Polynomial Operations Lesson #4:
Multiplication of Polynomials - Part One

Three Important Products

Complete the following using the distributive property (FOIL).

i) $(a+b)^2 = (a+b)(a+b)$ ii) $(a-b)^2 = (a-b)(a-b)$ iii) $(a-b)(a+b)$

$= a^2+ab+ab+b^2$ $= a^2-ab-ab+b^2$ $= a^2+ab-ab-b^2$

$= a^2+2ab+b^2$ $= a^2-2ab+b^2$ $= a^2-b^2$

1. **The square of a binomial can be found by squaring the first term, doubling the product of the two terms and squaring the last term.**
2. **The product of the sum and difference of the same two monomials results in the difference of the squares of the monomials.**
   This important result will be considered in more detail in future lessons on factoring.

Expand each of the following.

a) $(x+7)^2$

$(x+7)(x+7)$

$x^2+14x+49$

$2 \cdot 7 \cdot x$

b) $(3x-1)^2$

$9x^2-6x+1$

$2 \cdot 3x \cdot -1$

c) $(2m-3n)^2$

$4m^2-12mn+9n^2$

$2m \cdot 3n \cdot 2$

d) $(4-7x)(4+7x)$

$16-49x^2$

e) $(5a+3b)(5a-3b)$

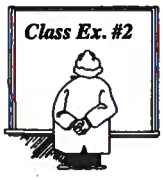
$25a^2-9b^2$

e) $(2p-9q)(2p-9q)$

$=(2p-9q)^2$

$4p^2-36pq+81q^2$

Complete Assignment Question #1



Expand and simplify.

a) $5(2x-3)(x-6)$

$5(2x^2-12x-3x+18)$

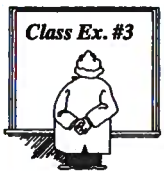
$5(2x^2-15x+18) = 10x^2-75x+90$

b) $-8(7p+3)^2$

$-8(7p+3)(7p+3)$

$-8(49p^2+42p+9)$

$= -392p^2-336p-72$



Expand and simplify.

a) $(x+5)(x-5)-(x+2)(x+8)$

$x^2-25-(x^2+8x+2x+16)$

$= x^2-25-(x^2+10x+16)$

$= x^2-25-x^2-10x-16$

$= -10x-41$

b) $(9a+4)(4a-9)-(6a-5)^2$

$(6a-5)(6a-5)$

$36a^2-81a+16a-36-(36a^2-60a+25$

$36a^2-65a-36-36a^2+60a-25$

$-5a-61$

Expand and simplify.

a) $5x(3x^2-7x+1)-(4x+3x^2)(5x-8)$

b) $4(2x-7)(3x+2)-8(x-1)(3x-1)$

$= 15x^3-35x^2+5x-(20x^2-32x+15x^3-24x^2)$

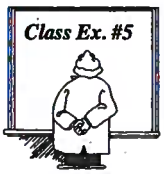
① Distribute all

$= 15x^3-35x^2+5x-(15x^3-4x^2-32x)$

② Group like terms (simplify)

$= -31x^2+37x$

③ Subtract



Given that, for every value of x, the polynomial $x^2+20x+50$ can be written in the form $(x+a)^2+b$, determine the values of a and b.

ax^2+bx+c

$a=1 \quad b=20 \quad c=50$

Complete Assignment Questions #2 - #7

Assignment (1-5) ac

1. Expand and simplify where possible.

a) $(x-8)^2$ **b)** $(x-9)(x+9)$ **c)** $(3x-y)^2$

d) $(5x+2y)^2$ **e)** $(3x-2)(3x+2)$ **f)** $(-2y+1)^2$

g) $(2p+7)^2$ **h)** $(4m+3n)(4m-3n)$ **i)** $(5a-6b)^2$

j) $(9-5x^2)(9+5x^2)$ **k)** $(6a-7b)(6a-7b)$ **l)** $(2a^3-7)(2a^3+7)$

2. Expand and simplify where possible.

a) $2(4x-3)(3x-4)$ **b)** $7(5x-2)(6x+1)$ **c)** $-3(a+8)(2a+9)$

d) $5(4x+1)^2$ **e)** $6(8x-3y)(2x+y)$ **f)** $-4(a+3b)(2a-5b)$

3. Expand and simplify where possible.

a) $(x-3)(x-6)+(x+2)(x+7)$ **b)** $(x-5)(x+4)-(x+1)(x-8)$

c) $(x-3)^2+(x+3)^2$ **d)** $(x-y)(x-4y)-(x+y)(x-y)$

4. Expand and simplify where possible.

a) $(3x-1)(x-3)-2x(x-1)$ **b)** $(4x+1)(2x+3)-(3x-7)(2x-5)$

c) $(9x-1)(x-4)-(3x+1)(3x-1)$ **d)** $8(5-3x)(2+5x)-3(1+x)^2$

e) $5(2x-3)(2x+5)+3(x+7)(x+2)$ **f)** $4(2p+3q)^2-(5p-q)(7p+11q)$

5. Expand and simplify where possible.

a) $(x+4)^2+(x+2)^2$ **b)** $(3a-b)^2-(2a+5b)^2$ **c)** $3(y-1)^2-2(2y-1)^2$

d) $9-2(x-1)(x+7)+(2x-5)(x-3)$ **e)** $3(1+3y)(4-y)-(3y-2)(3y-5)$

Use the following information to answer the next question .

A student provides the following expansions for four binomial products.

$(x+3)^2 = x^2 + 9$ $\qquad$ $(3x-y)(3x-y) = 9x^2 - 6xy - y^2$

$(2x+y)(2x-y) = 2x^2 - y^2$ $\qquad$ $(5x+7)^2 = 25x^2 + 35x + 49$

Multiple Choice **6.** How many of the student's expansions are incorrect?

A. One
B. Two
C. Three
D. Four

Numerical Response **7.** Given that for every value of x, $x^2 - 10x + 39 = (x-a)^2 + b$, then the value of b must be _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x^2 - 16x + 64$ **b)** $x^2 - 81$ **c)** $9x^2 - 6xy + y^2$
d) $25x^2 + 20xy + 4y^2$ **e)** $9x^2 - 4$ **f)** $4y^2 - 4y + 1$
g) $4p^2 + 28p + 49$ **h)** $16m^2 - 9n^2$ **i)** $25a^2 - 60ab + 36b^2$
j) $81 - 25x^4$ **k)** $36a^2 - 84ab + 49b^2$ **l)** $4a^6 - 49$

2. a) $24x^2 - 50x + 24$ **b)** $210x^2 - 49x - 14$ **c)** $-6a^2 - 75a - 216$
d) $80x^2 + 40x + 5$ **e)** $96x^2 + 12xy - 18y^2$ **f)** $-8a^2 - 4ab + 60b^2$

3. a) $2x^2 + 32$ **b)** $6x - 12$ **c)** $2x^2 + 18$ **d)** $-5xy + 5y^2$

4. a) $x^2 - 8x + 3$ **b)** $2x^2 + 43x - 32$ **c)** $-37x + 5$
d) $77 + 146x - 123x^2$ **e)** $23x^2 + 47x - 33$ **f)** $-19p^2 + 47q^2$

5. a) $2x^2 + 12x + 20$ **b)** $5a^2 - 26ab - 24b^2$ **c)** $-5y^2 + 2y + 1$
d) $-23x + 38$ **e)** $-18y^2 + 54y + 2$

6. D **7.**

1	4		

Polynomial Operations Lesson #5:
Multiplication of Polynomials - Part Two

In this lesson, we deal with more involved polynomial multiplication including multiplying a binomial by a trinomial and the product of three binomials.

Product of a Binomial and a Trinomial

A rectangle has length $(5x+2)$ cm and width (x^2+2x+1) cm.
Determine the area of the rectangle (in cm^2) by completing each of the following solutions.

Area = length × width = $(5x+2)(x^2+2x+1)$

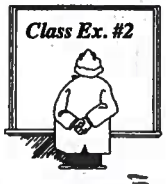
i) *use a diagram*

	$5x$	2
x^2	$5x^3$	$2x^2$
$2x$	$10x^2$	$4x$
1	$5x$	2

$5x^3+12x^2+9x+2$

ii) *use the distributive property*

$(5x+2)(x^2+2x+1)$
$= 5x(x^2+2x+1) + 2(x^2+2x+1)$
$= 5x^3+10x^2+5x+2x^2+4x+2$
$= 5x^3+12x^2+9x+2$



Expand and simplify.

a) $(x^2-4)(2x^3+x-5)$

$= 2x^5+x^3-5x^2-8x^3-4x+20$

$= 2x^5-7x^3-5x^2-4x+20$

b) $(2y^2-3y-7)(y-6)$ $= 2y^3-15y^2+11y+42$

	$2y^2$	$-3y$	-7
y	$2y^3$	$-3y^2$	$-7y$
-6	$-12y^2$	$18y$	42

Expand and simplify $4(a-4)(a^2-3a-6)-(4a-3)(4a+3)$.

$-(16a^2-9)$

$4(a^3-3a^2-6a-4a^2+12a+24)-(16a^2+12a-12a-9)$
$= 4(a^3-7a^2+6a+24)-(16a^2-9)$
$= 4a^3-28a^2+24a+96-16a^2+9$
$= 4a^3-44a^2+24a+105$

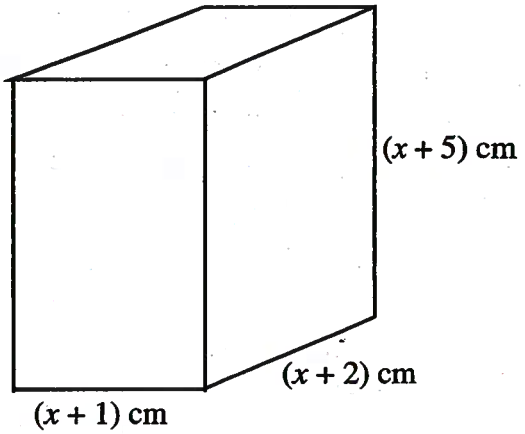
Complete Assignment Questions #1 - #3

Product of Three Binomials

In this section, we extend the multiplication of binomials to consider three factors. This leads to applications involving the volume of a rectangular prism.

Class Ex. #4

Consider the rectangular prism shown.



a) Write an expression which represents the volume of the prism in cm^3.

$(x+1)(x+2)(x+5)$

b) Expand the expression in a) to write the volume in the form $V = ax^3 + bx^2 + cx + d$.

$= (x+1)(x+2)(x+5)$

$= (x^2+2x+x+2)(x+5)$

$= (x^2+3x+2)(x+5)$

$= x^3+3x^2+2x+5x^2+15x+10$

$= x^3+8x^2+17x+10$

ax^3+bx^2+cx+d

Class Ex. #5

Expand and simplify.

a) $(x-3)(x+4)(2x-1)$

	x	-3
x	x^2	$-3x$
4	$4x$	-12

$(x^2+x-12)(2x-1)$

b) $(2x-1)(x-3)(x+4)$

	x^2	x	-12
$2x$	$2x^3$	$2x^2$	$-24x$
-1	$-x^2$	$-x$	12

$= 2x^3+x^2-25x+12$

c) Comment on the results to a) and b).

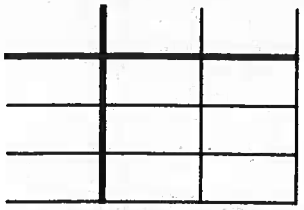
You can distribute any of them first

Complete Assignment Questions #4 - #9

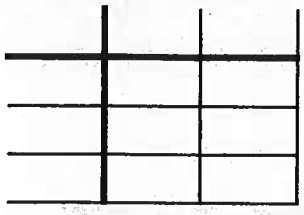
Assignment 4ac, 5, 7a

1. Use a diagram to determine the expansion.

a) $(y-5)(y^2+2y+4)$



b) $(3m+7)(m^2-3m+6)$



2. Use the distributive law to determine the expansion.

a) $(x-4)(x^2-6x+3)$

b) $(2a+5)(a^2-7a-9)$

3. Expand and simplify.

a) $(x^2-7)(2x^3+4x-1)$

b) $(-m^2-m+1)(m+1)$

c) $(a-3b)(4a^2-3ab-2b^2)$

d) $2(5x+2)(3x^2+x-4)$

4. Expand and simplify.

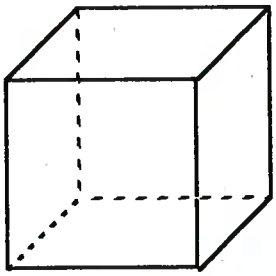
a) $(x+1)(x+2)(3x+5)$

b) $(h-4)(2h-3)(3h-1)$

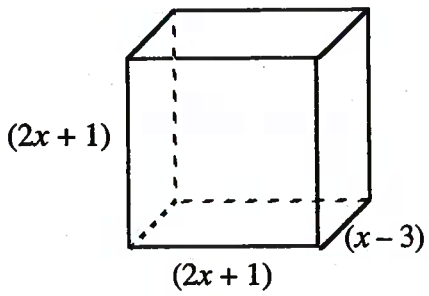
c) $(a+3b)(2a-5b)(2a+5b)$

d) $(3x+7y)(4x-3y)(x-4y)$

5. Calculate the volume of the cube shown below.



6. Calculate the volume of the rectangular prism illustrated.



7. Simplify

a) $-3(a^2+2)(3a^2-a-1)$

b) $(-2x^2-3x+1)(x^2-x-3)$

c) $2(4x-1)^2-(3x-2)^3$

Use the following information to answer the next question.

A student attempts to expand $(a+2)^3$.
His work is shown below.

$(a+2)^3$

$= (a+2)(a+2)(a+2)$ Line 1

$= (a+2)(a^2+4)$ Line 2

$= a^3+2a^2+4a+8$ Line 3

Multiple Choice **8.** Which of the following statements is true?

A. The student made an error in Line 1.

B. The student made an error in Line 2.

C. The student made an error in Line 3.

D. The student's expansion is correct.

Numerical Response

9. Subtracting the product of $(3x-1)$ and $(2x^2-4x+3)$ from the sum of $(2x^3-7x^2-6)$ and (x^2+6x-3) results in a polynomial of the form ax^3+bx^2+cx+d. The value of $b-2c$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $y^3-3y^2-6y-20$ b) $3m^3-2m^2-3m+42$

2. a) $x^3-10x^2+27x-12$ b) $2a^3-9a^2-53a-45$

3. a) $2x^5-10x^3-x^2-28x+7$ b) $-m^3-2m^2+1$
c) $4a^3-15a^2b+7ab^2+6b^3$ d) $30x^3+22x^2-36x-16$

4. a) $3x^3+14x^2+21x+10$ b) $6h^3-35h^2+47h-12$
c) $4a^3+12a^2b-25ab^2-75b^3$ d) $12x^3-29x^2y-97xy^2+84y^3$

5. $8z^3+36z^2+54z+27$

6. $4x^3-8x^2-11x-3$

7. a) $-9a^4+3a^3-15a^2+6a+6$ b) $-2x^4-x^3+10x^2+8x-3$
c) $-27x^3+86x^2-52x+10$

8. B **9.**

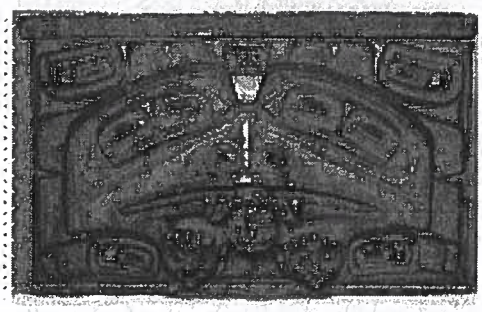
2	2		

Polynomial Operations Lesson #6:
Problem Solving with Polynomial Products

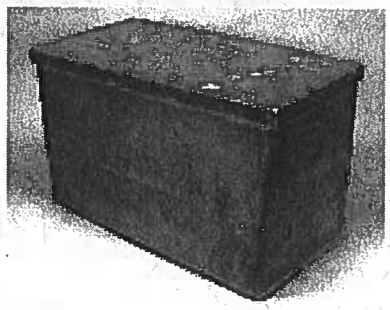
Bentwood boxes are containers traditionally constructed by the Northwest Coast Aboriginal peoples. An artist typically uses a single piece of wood that is bent to form a box in the shape of a rectangular prism using only steam and strategically placed grooves. Traditional uses of these boxes range from food and clothing storage to burials.

The images below are examples of a bentwood box and can be referenced at the following website https://www.rrncommunity.org/items/3442#

Front View



Back and Side View



Additional information on Bentwood art can be found at https://en.wikipedia.org/wiki/Bentwood

$V = lwh$

A Haida artist constructs a bentwood box with the following dimensions: length $(5x-2)$ cm, width $(3x-1)$ cm, and height $(3x+1)$ cm.

a) Write and simplify an expression for the volume of the bentwood box in cm^3.

$(5x-2)(3x-1)(3x+1) = (5x-2)(9x^2-1)$

$= (45x^3 - 5x - 18x^2 + 2)\,cm^3$

b) Write and simplify an expression for the surface area of the bentwood box in cm^2.

c) If $x = 20$, calculate the volume and surface area of the bentwood box.

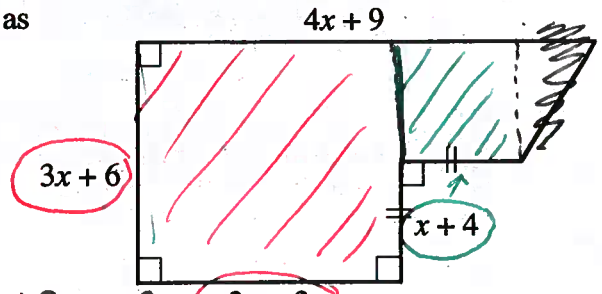
$45(20)^3 - 5(20) - 18(20)^2 + 2$

$= 352702\,cm^3$

$45x^3 - 5x - 18x^2 + 2$

Class Ex. #2

a) The area of the given figure can be written as a trinomial in the form $ax^2 + bx + c$. Determine the values of a, b, and c.



$(3x+6)(2x+3) = 6x^2+9x+12x+18$

$= (6x^2+21x+18)\,cm^2$

$3x+6 \;|\; (3x+6)-(x+4) = 2x+2$

$x+4$

$2x+2$

$x+4$

$(x+4)(2x+2)$

$= 2x^2+2x+8x+8 = 2x^2+10x+8$

b) Calculate the area if $x = 2.5$. ~~153.5~~

$8(2.5)^2+31(2.5)+26 = 156$ $\quad 8x^2+31x+26$

both shapes added together

Class Ex. #3

The hypotenuse of a right triangle is $(5x + 5)$ cm long and the lengths of the other two sides are $(4x + 8)$ cm and $(3x - 5)$ cm.

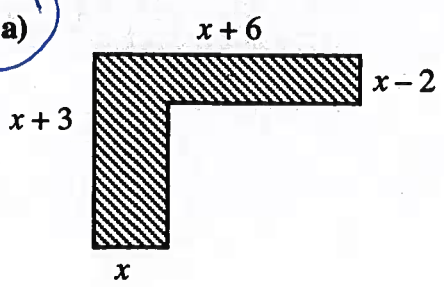
Form an equation and solve it to determine the lengths of the three sides of the triangle.

Complete Assignment Questions #1 - #15

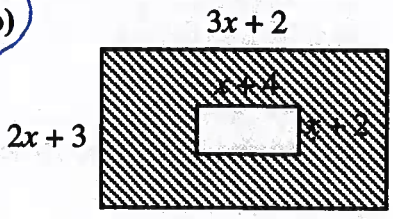
Assignment lab, 5

1. In each case, the figures consist of a series of horizontal and vertical lines.
The area of each figure can be written as a trinomial in the form $ax^2 + bx + c$.
Determine the values of a, b, and c, and calculate the area when $x = 2.4$.

a)

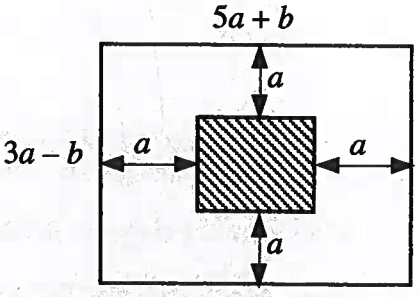


b)



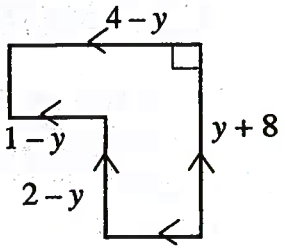
2. The figure consists of a rectangle within a rectangle.

a) Determine a simplified expression of the shaded area in terms of a and b.



b) Calculate the area when $a = 2.8$ and $b = -3.5$.

3. a) Determine the area of the figure in the form $ay^2 + by + c$.



b) Determine the area of the figure when $y = -2$.

4. A square metal plate of side 25 cm is heated so that each side increases in length by x cm.

a) Write and simplify an expression for the area of the heated plate.

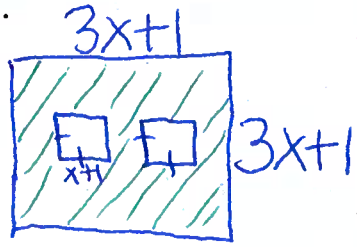
b) Write and simplify an expression for the increase in area of the plate.

c) If $x = 0.2$, calculate the increase in area.

5. A square garden with a side length of $(3x + 1)$ m contains two square flower beds each with a side length of $(x + 1)$ m. The remainder of the garden is grass.

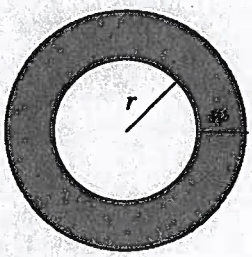
✓ **a)** Draw a diagram to illustrate this information.

b) Write and simplify an expression for the area of grass in the garden.



total area – garden areas

6. A metal washer has internal radius r mm and width w mm as shown.



a) Write an expression for the outer radius of the washer.

b) Show that the area of the washer, A mm^2, is given by $A = 2\pi rw + \pi w^2$.

7. Solve the following equations where the variable is in the set of real numbers.

a) $(3x - 1)(x - 1) = 3x(x + 1)$

b) $(y + 2)^2 = y^2 + 2$

c) $t^2 - (t - 9)^2 = 9$

d) $2a^2 - (a - 3)^2 = (a + 2)(a - 1)$

8. The hypotenuse of a right triangle is $(5x - 6)$ cm long and the lengths of the other two sides are $(4x - 7)$ cm and $(3x - 1)$ cm. Form an equation and solve it to determine the value of x and the lengths of the three sides of the triangle.

9. Consider a set of rectangles with sides $(4a-3)$ cm and $(2a+7)$ cm.

a) Write and simplify an expression in a for the area of one of these rectangles.

b) If one of these rectangles has a perimeter of 50 cm, determine the length and width of this rectangle.

c) If another of these rectangles is a square, determine the length of each side.

10. A rectangle has length (x^2+4x-1) cm and width $(3x-2)$ cm.

a) Write and simplify an expression for the area of the rectangle in cm^2.

b) If $x=2.5$, calculate the area of the rectangle.

11. Dice for a children's board game are cubes with an edge length of $(3x-2)$ mm.

a) Write and simplify an expression for the volume of a die in mm^3.

b) The manufacturer packages dice in cubic containers containing 64 dice. Determine the volume of the container in cm^3 if $x=4$.

12. A rectangular garden with length $(8-3a)$ m and width $(a+8)$ m contains three square flower beds, each with a side length of $(2a+5)$ m. The remainder of the garden is grass.

a) Draw a diagram to illustrate this information.

b) Write and simplify an expression for the area of grass in the garden.

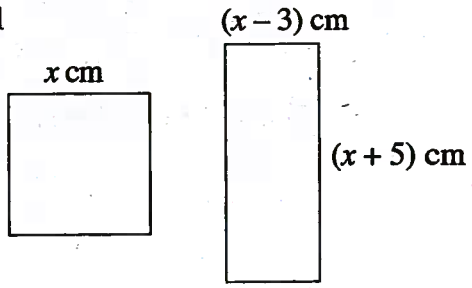
c) Determine the area of grass if $a=-1.5$.

Multiple Choice

13. A box is in the shape of a rectangular prism. The length of the box is y cm. The width is 2 cm less than the length, and the height is 2 cm more than the length. If the volume of the box can be written in the form $V=ay^3+by^2+cy+d$ where $a, b, c,$ and d are integers, then how many of the parameters a, b, c, and d are equal to zero?

A. 0
B. 1
C. 2
D. 3

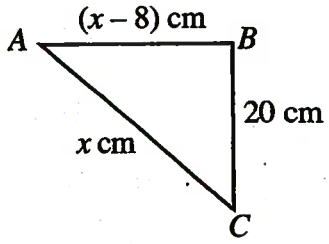
14. The square and the rectangle in the diagram are equal in area.
The value of x, to the nearest tenth, is _____.



(Record your answer in the numerical response box from left to right)

15. The diagram shows the lengths of the sides of right triangle ABC.

The perimeter (to the nearest tenth of a cm) of triangle ABC is _____.



(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $a = 1,\ b = 9,\ c = -12$, area = 15.36 units^2 **b)** $a = 5,\ b = 7,\ c = -2$, area = 43.6 units^2
2. a) $3a^2 - 2ab - b^2$ **b)** 30.87 units^2 **3. a)** $30 - y - 2y^2$ **b)** 24 units^2
4. a) $625 + 50x + x^2$ cm^2 **b)** $50x + x^2$ cm^2 **c)** 10.04 cm^2
5. b) $(3x+1)^2 - 2(x+1)^2 = 7x^2 + 2x - 1$ m^2 **6. a)** $(r + w)\ mm$
7. a) $\frac{1}{7}$ **b)** $-\frac{1}{2}$ **c)** 5 **d)** $\frac{7}{5}$
8. $(5x-6)^2 = (4x-7)^2 + (3x-1)^2$; $x = 7$; 29 cm, 21 cm 20 cm
9. a) $(4a-3)(2a+7) = 8a^2 + 22a - 21$ cm^2 **b)** 11 cm by 14 cm **c)** 17 cm
10.a) $(x^2 + 4x - 1)(3x - 2) = 3x^3 + 10x^2 - 11x + 2$ cm^2 **b)** 83.875 cm^2
11.a) $(3x-2)^3 = 27x^3 - 54x^2 + 36x - 8$ mm^3 **b)** 64 cm^3
12.b) $-15a^2 - 76a - 11$ m^2 **c)** 69.25 m^2
c) volume = 2574 cm^3, surface area = 1150 cm^2

13. C **14.** | 7 | . | 5 | | **15.** | 7 | 0 | . | 0 |

Unit 3 Test Review

PART I
UNIT 3

NAME: _____
BLK: _____

/24 = ____ %

/22

1 ① State whether the following is a polynomial.

a) $\sqrt{5}$

b) $\frac{1}{5x}$

c) $4m^{-2}$

d) $\frac{5}{6}x^2 - 3x$

e) 18

/9 ② What is the

	name	leading coefficient	degree
a) $8m^2n^5p$			
b) $2x - 3y^2 + 9$			
c) $-\frac{1}{2}x^2 + p^4r^3$			

/2 ③ collect like terms

a) $4a + 3B - 10c + 6a - 2c + B$

b) $(2x^3 - 9x^2 + 4) - (3x^2 + 4x - 5)$

/3 ④ Expand and simplify.

a) $2(3x + 3)$

b) $(x + 5)(x - 4)$

c) $(m + 2)(2m^2 - 1) - (m + 3)(m - 8)$

Area ( ) =

4 ⑤ Expand and Simplify

a) $(x-y)(2x+4)(x+7y)$

b) $(x+11)(x^2+2x-3)$

c) $(2y-5)^2$

d) $(y+4)^2-(2y-1)^2$

3 ⑥ P. 171 #12

5 ⑦ p. 178 #4

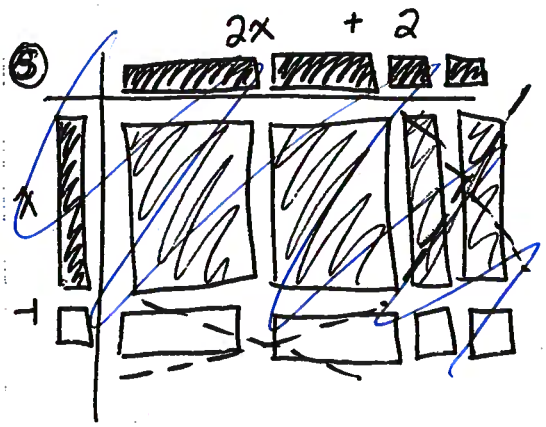
Answers:

① A, D, E

② a) monomial, 8, 8th degree
b) trinomial, 3, 2nd degree
c) binomial, 1, 7th degree

③ a) $10a+4B-12C$
b) $2x^3-12x^2-4x+9$

④ a) $6x+6$
b) x^2+x-20
c) $2m^3+3m^2+4m+22$



$(2x+2)(x-1) = 2x^2-2$

6 a) $2x^3+4x^2+12x^2y+24xy-14xy^2-28y^2$

b) $x^3+14x^2+19x-33$

c) $4y^2-20y+25$

d) $-3y^2+12y+15$