

# the function machine

domain

x-values

independent

Domain  
(left-right)  
all possible  
x-values

Range: (up/down)  
set of all possible  
y-values (output)

range

y value

dependent

A function is a  
relationship  
with one or more  
variable

where each  
input

has a single  
output

# functions

Each x-value is only allowed to correspond to ONE y value

This relation is a function because none of the input values (x-values) has more than one different output (y-value).

x	0	1	2	3	4
y	0	3	6	9	12

This relation is NOT a function because at least one of the input values (x-values) has more than one different output (y-value).

x	-3	2	-3	5	8
y	12	5	-1	2	-2

## notation

When dealing with functions, you will see f(x) in place of y.

How to say it out loud: "fof x"

## evaluating

To evaluate a function for a particular x-value, just substitute and then simplify!

**Example:** If  $f(x) = 2x + 1$ , find  $f(3)$ .

Work:  $f(3) = 2(3) + 1$

Answer:

7  
output

When we put something in,  
we can always expect a consistent result  
to come back out.

# determining whether a relation is a function

Add circles, arrows, lines, etc. to demonstrate why each relation is/isn't a function.

## table

Review the columns. The relation will not be a function if any x-value corresponds to more than one different y-value.

A

x	y
-1	1
0	0
1	1

B

x	y
3	-6
0	1
3	6

## set notation

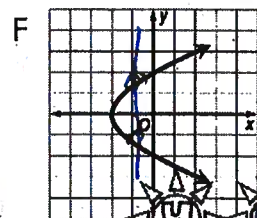
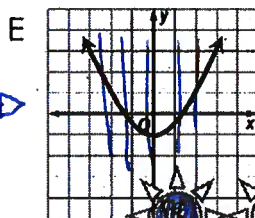
Review each ordered pair. The relation will not be a function if any x-coordinate corresponds to more than one different y-coordinate.

C  
 {(3, 3), (4, -1), (2, 3)}

D  
 {(1, 8), (0, -2), (1, -3)}

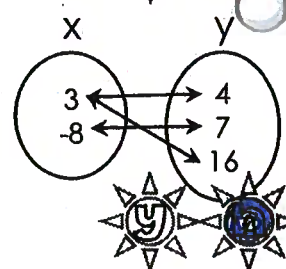
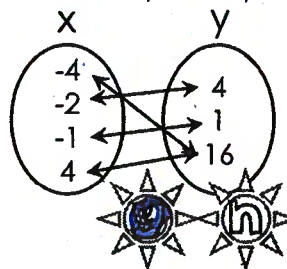
## graph

Use the vertical line test. The relation will not be a function if a vertical line ever passes through more than one point.



## mapping diagram

Review the arrows. The relation will not be a function if any x-values maps to more than one different y-value.



## finding domain and range

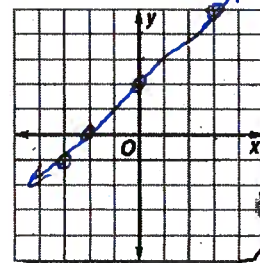
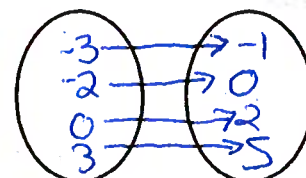
The domain is the set of all possible x-values.  
 The range is the set of all possible y-values.

Identify the domain and range of the relations in the "table," "set notation," and "graph" examples above.

	Domain	Range
A	$\{-1, 0, 1\}$	$\{0, 1\}$
B		
C		
D		
E	$\{x \in \mathbb{R}\}$	$\{y \geq -1, y \in \mathbb{R}\}$
F	$\{x \geq -2, x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$

## Try it

Create a mapping diagram and a graph that each represent functions.



# Functions Lesson #4: Function Notation and Problem Solving

## Using Function Notation

In the previous unit we solved problems about relations defined by an equation. In this lesson we will solve problems where function notation is used to define the relation.

On page 256, assignment question #6, we had the following scenario.

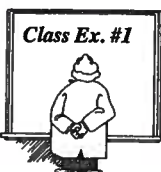
“A candle manufacturer found that their “Long-Last” candles melted according to the formula  $h = -2t + 12$ , where  $h$  is the height of the candle, in cm, after  $t$  hours.”

The relation between height and time is described by an equation.

The relation is a function because for each input there is only one output, and so it can be described using the **function notation** below.

“A candle manufacturer found that their “Long-Last” candles melted according to the formula  $h(t) = -2t + 12$ , where  $h$  is the height of the candle, in cm, after  $t$  hours.”

In this example, the notation  $h(4)$  is a simplified way of representing the height of the candle after four hours.



A candle manufacturer found that their “Long-Last” candles melted according to the formula  $h(t) = -2t + 12$ , where  $h$  is the height of the candle, in cm, after  $t$  hours.

a) Use a graphing calculator to sketch the graph of the function and show the graph on the grid.

b) Determine the value of  $h(5)$ .

c) Write in words the meaning of  $h(5)$ .

after 5 hours, my candle is 3cm

d) Evaluate the following, and explain the significance of each.

i)  $h(0) = 12$

new candle

ii)  $h(6) = 0$

burnt out candle

iii)  $h(8)$

does not exist, candle is done already.

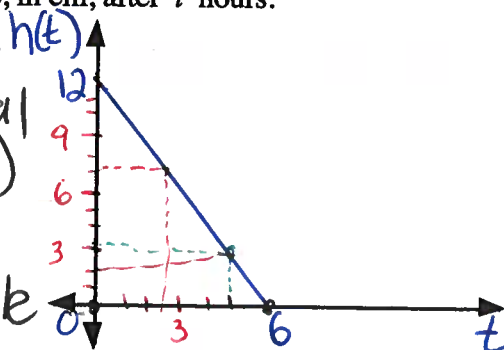
e) How long will it take for the candle to burn down to a height of 7 cm?

2.5 hours

f) Suggest an appropriate domain and range for the function.

$d: \{0 \leq t \leq 6\}$

$r: \{0 \leq h(t) \leq 12\}$



## Complete Assignment Questions #1 - #4

**Assignment**

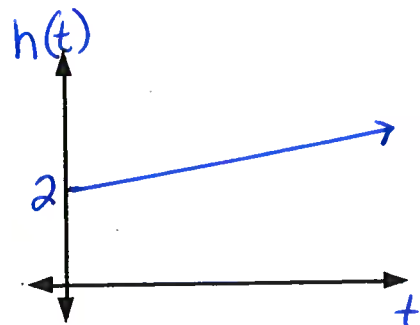
#1

1. Ivory the botanist treated a 2 cm plant with a special growth fertilizer. With this fertilizer, the plant grew at a rate modelled by the function  $H(t) = \frac{5}{3}t + 2$ , where  $H(t)$  represents the height of the plant in cm after  $t$  days.

a) ~~Use a graphing calculator to sketch the graph of the function and show the graph on the grid.~~

b) Determine the value of  $H(3)$ .

c) Write in words the meaning of  $H(3)$ .



d) Evaluate the following.

i)  $H(0)$

ii)  $H(6)$

iii)  $H(21)$

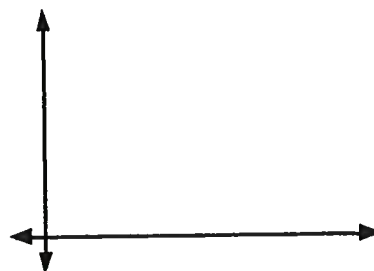
e) How long will it take for the plant to reach a height of 21 cm?

f) It takes 27 days for the plant to mature (to reach maximum height). State the domain and range of the function  $H(t)$ .

2. The cost to Inner Technology of producing IT graphing calculators can be modelled by the function  $C(n) = 11750 + 32n$ , where  $C(n)$  represents the cost in dollars of producing  $n$  calculators.

a) Sketch the graph of the function for a maximum of 4000 calculators.

b) Determine the value of  $C(30)$ .



c) Write in words the meaning of  $C(30)$ .

d) Evaluate  $C(0)$  and explain its significance.

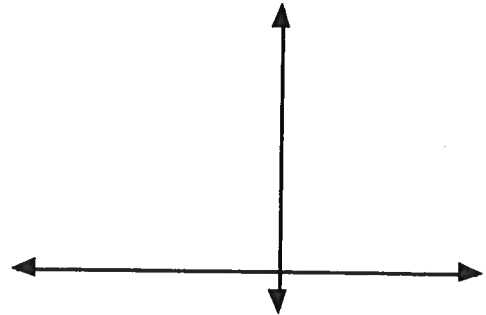
- e) How many calculators can be produced for \$31 270?
- f) Last month IT produced 2 600 calculators and spent \$14 000 on advertising. If there are other fixed monthly costs of \$24 500, and each calculator sells for \$165, how much profit would be made if all the calculators are sold?

3. Over the last 10 years, data was recorded for the number of cups of hot chocolate sold at BGB Senior High School. It was found from the data that the warmer the weather, the less cups of hot chocolate were sold. The data can be modelled by the formula  $N(t) = 150 - 10t$ , where  $N(t)$  is the daily number of cups of hot chocolates sold when the average daily temperature is  $t$  °C.

a) Sketch the graph of the function on the grid provided.

b) Determine the value of  $N(-5)$ .

c) Write in words the meaning of  $N(-5)$ .



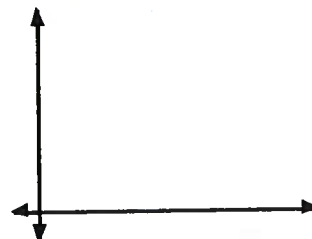
d) What was the average temperature if 190 cups of hot chocolate were sold?

e) Explain how to estimate the lower limit of the domain of the relation.

f) Suggest an appropriate domain and range for the function  $N(t)$  if BGB High School is located in the Okanagan, British Columbia.

4. A special type of weather balloon follows a path which can be represented by the formula  $h(t) = -9t^2 + 900t$ , where  $h(t)$  is the height in cm after  $t$  minutes.

- a) Sketch the graph of the function on the grid.  
b) Determine the value of  $h(30)$  and  $h(70)$ .



- c) Does  $h(30) = h(70)$ ? Do they mean the same thing? Explain.

- d) Evaluate the following, and explain their significance in the context of the question.

i)  $h(0)$                       ii)  $h(100)$                       iii)  $h(110)$

- e) What is the highest point the balloon will reach?

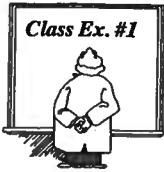
- f) When will the balloon land?

- g) Suggest an appropriate domain and range for the function  $h(t)$ ?

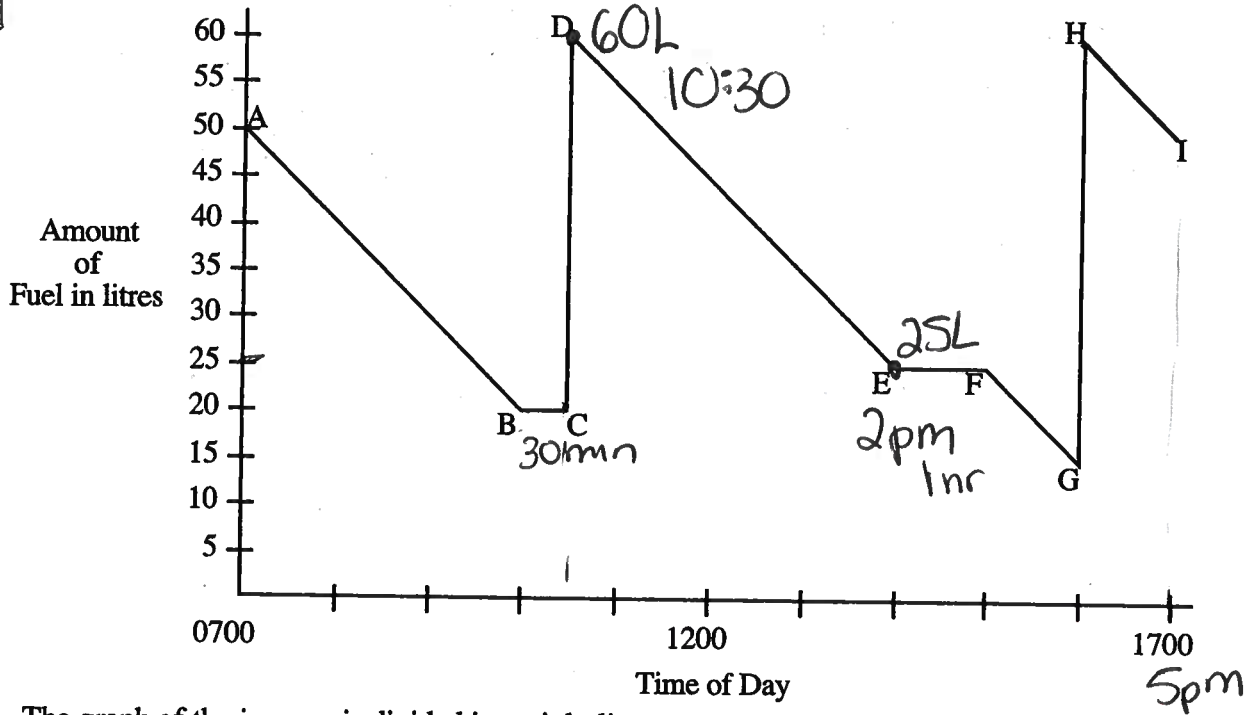
### Answer Key

1. b) 7                      c) After 3 days the height is 7 cm.                      d) i) 2 ii) 12 iii) 37                      e) 11.4 days  
f) domain  $\{t \mid 0 \leq t \leq 27, t \in R\}$                       range  $\{H(t) \mid 2 \leq H(t) \leq 47, H(t) \in R\}$
2. b) 12710                      c) It costs \$12 710 to produce 30 calculators.  
d)  $C(0) = 11\,750$ . There are fixed costs of \$11750 before any calculators are produced.  
e) 610                      f) \$295 550
3. b) 200                      c) 200 cups are sold when the average temperature is  $-5^\circ\text{C}$ .                      d)  $-4^\circ\text{C}$   
e) Estimate the minimum average daily temperature.  
f) Answers may vary. domain  $\{t \mid -20 \leq t \leq 15, t \in R\}$                       range  $\{N(t) \mid 0 \leq N(t) \leq 350, N(t) \in W\}$
4. b) both = 18 900  
c) They are equal but do not represent the same thing.  $h(30)$  is the height after 30 minutes. and  $h(70)$  is the height after 70 minutes  
d) i) 0 Initial height = 0 m                      ii) 0 After 100 min the balloon has landed on the ground.  
                    iii)  $-9900$  this has no meaning since the balloon has already landed  
e) 22 500 cm = 225 m                      f) after 100 min  
g) domain  $\{t \mid 0 \leq t \leq 100, t \in R\}$                       range  $\{h(t) \mid 0 \leq h(t) \leq 22\,500, h(t) \in R\}$

# Functions Lesson #5: Interpreting Graphs of Functions



The Carter Family are driving to the Yukon for a family vacation. The graph represents the amount of fuel (in litres) in the gas tank of their car on the first day of their journey.



The graph of the journey is divided into eight line segments.

a) With reference to the journey, explain what is happening between:

- A and B driving constantly for 3 hours
- B and C 30 minute break
- C and D filled up the tank

b) What is the rate of fuel consumption (in litres per hour) between D and E?

$$60 - 25 = 35L$$

$$2 - 10:30 = 3.5hr$$

$$35L / 3.5hr = 10L/hr$$

c) Which line segment represents the car being refueled for the second time?

d) Calculate the total time when the car was driven.

8.5 hrs. 7am → 5pm with 1.5hr break

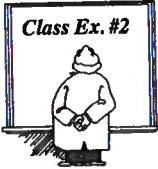
e) If fuel costs 85¢ per litre, calculate the cost of the fuel used for the first day of the journey.

fuel used AB + DE + FG + HI

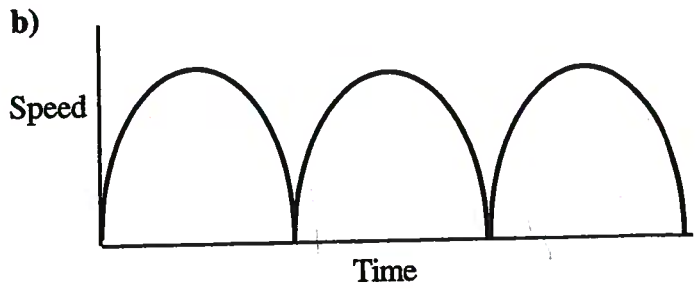
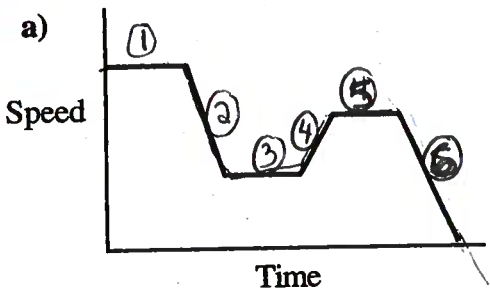
$$30L + 35L + 10L + 10L = 85L$$

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Cost 85L x 85 = (\$72.25)



Suggest a possible scenario for each of the following graphs:



- ① boat at constant speed
- ② boat slows down
- ③ boat at constant speed
- ④ speed up
- ⑤ constant
- ⑥ slow down to a stop

A pendulum swinging back and forth

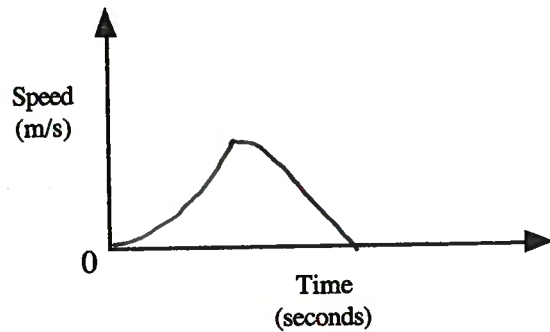
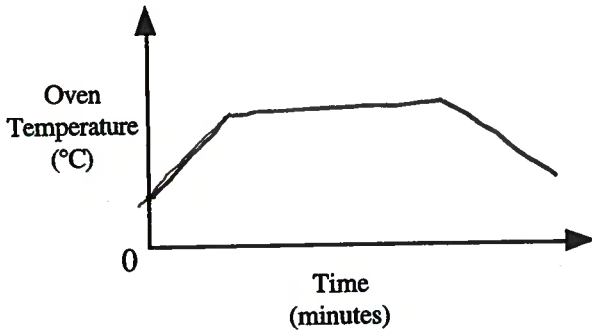
**Sketching a Graph**



Sketch a graph with no scale for each of the following

a) the oven temperature when baking a pie

b) Ben taking part in a 100 m sprint

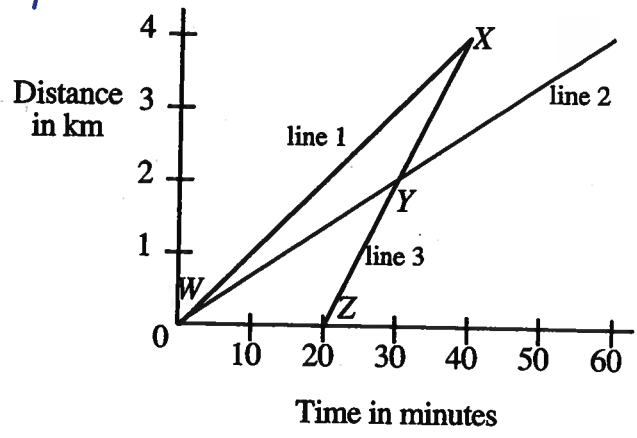


**Complete Assignment Questions #1 - #12**



# Assignment #1, 2, 3, 5

1. Amanda, Brittany, and Chelsea, each follow the same route to school. One morning Amanda cycles to school, Brittany walks to school, and Chelsea runs to school. Lines 1, 2, and 3, on the graph represent the three routes.



a) Complete the table below.

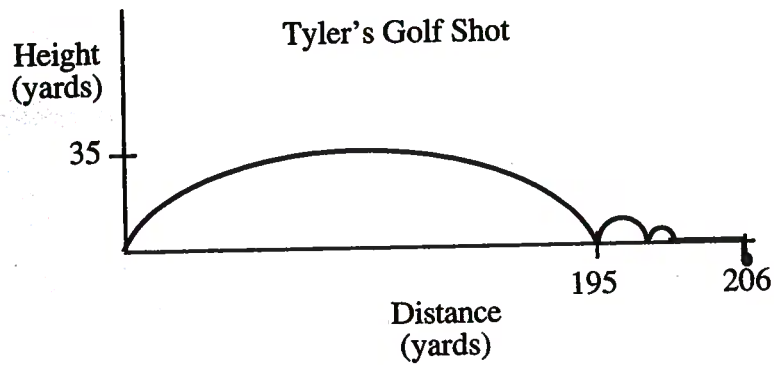
	Line 1	Line 2	Line 3
Distance (km)	4		
Time (hrs)	$\frac{2}{3}$		
Rate (km/hr)	6		
Student			

b) Explain what is happening at the following points.

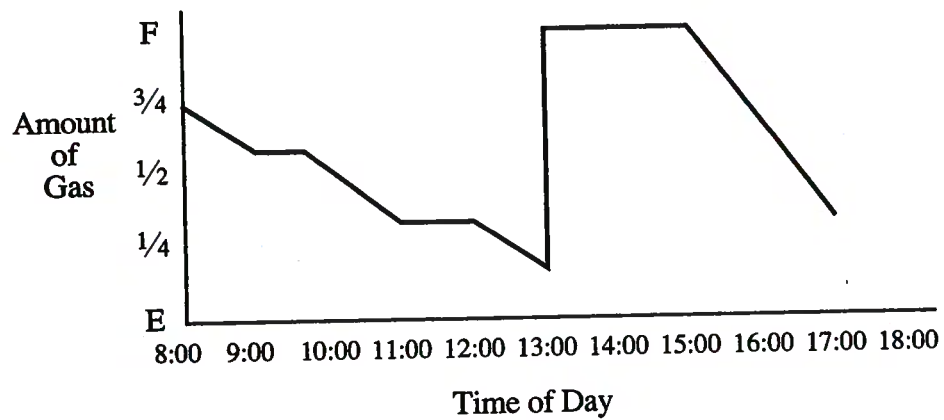
- i) W
- ii) X
- iii) Y
- iv) Z

c) How can you tell from the steepness of the lines which line represents the route of each student?

2. Tyler, a member of St. Andrews High School golf team, hits a golf ball. The graph shows the path of the ball. Describe Tyler's golf shot.

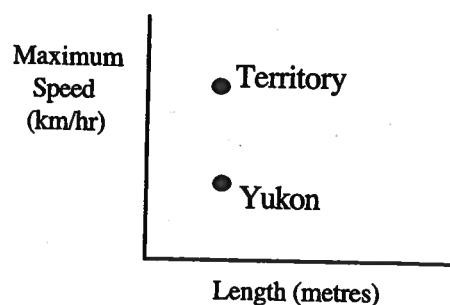
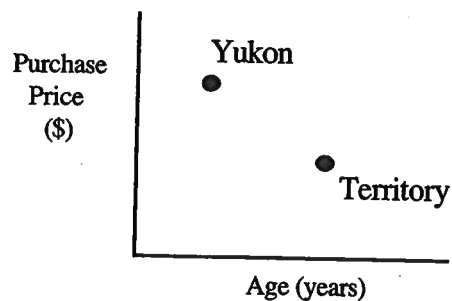


3. Dar sells medical supplies. The graph shows the amount of gasoline in his car during a particular day.

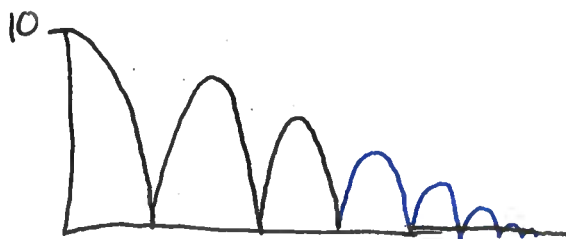


Describe how Dar may have spent the day.

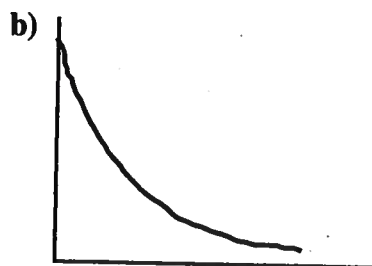
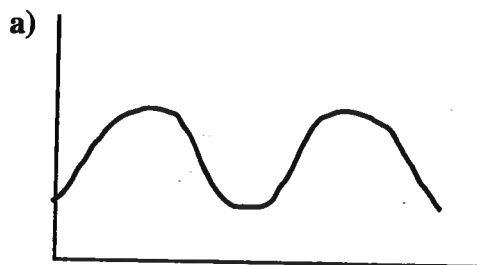
4. The two graphs shown compare two yachts: the Yukon and the Territory. The first graph compares the yachts by age and cost. The second graph compares the boats by speed and length. Describe the comparison between the two yachts.



5. A super ball is dropped from a 10 m building. On each bounce, it bounces back to 80% of its previous height. Create a graph of height as a function of time.

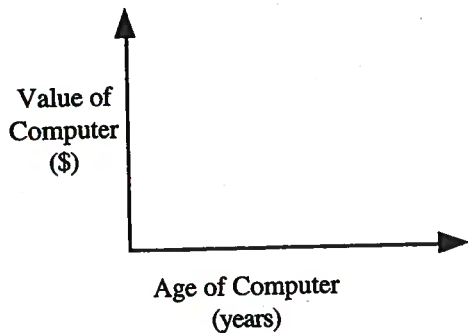


6. Suggest a possible scenario for each of the following graphs.

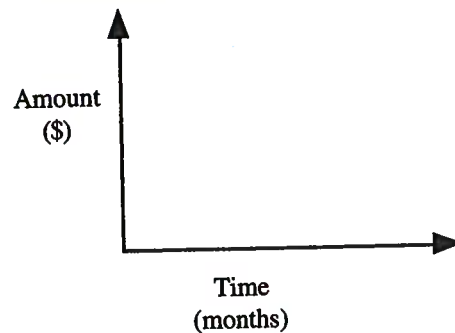


7. Sketch a graph with no scale to represent each of the following.

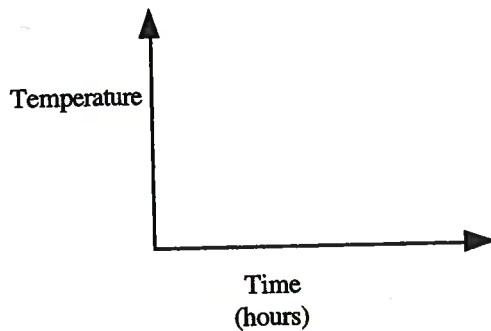
a) A computer's value compared to its age in years.



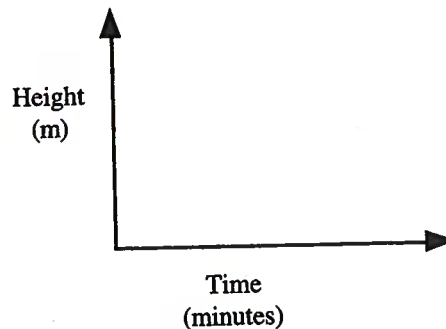
b) The amount of your savings if you save \$10 every month for a period of six months.



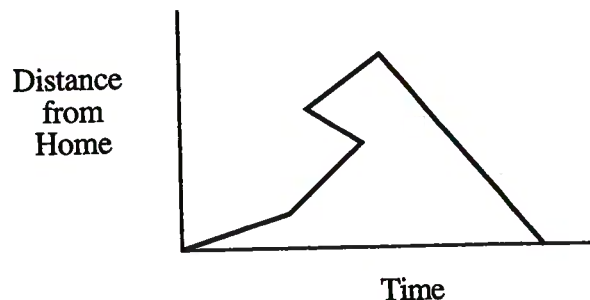
c) The air temperature during a spring day from 6:00 a.m. to 6:00 p.m.



d) You are sitting in the bottom chair of a ferris wheel. Graph your height above the ground during two rotations of the wheel.



8. A student drew the following graph to represent a journey. Explain why the graph must be incorrect.



**Matching**

Match each description on the left with the best graph on the right. Each graph may be used once, more than once, or not at all.

Description

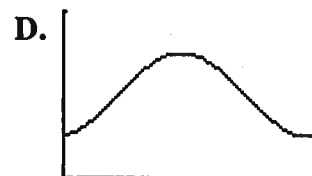
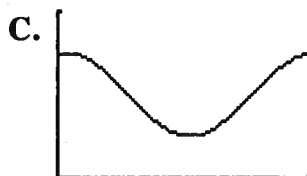
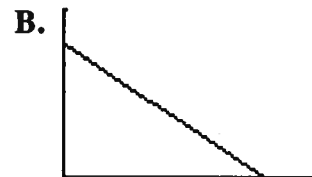
Graph

9. Sketch a graph of a person's height as a function of their age.

10. The number of hours of daylight in a given town in northern BC depends on the day of the year. Sketch a graph of the number of hours of daylight as a function of day of the year.

11. Sketch a graph of the area of a circle as a function of its radius.

12. You start driving at a constant speed with a full tank of gas. Sketch a graph of litres of gas in the tank as a function of distance travelled.



**Answer Key** (Answers may vary)

1. a) see table below

	Line 1	Line 2	Line 3
Distance (km)	4	4	4
Time (hrs)	$\frac{2}{3}$	1	$\frac{1}{3}$
Rate (km/hr)	6	4	12
Student	Chelsea	Brittany	Amanda

b) i) Chelsea and Brittany leave home at the same time.

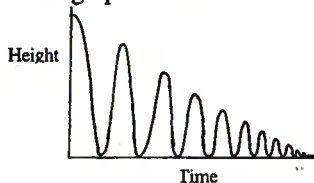
ii) Chelsea and Amanda arrive at school.

iii) Amanda overtakes Brittany.

iv) Amanda leaves home 20 minutes after Brittany and Chelsea.

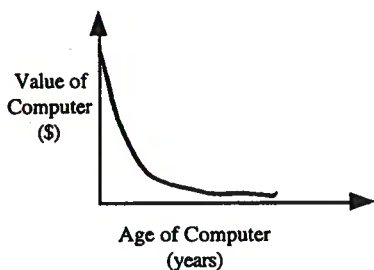
c) The steeper the graph, the less time is taken to travel to school. The steepest slope represents the cyclist Amanda, the next steepest slope represents the runner Chelsea, and the remaining line represents the walker Brittany.

2. Tyler hits the ball through the air for a distance of 195 yards. The ball bounces twice and rolls into the hole. The golf shot travelled a total of 206 yards, and had a maximum height of 35 yards.
3. Dar left home at 8:00 AM with  $\frac{3}{4}$  tank of gas in his car. He drove for about one hour, had a meeting for about  $\frac{1}{2}$  hour, drove for about  $1\frac{1}{2}$  hours, had a second meeting for one hour, and drove for about one hour. He refueled at 1 pm and had a lunch meeting for about 2 hours. He then drove home and arrived about 5 p.m. with a quarter tank of gas left.
4. The Territory is older than the Yukon, and its purchase price was less. Both the Territory and the Yukon are the same length, but the Territory can achieve a greater maximum speed.
5. see graph below

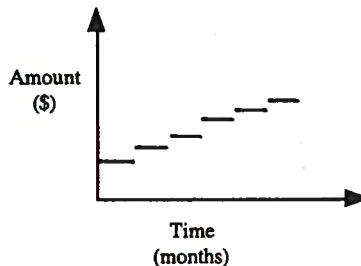


6. a) The number of hours of daylight per day over a period of two years for a location in the northern hemisphere.  
b) The value of a car depreciating over time.

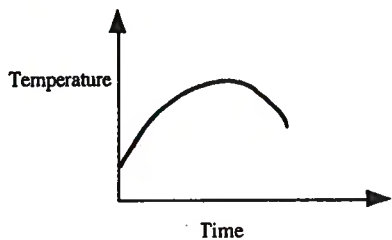
7. a)



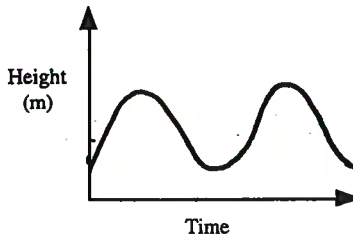
b)



c)



d)



8. The graph is not a function. The person cannot be at three different places at the same time.
9. E      10. D      11. F      12. B