{1, 4, 9, 16\}$ and range $R = \{1, 2, 3, 4\}$.

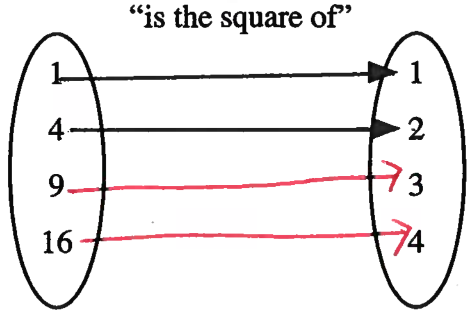
i) "is a multiple of" **ii)** "is the square of"

a) Complete the arrow diagrams.

i) "is a multiple of"



ii) "is the square of"



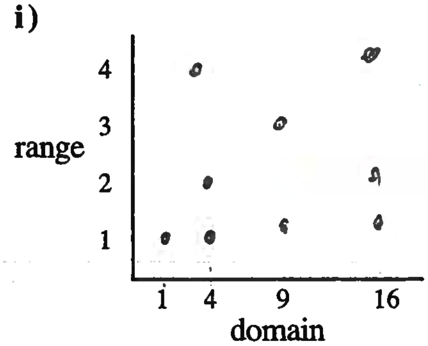
b) Complete the set of ordered pairs.

i) (1, 1), (4, 1), (4,2) (4,4)(9,1)(9,3) (16, 1), (16,2), (16,4)

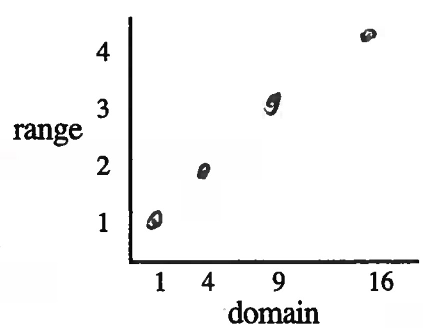
ii) (1,1), (4,2), (9,3) (16,4)

c) Plot the ordered pairs on the grid.

i)



ii)



Function

A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Class Ex. #1

In the exploration on the previous page, one of the relations is a function, and the other relation is not a function.

Explain how we can determine which relation is a function by looking at the following:

a) arrow diagrams

the domain can only point to one range

b) ordered pairs

the x value can not be repeated

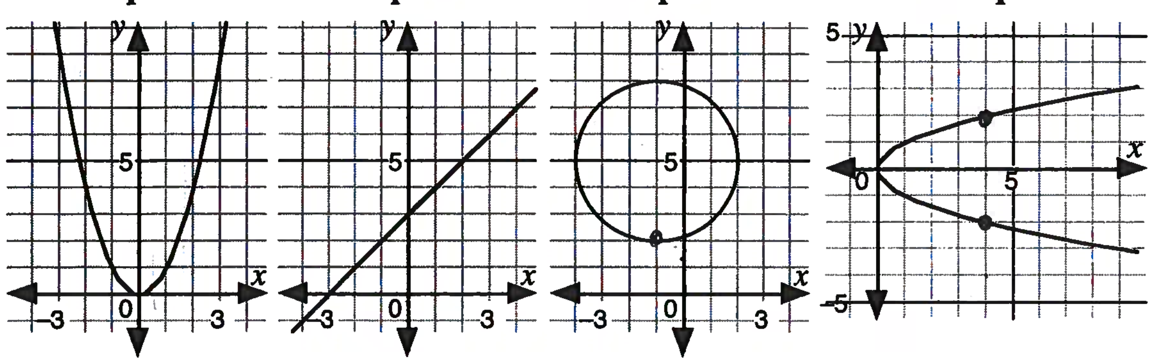
c) graphs

horizontal would only have one vertical output

Class Ex. #2

Each of the following is the graph of a relation.

Graph A | Graph B | Graph C | Graph D



a) Classify the following statements as true (T) or false (F).

	A	B	C	D
• For each input value there is only one output.	T	T	F	F
• For each output value there is only one input.	F	T	F	T
• The relation is a function.	T	T	F	F

b) From graph C, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

x = −1 (−1, 8) (−1, 2)

c) From graph D, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

x = 4 (4, −2), (4, 2)

d) On graphs A and B draw a series of vertical lines. Do any of these lines intersect the graph of the relation at more than one point?

Vertical Line Test

The vertical line test can be used on the graph of a relation to determine whether the relation is a function or not.

→ vertical line test is like moving your pen vertically across the graph

- If every vertical line, drawn on the domain of the relation, intersects the graph exactly once, then the relation is a function.
- If any vertical line intersects the graph more than once, then it is **not** a function.

Class Ex. #3

Determine which of the following are functions. Explain your answers.

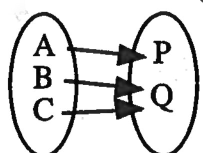
a) (5, 8), (6, 7), (−5, 3), (2, 3), (6, 8)

not a function
input 6 gives two outputs

b) (3, 3), (2, 3), (4, 5), (−3, 2)

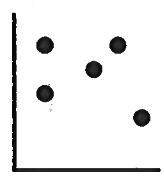
Function ✓

c)



Function ✓

d)

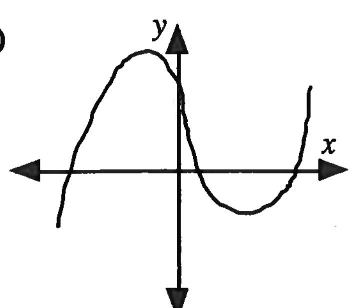


not a function
fails vertical line test

e) The relation connecting the provinces and territories of Canada with their capital cities.

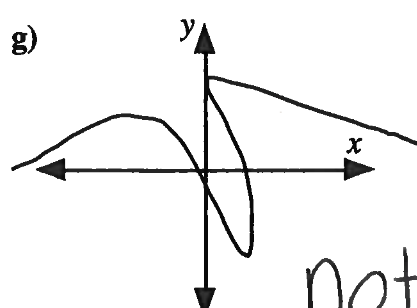
Function

f)



Function ✓

g)



not a function

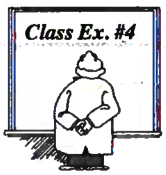
A Function as a Mapping

A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.

If the function f maps an element x in the domain to an element y in the range, we write $f: x \to y$.

Complete the following for the function "is the square of" on the first page of this lesson.

$1 \to$ 1 $\quad 4 \to$ 2 $\quad 9 \to$ 3 $\quad$ 16 → 4



Consider the function $f: x \to 3x + 1$, for domain $\{-1, 0, 1, 2\}$.

a) Complete $\quad -1 \to$ −2 $\quad$ 0 → 1 $\quad$ 1 → 4 $\quad$ 2 → 7

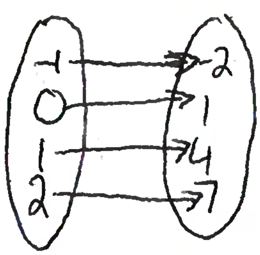
3x+1
3(−1)+1
−3+1 = −2

b) List the elements of the range of the function.

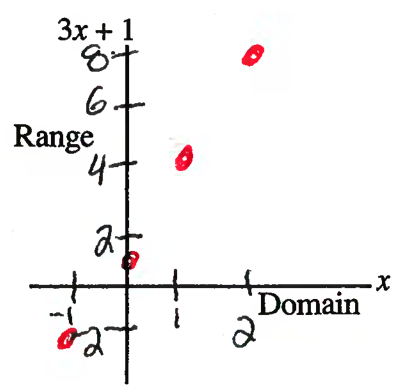
Range: {−2, 1, 4, 7}

c) Show the function as:

i) an arrow diagram $\quad$ **ii)** a set of ordered pairs $\quad$ **iii)** a Cartesian graph.



(−1, −2)
(0, 1)
(1, 4)
(2, 7)



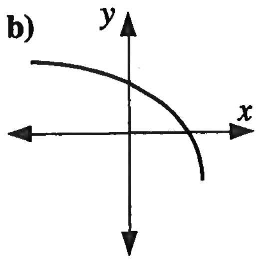
At this time we label the top of the vertical axis with $3x + 1$. In the next lesson we will learn function notation which is more commonly used.

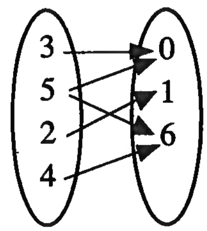
Complete Assignment Questions #1 - #12

Assignment #1-3, 5, 7

1. Determine which of the following relations are functions. Give reasons for your answers.

a) (–1, 3), (–2, 1), (5, 2), (7, 3)

b)


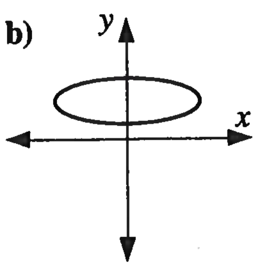
c)


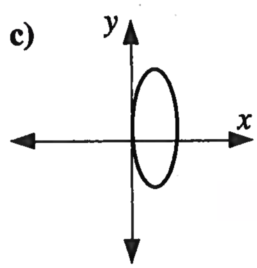
d)

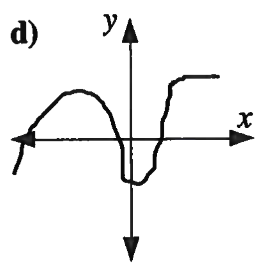
Input (x)	Output (y)
2	3
0	4
–3	5
2	6

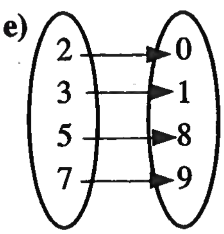
2. State which of the following relations are functions.

a) (0, 0), (1, 2), (2, 3), (3, 4), (4, 3)

b)


c)


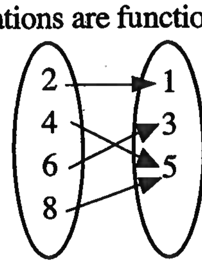
d)


e)


3. State which of the following relations are functions.

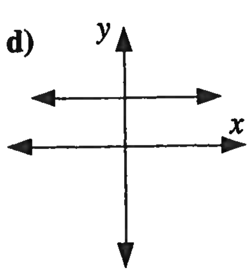
a)

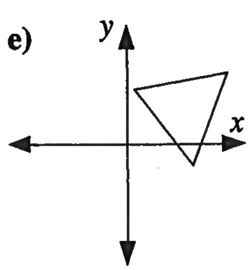
Input (x)	Output (y)
0	3
2	4
4	5
6	3

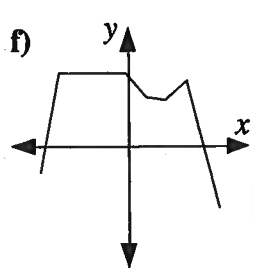
b)


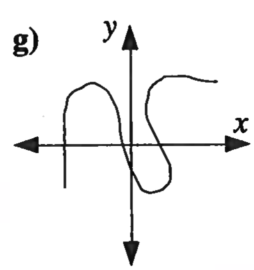
c)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

d)


e)


f)


g)


4. Mr. A has a son Jim and a daughter Kristen. Mr. B has three daughters, Lauren, Melanie, and Noreen.

a) Draw an arrow diagram to illustrate the relation "is the father of" from the set of fathers to the set of children. Is the relation "is the father of" a function?

b) Draw an arrow diagram to illustrate the relation "is the child of" from the set of children to the set of fathers. Is the relation "is the child of" a function?

5. The function $f: x \to 2x + 5$ has domain $\{0, 1, 2, 3\}$.

a) List the elements of the range of the function.

b) Show the function f in a Cartesian graph.

6. The function $g: x \to x^3$ has domain $\{-2, -1, 0, 1, 2\}$.

a) List the elements of the range of the function.

b) Show the function g in a Cartesian graph.

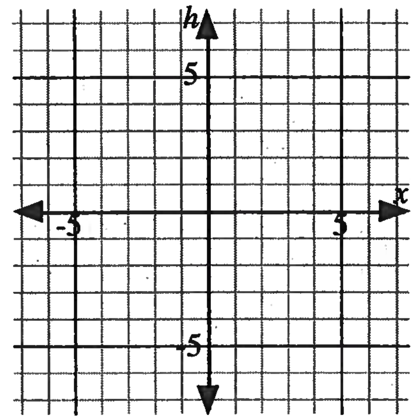
7. Consider the function $f: x \to x^2 - 4$.

a) Complete the following table of values.

Elements of Domain	3	2	1	−1	−2	−3
Elements of Range						

b) Plot the ordered pairs on a Cartesian graph.

c) Draw a smooth curve through the points to illustrate the function $f: x \to x^2 - 4,\ x \in R$.



8. The domain of the function $h: x \to 6$ is $\{0, 10, 20\}$.

a) List the ordered pairs of the graph of the function.

b) Show the function h in a Cartesian graph.

Multiple Choice

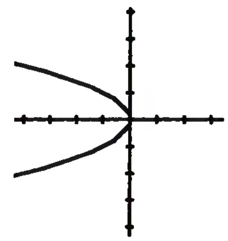
9. The function $f: x \to 6 - 2x$ has domain $\{0, 2, 4, 6, 8\}$.
Which of the following is **not** an element of the range of the function?

A. −10
B. 2
C. 4
D. −6

10. Which of the following statements is not always true for a function?

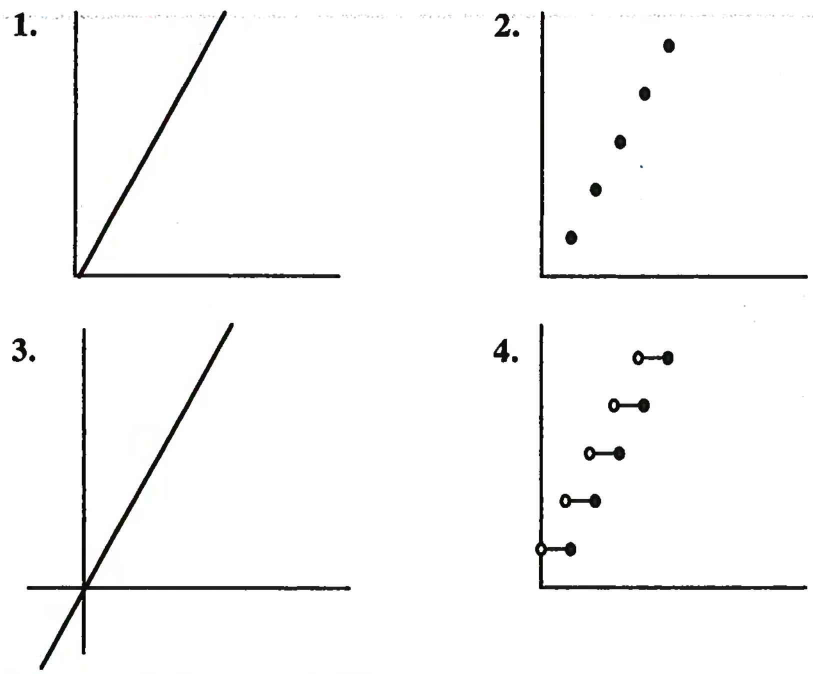
A. A function is a set of ordered pairs (x, y) in which for every x there is only one y.
B. A vertical line must not intersect the graph of a function in more than one point.
C. For every output there is only one input.
D. For every element in the domain, there is only one element in the range.

11. Which of the following represents a function?

1	"multiply the number by 3 and add 5."	2	$y = -x^2$
3	$(9,3), (4,2), (1,1), (0,0). (1,-1), (4,-2), (9,-3)$	4	

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. some other combination of 1 - 4

Numerical Response **12.** Partial graphs of four functions are shown.



The functions are described as follows:

A: Coffee costs \$8 per jar. Graph cost as a function of the number of jars purchased.

B: Distance cycled at a constant speed of 8 km/h. Graph distance as a function of time.

C: Parking costs \$8 per hour (or part of an hour). Graph cost as a function of time.

D: Set of ordered pairs which satisfy the equation $y = 8x, x \in R$.
Graph y as a function of x.

Place the graph number for function A in the first box.
Place the graph number for function B in the second box.
Place the graph number for function C in the third box.
Place the graph number for function D in the last box.
(Record your answer in the numerical response box from left to right)

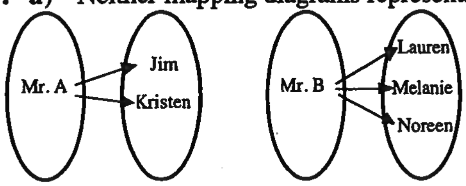
Answer Key

1. a) function: each first coordinate has only one second coordinate
   b) function: vertical lines intersects the graph exactly once
   c) not a function: the input 5 has two outputs
   d) not a function: the input 2 has two outputs

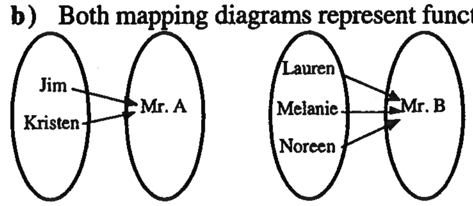
2. a) function b) not a function c) not a function d) not a function e) function

3. a) function b) function c) function d) function e) not a function
   f) function g) not a function

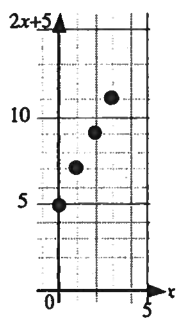
4. a) Neither mapping diagrams represents a function



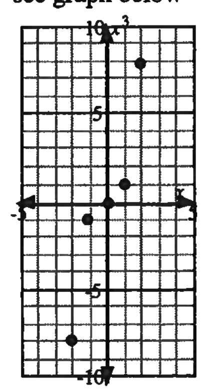
   b) Both mapping diagrams represent functions



5. a) $\{5, 7, 9, 11\}$
   b) see graph below

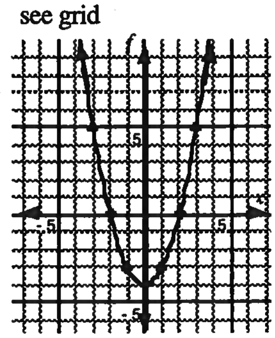


6. a) $\{-8, -1, 0, 1, 8\}$
   b) see graph below

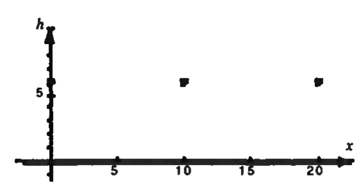


7. a) see table b) see grid c) see grid

Elements of Domain	3	2	1	–1	–2	–3
Elements of Range	5	0	–3	–3	0	5



8. a) $\{(0, 6), (10, 6), (20, 6)\}$
   b)



9. C 10. C 11. B 12.

2	1	4	3

Functions Lesson #2:
Function Notation - Part One

Mapping Notation

In the previous lesson we discovered some ways in which functions can be represented:

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (this unit)

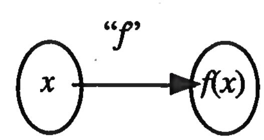
A **function** was defined in mapping notation as follows:

"A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.
If the function f maps an element x in the domain to an element y in the range, we write $f: x \rightarrow y$."

Consider the function $f: x \rightarrow 2x + 3$ defined on the set of real numbers.
Under this function we know that $5 \rightarrow 2(5) + 3$ ie $5 \rightarrow 13$.
We say that under the function f, the **image** of 5 is 13.
We also say that the **value of the function** is 13 when $x = 5$.

Function Notation

In most math courses, function notation is used to replace the mapping notation $f: x \rightarrow 2x+3$. Under a function f, the image of an element x in the domain is denoted by $f(x)$, which is read "f of x".



In the example above, the function f can be defined by the formula $f(x) = 2x + 3$.
The notation $f(x) = 2x + 3$ is called **function notation**.

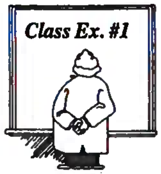
We showed above, that, under the function f, the image of 5 is 13. We write $f(5) = 13$.

mapping notation	function notation	equation of graph of function
$f: x \rightarrow 2x+3$	$f(x) = 2x + 3$	$y = 2x + 3$
$f: 5 \rightarrow 2(5)+3$	$f(5) = 2(5) + 3$	$y = 2(5) + 3$
$f: 5 \rightarrow 13$	$f(5) = 13$	$y = 13$

The symbol $f(x)$ is read as "f at x" or "f of x".
$f(x)$ provides a formula for the function f, and also represents the value of the function for a given value of x.

In function notation:

- $f(x)$ does not mean f times x.
- The "name" of the function is f.
- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**.
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**.



Consider the function $f(x) = x^2 + 5$ and $g(x) = 4 - x$. Evaluate:

a) $f(3)$

f of 3
function at 3
$f(3) = 3^2 + 5 = 14$

b) $g(1)$

$g(1) = 4 - (1)$
$= 3$

c) $f(-2)$

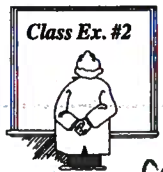
$f(-2) = (-2)^2 + 5$
$= 9$

d) $g(-2)$

$g(-2) = 4 - (-2)$
$= 6$

e) $f(0) - g(0)$

$0^2 + 5 - 4 - 0$
$5 - 4 = 1$



Consider the function f defined by $f(x) = 5x^3 - 2x, x \in R$. Determine:

a) $f(-3)$

$f(-3) = 5(-3)^3 - 2(-3)$
$= -135 + 6 = -129$

b) the value of f when $x = 2$

$f(2) = 5(2)^3 - 2(2)$
$= 40 - 4$
$= 36$

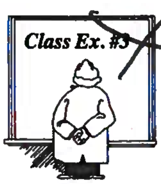
~~**c)** the image of 7 under f~~

e) an expression for $f(a)$

$5a^3 - 2a$

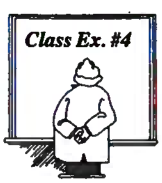
f) an expression for $f(2x)$

$5(2x)^3 - 2(2x)$
$40x^3 - 4x$



If $P(x) = 4x^2 - 6x + 1$, determine a simplified expression for $P(x - 3)$.

Complete Assignment Questions #1 - #7



Consider the function $f(x) = 10x - 3$, $x \in R$.

a) Determine the value of x if $f(x) = 47$.

*Solve for x

$47 = 10x - 3$ (+3, +3)
$50 = 10x$
$\frac{50}{10} = \frac{10x}{10}$
$x = 5$

b) Solve the equation $f(x) = -23$.

$-23 = 10x - 3$ (+3, +3)
$-20 = 10x$
$\frac{-20}{10} = \frac{10x}{10}$
$x = -2$

Class Ex. #5

Consider the function $f(x) = x^2 - 5, x \in R$.

a) Evaluate $f(4)$. input

$f(4) = 4^2 - 5$
$= 16 - 5$
$= 11$

b) Solve the equation $f(x) = 4$. output

$4 = x^2 - 5$
$+5 \quad +5$
$\sqrt{9} = \sqrt{x^2}$
$x = \pm 3$

c) Solve the equation $f(t) = 75$, where $t > 0$. Answer as an exact ~~value and as a~~ decimal to the nearest hundredth.

$75 = t^2 - 5$
$80 = t^2$
$t = \pm\sqrt{80}$

$t \neq -\sqrt{80}$ has to be greater than zero

$t = \sqrt{80}$
$t = 8.94$

Complete Assignment Questions #8 - #13 #1, (2-6)b, 8, 9

Assignment

1. Each statement refers to the function f whose graph has equation $y = f(x)$. Circle the correct choice.

a) f is the *name / value* of the function.
b) The values of x represent the *inputs / outputs* of the function.
c) The values of $f(x)$ represent the *inputs / outputs* of the function.
d) The values of y represent the *inputs / outputs* of the function.
e) x represents the *independent / dependent* variable of the function.
f) $f(x)$ represents the *independent / dependent* variable of the function.
g) y represents the *independent / dependent* variable of the function.

2. If $f(x) = 5x - 7$, determine:

a) $f(2)$

$f(2) = 5(2) - 7$
$f(2) =$

b) $f(-3)$

c) $f(0)$

3. Function g is defined by $g(x) = 6 - x^2$. Evaluate

a) $g(4)$ **b)** $g(-6)$ **c)** $g(\sqrt{3})$

4. A function f is defined by the formula $f(x) = x^3 + 1$. Find

a) the image of 2 under f **b)** the value of f at -7. **c)** an expression for $f(a)$

5. If $f(x) = x^3 - 2x^2 - x - 5$, evaluate

a) $f(5)$ **b)** $f(-3)$

6. Consider the function f defined by $f(x) = 8 - 2x$, $x \in R$. Determine

a) $f(4)$ **b)** the value of f when $x = -4$ **c)** the image of 0.5 under f

d) an expression for $f(2t)$ **e)** an expression for $f(a + 3)$

7. If $F(x) = 3x^2 - 2x - 9$, determine a simplified expression for

a) $F(-x)$ **b)** $F(x - 5)$

8. a) If $f(x) = 5x - 7$, then determine the value of x if $f(x) = 43$.

b) If $g(x) = 6x + 3$, then determine the value of x if $g(x) = -24$.

c) If $g(t) = 56 - 3t$, then determine the value of t if $g(t) = 11$.

d) If $h(x) = -3x + 1$, then determine the value of x if $h(x) = 22$.

e) If $P(x) = 50 - 3x^2$, then determine the values of x if $P(x) = -25$.

9. Consider the function f defined by $f(x) = 6x - 15$. Find

a) $f(0)$ **b)** an expression for $f(2x + 1)$ **c)** the solution to the equation $f(x) = 27$

10. A function C is defined by $C(x) = \sqrt{x}$ where $x \geq 0$.

a) Evaluate

i) $C(16)$ **ii)** $C\left(\frac{1}{36}\right)$ **iii)** $\frac{C(100)}{C(4)}$

b) If $C(x) = 9$, find x.

11. A function g is defined by the formula $g(t) = t + 12$.

a) Calculate the value of $g(4) + g(-2)$.

b) If $g(a^2) = 48$, determine all possible values of a.

Multiple Choice **12.** If $f(x) = 3x - 1$ and $f(t) = 8$, then $t =$

A. $\frac{7}{3}$

B. 3

C. $\frac{11}{3}$

D. 23

Numerical Response **13.** A function f is defined by the formula $f(x) = 8\sqrt{x}$, $x \in R$.
The value of $f(144)$ is _____ .

(Record your answer in the numerical response box from left to right)

Further assignment questions on Function Notation - Part One will appear in the assignment of the next lesson, Function Notation - Part Two.

Answer Key

1. a) name **b)** inputs **c)** outputs **d)** outputs
e) independent **f)** dependent **g)** dependent

2. a) 3 **b)** -22 **c)** -7 **3. a)** -10 **b)** -30 **c)** 3

4. a) 9 **b)** -342 **c)** $a^3 + 1$ **5. a)** 65 **b)** -47

6. a) 0 **b)** 16 **c)** 7 **d)** $8 - 4t$ **e)** $2 - 2a$

7. a) $3x^2 + 2x - 9$ **b)** $3x^2 - 32x + 76$

8. a) 10 **b)** $-\frac{9}{2}$ **c)** 15 **d)** -7 **e)** ± 5

9. a) -15 **b)** $12x - 9$ **c)** $x = 7$ **10.a) i)** 4 **ii)** $\frac{1}{6}$ **iii)** 5 **b)** 81

11.a) 26 **b)** ± 6 **12.** B **13.**

9	6		

Functions Lesson #3:
Function Notation - Part Two

Graphing a Function

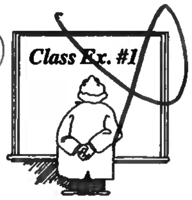
Consider the function $f(x) = 3x + 1$. The values of x represent the inputs and make up the domain of the function. The values of $f(x)$ represent the outputs and make up the range of the function.

In previous lessons, we have used y to represent the outputs and the range of a relation. We can therefore write the function $f(x) = 3x + 1$ in x-y notation as $y = 3x + 1$.

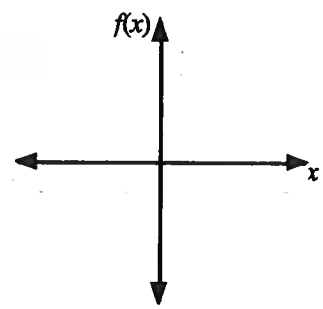
The function $f(x) = 3x + 1$ can be written in x-y notation as shown.

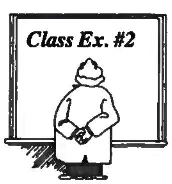
<u>Function notation</u>	<u>x-y notation</u>
$f(x) = 3x + 1$	$y = 3x + 1$

- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis.**
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis.**



Use a graphing calculator to sketch the graph of the function $f(x) = 3x + 1$.





a) In each case, express the relation given in function notation as an equation in two variables.

i) $f(x) = 7x - 23$ **ii)** $g(t) = t^2 - 2t + 35$

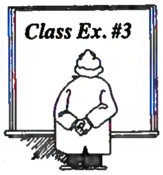
$y = 7x - 23$ $y = t^2 - 2t + 35$

b) Express the relation $y = 11x - 15$ in function notation.

$f(x) = 11x - 15$

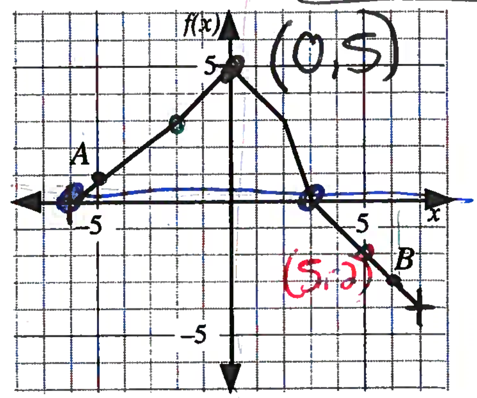
c) The graph of the function defined by $y = f(x)$ has equation $y = 4 - 3x$. Express the equation in function notation.

$f(x) = 4 - 3x$



Class Ex. #3

The graph of a function f is shown.



a) Complete

i) $f(5) =$ ii) $f(-2) =$ iii) $f(4) =$

b) Write the ordered pairs associated in **a)**.

i) ii) iii)

c) State the value(s) of x if

i) $f(x) = -1$ ii) $f(x) = 3$ iii) $f(x) = 4$

d) Use the notation in **a)** to make a statement about the points A and B on the graph.

e) Write the x- and y- intercepts of the graph using function notation.

f) Complete the following statements.

- The domain of f is $\{x \mid \_\_\_\_ \le x \le \_\_\_\_, x \in R\}$
- The range of f is $\{f(x) \mid \_\_\_\_ \le f(x) \le \_\_\_\_, f(x) \in R\}$

Complete Assignment Questions #1 - #12

Assignment

1. In each case, express the relation given in function notation as an equation in two variables.

a) $f(x) = 10 - 3x$ **b)** $g(x) = 12x^2 - 5$ **c)** $P(t) = 2t + 9$

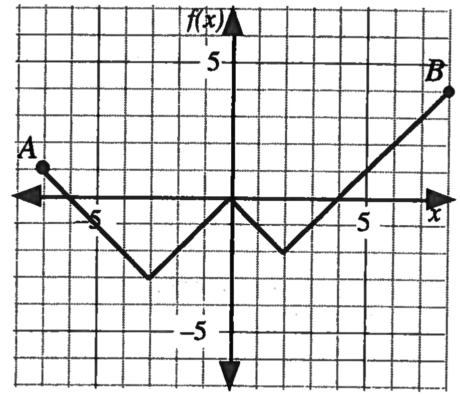
2. Express the following relations in function notation.

a) $y = 17x - 9$ **b)** $y = 4v + 25$ **c)** $x + 2y + 6 = 0$

3. a) The graph of the function defined by $y = f(x)$ has equation $y = 0.5x - 0.25$
Express the equation in function notation.

b) The graph of the velocity function defined by $v = f(t)$ has equation $v = 4.9t^2$.
Express the equation in function notation.

4. The graph of a function f is shown.



a) Complete
i) $f(3) =$ **ii)** $f(-3) =$ **iii)** $f(-6) =$

b) Write the ordered pairs associated with **a)**.
i) **ii)** **iii)**

c) State the value(s) of x if
i) $f(x) = 3$ **ii)** $f(x) = -2$ **iii)** $f(x) = -4$

d) Use the notation in a) to make a statement about the points A and B on the graph.

e) Write the x- and y- intercepts of the graph using function notation.

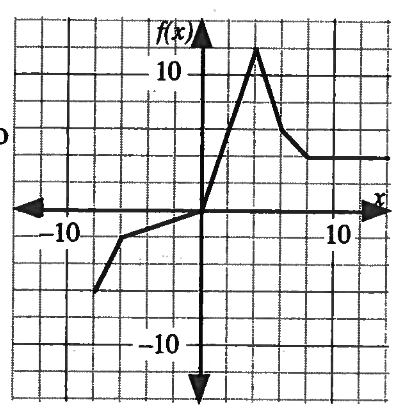
f) Complete the following statements.
- The domain of f is $\{x \mid \_\_\_\_ \le x \le \_\_\_\_, x \in R\}$
- The range of f is $\{f(x) \mid \_\_\_\_ \le f(x) \le \_\_\_\_, f(x) \in R\}$

5. The function $g(x) = 3x^2 - 4$ has a domain $\{-2, -1, 0, 1, 2\}$.
a) State the range of g. **b)** Solve the equation $g(x) = -1$.

6. Consider the graph of the function f shown below.

a) Complete the table.

b) Explain why the solution to the equation $f(x) = 4$ has an infinite number of solutions.



x	$f(x)$	Ordered Pair
		(2,)
	0	
–6		
8		
	–6	
10		

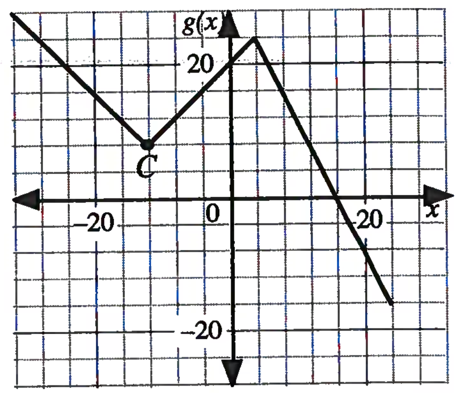
7. Given that $f(x) = 9 - 2x$

a) evaluate $f(-3)$

b) find the value of $f(t) + f(-t)$

c) calculate the x-intercept and the y-intercept on the graph of f.

8. The graph of a function is shown.



a) A student is asked to make a statement about point C on the graph. The student states that $f(-3) = 2$.

i) Explain **two** errors in the student's statement.

ii) Write a correct statement using function notation about point C.

b) Give the solution to the following equations.

i) $g(x) = -8$ **ii)** $g(x) = 16$.

c) State the value of **i)** $g(-8)$ **ii)** $g(16)$

d) State the domain and range of the function.

e) The equation $g(a) = b$ has **exactly two** solutions. Explain clearly how to use the graph to determine values of a and b, and provide two sets of answers to the problem.

9. Consider the function $f(x) = 1 - x^2$, where x is an integer.

a) Evaluate $f(2) - f(-1)$ **b)** Given that $f(a) = -8$, calculate all possible values of a.

Multiple Choice

10. The graph of the function $f(x) = 4^x$, $x \in R$, intersects the y-axis at

A. $(0, 0)$
B. $(0, 1)$
C. $(0, 4)$
D. no point

Use the following information to answer the next question.

Function P is such that $P(5) = -1$.
Two students each make a statement about the function P.

- Rose states "When the domain value is 5, the related range value is -1."
- Susan states "The point $(-1, 5)$ is on the graph of $y = P(x)$."

11. Which of the following is true?

A. Both statements are correct.
B. Both statements are incorrect.
C. Rose is correct and Susan is incorrect.
D. Susan is correct and Rose is incorrect.

Numerical Response **12.** Consider the graph of the function $f(x) = 5x - 11$. The x-intercept of the graph of f is located at $(a, 0)$. The value of a is ______ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $y = 10 - 3x$ **b)** $y = 12x^2 - 5$ **c)** $y = 2t + 9$

2. a) $f(x) = 17x - 9$ **b)** $f(v) = 4v + 25$ **c)** $f(x) = -\frac{1}{2}x - 3$

3. a) $f(x) = 0.5x - 0.25$ **b)** $f(t) = 4.9t^2$

4. a) i) -1 **ii)** -3 **iii)** 0

b) i) $(3, -1)$ **ii)** $(-3, -3)$ **iii)** $(-6, 0)$

c) i) 7 **ii)** $-4, -2, 2$ **iii)** no solution **d)** A is $f(-7) = 1$, B is $f(8) = 4$

e) x-intercepts can be represented in function notation by; $f(-6) = 0, f(0) = 0, f(4) = 0$
y-intercept can be represented in function notation by $f(0) = 0$

f) $-7 \le x \le 8, \ -3 \le f(x) \le 4$

5. a) Range $= \{-4, -1, 8\}$ **b)** $x = \pm 1$

6. See table below.

x	$f(x)$	Ordered Pair
2	6	(2, 6)
0	0	(0, 0)
-6	-2	(-6, -2)
8	4	(8, 4)
-8	-6	(-8, -6)
10	4	(10, 4)

b) The horizontal line where $f(x) = 4$ has an infinite number of input values between 8 and 14.

7. a) 15 **b)** 18 **c)** x-int $= \frac{9}{2}$, y-int $= 9$

8. a) i) The name of the function is g not f. The scale is 4 units per box, not 1 unit per box.

ii) $g(-12) = 8$

b) i) $x = 20$ **ii)** $x = -20, -4, 8$

c) i) 12 **ii)** 0

d) Domain $= \{x \mid -32 \le x \le 24, x \in R\}$, $\{g(x) \mid -16 \le g(x) \le 28\}, g(x) \in R$

e) A horizontal line must intersect the graph at exactly two points.
This occurs when $g(x) = 24$ and when $g(x) = 8$.
Solution 1: $b = 24$ when $a = -28$ or 4.
Solution 2: $b = 8$ when $a = -12$ or 12

9. a) -3 **b)** ± 3 **10.** B **11.** C **12.**

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