Polynomial Operations Lesson #1:
Review and Preview

Overview of Unit

In this unit we study algebraic expressions called polynomials. We review the classification of polynomials, addition and subtraction of polynomials, and multiplication by a monomial. We introduce the product of two binomials (concretely, pictorially, and symbolically) and extend this to the multiplication of polynomials. We also solve problems involving polynomial expressions.

Review

In algebra, a letter that represents one or more numbers is called a **variable**.

no equal sign

Expressions like $2a - b + 4$ or $\frac{5}{x} + 3$ are called **algebraic expressions**.
Certain algebraic expressions are called **polynomials** as explained below.

A **monomial** is a number or a variable or the product of numbers and variables. (Note that the exponent of any variable must be a positive integer in the numerator of the monomial.)

eg. 6, x, $6x$, $-\frac{1}{2}xy$, $0.25x^3$, abc, $2p^4q^2$ etc. are all monomials.

The number that multiplies the variable is called the **numerical coefficient**.

A **polynomial** is a monomial or a sum or difference of monomials.

- 6, x, $6 + x$, $2y + 7z$, $x^2 - 5x - 9$ etc. are all examples of polynomials.

Explain why $\frac{5}{x} + 3$ is not a polynomial.

$5x^{-1} + 3$, not a polynomial because it is a negative exponent on variable

Classifying Polynomials

Polynomials may be classified in two different ways as shown below.

Ways to Classify Polynomials

- number of terms in the polynomial
- degree of the polynomial

Continued on the next two pages

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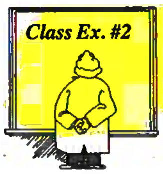
Classifying Polynomials by The Number of Terms

A polynomial may be classified by the number of **terms** it contains.

- A term can be a number, a variable, or the product of a number and variable(s).
- When there is more than one term, the terms are connected by + or – signs.

A polynomial with 1 term is a **monomial** (eg. $4x$).
A polynomial with 2 terms is a **binomial** (eg. $x+4$).
A polynomial with 3 terms is a **trinomial** (eg. x^2+x+4).
A polynomial with 4 or more terms is simply called a polynomial when classifying by the number of terms.



Consider the following algebraic expressions. In each case:

- State whether the expression represents a polynomial or not.
- If the expression does not represent a polynomial, explain why.
- If the expression does represent a polynomial, state whether the polynomial is a monomial, a binomial, or a trinomial.

a) $\frac{1}{4}xy-10$

-yes, polynomial
-binomial

b) $3pq^{\frac{1}{2}}$

-no
-exponent is not a positive whole number

c) $\sqrt{7}x^4-x^3+1$

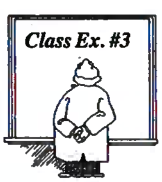
- yes, poly
- trinomial

d) $3x^2+9x-4x^{0.2}$

-no, not poly
-exponent is not a whole #

e) $\frac{7}{a}$

-no
-exponent is negative



Complete the following table.

letters (the unknowns)

Polynomial Expression	# Variables	# Terms	Classification by # Terms
$4x+3yz$	3 → xyz	2	binomial
$2a-4b+7c$	3 → abc	3	trinomial
x^2+3x+4	1 → x	3	trinomial
$\sqrt{2}x$	1 → x	1	monomial
$2x^3+3x^2y+3y^2-8$	2 → xy	4	polynomial

Classifying Polynomials by The Degree of The Polynomial

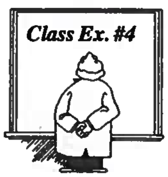
Polynomials can also be classified according to **degree** of the polynomial.
The **degree of a monomial** is the sum of the exponents of its variable(s).

eg. $2x^5$ has degree 5 $-\frac{2}{3}ab^3c^2$ has degree 6 ← (1+3+2)

Recall from the lesson "Whole Number Exponents" on page 20 that a variable raised to the power zero is equal to 1.
For example, the monomial $7x^0$ can be written as $7(1)$ or 7.
Therefore the degree of a monomial with no variable present is 0.

The **degree of a polynomial** is given by the **term** or **monomial** with highest degree.
eg. $3x^2y^2 - 2x^4 + xy^4 - 2$ has degree 5 ← (1+4) * pick highest degree
(4) (4) (5)

If a polynomial has a term with no variable present, this term is called a **constant term**.
In the polynomial $3x^2y^2 - 2x^4 + xy^4 - 2$, the constant term is −2.



State the degree of the following polynomials.

a) $3x^2 - 10x^4 - 9$ degree 4

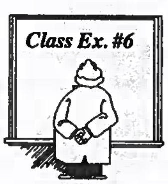
b) $7p^2q^3 - 8p^7q - 2q^7$ (5) (8) (7) degree 8

Give an example of
a) a binomial of degree 1 in one variable. 7x+7
b) a trinomial in two variables with a constant term. x+y+7
c) a monomial of degree 6 with a (numerical) coefficient of 9. $9x^6$
d) a binomial of degree 8 with each term containing two variables. $x^4y^4 + xy$

<u>The following list classifies polynomials by using the degree of the polynomial.</u>

- A **Constant** Polynomial has a degree of 0 eg. 8
- A **Linear** Polynomial has a degree of 1 eg. $x + 3$
- A **Quadratic** Polynomial has a degree of 2 eg. $x^2 - 2x + 5$
- A **Cubic** Polynomial has a degree of 3 eg. $x^3 - 8x^2 + x + 1$
- A **Quartic** Polynomial has a degree of 4 eg. $x^4 - 61x + 9$
- A **Quintic** Polynomial has a degree of 5 eg. $x^5 - 17$

There are names for polynomials of higher degree that are beyond the scope of this course.



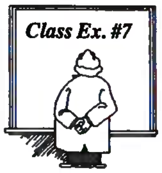
Complete the following table.

Polynomial Expression	Degree	Classification by Degree	Constant Term
$4x^1y^1 - 6$	2	Quadratic	−6
$9y^2 - 8y^3$	3	Cubic	0

Polynomials in a Single Variable

Polynomials in a single variable are usually arranged in **ascending** or **descending** order of the powers of the variable.

The **leading coefficient** of a polynomial in a single variable is the coefficient of the term with highest power of the variable.



Consider the polynomial expression $2x - 4x^3 - 7 + \frac{6x^2}{5}$.

a) Write the polynomial in descending powers of x. $-4x^3 + \frac{6x^2}{5} + 2x - 7$

b) Write the polynomial in ascending powers of x. $-7 + 2x + \frac{6x^2}{5} - 4x^3$

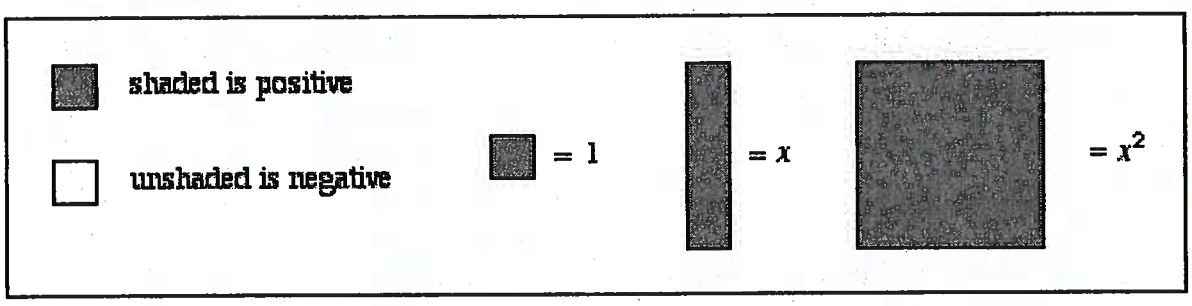
c) State the leading coefficient and the constant term. a) -4 b) -7

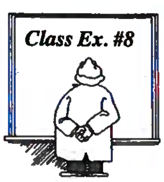
d) State the numerical coefficient of the term in x^2. $\frac{6}{5}$

Complete Assignment Questions #1 - #9

Representing Polynomials Using Algebra Tiles

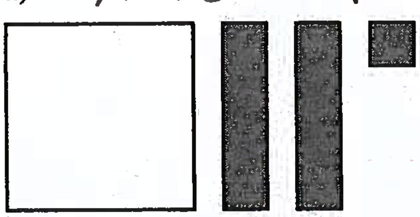
The following legend will be used for algebra tiles in this workbook.



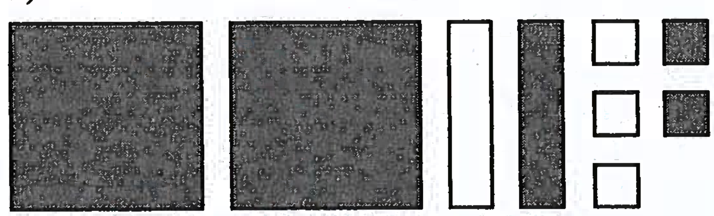


State the polynomial expression which describes each diagram.

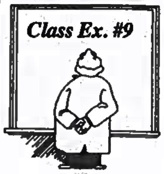
a) $-x^2 + 2x + 1$



b)



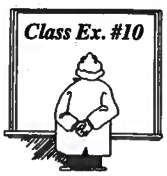
Addition and Subtraction Using Algebra Tiles



Use algebra tiles to determine the result of the addition $(2x^2 + 1) + (x^2 - 2x - 3)$.

Subtracting a polynomial is equivalent to adding the inverse polynomial,

eg. $(4x + 3) - (2x - 5)$ is equivalent to $(4x + 3) + (-2x + 5)$



Use algebra tiles to determine the result of the subtraction $(-x^2 + 3x - 2) - (2x^2 - x - 1)$.

Addition and Subtraction of Polynomial Expressions

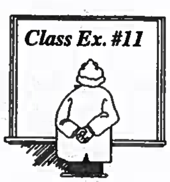
Like terms are terms with the same variable raised to the same exponent.

eg. $3a$, $7a$ and a are like terms. $2x^3$, $\frac{1}{5}x^3$ and $-4x^3$ are like terms.

Unlike terms have different variables or the same variable raised to different exponents.

eg. $2x^3$, $\frac{1}{5}x^2$ and $-4x$ are unlike terms. $4x$ and $4y$ are unlike terms.

Like terms can be added or subtracted to produce a single term.



Simplify the following polynomials by collecting like terms.

a) $(3a - 4b + c) + (3b - 5c - 3a)$

$3a - 3a = 0$

$-4b + 3a = -b$

$c + -5a = -4c$

$-b - 4c$

b) $(4x^2 - 9x + 6)$

$-\ (2x^2 - 3x - 1)$

$2x^2 - 6x + 7$

[redacted]

Class Ex. #12

Simplify

a) $4x - 2x^2 + 3 - 6x^2 + 5 - x$

$-8x^2 + 3x + 8$

b) $a^2b - ab^2 + 4a^3b - 7ab^2 + 5a^2b$

$6a^2b - 8ab^2 + 4a^3b$

Complete Assignment Questions #10 - #20

Assignment

1, 2, 3, 5, 7ac, 9, 13ade, 14acde

1. Identify as a monomial, a binomial, or a trinomial.

a) $x + 1$ b) $3x^3$ c) $2x^2 + 2x + 2$

2. State the degree of each monomial.

a) $5a$ b) $3x^3y$ c) 10 d) $-2a^2b^2$ e) $3xy^2z^3$

3. State whether or not the following are polynomial expressions. If they are not polynomial expressions, explain why not.

a) $\frac{1}{2}x^2 - 3x$ b) $8m^{-2}$ c) $\sqrt{6}$

d) $\frac{7}{x^3}$ e) $\frac{8x^2}{3}$ f) $x^4 + 3x^{1.5}$

4. Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by Number of Terms	Degree
$2y^3 + y^4 - y + 13$				
$9ab - 4x + 11c$				
25				
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$				

5. Complete the following table for the single variable polynomials.

Polynomial Expression	Leading Coefficient	Constant Term	Degree	Classification by Degree
$y^4 - y + 13$				
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$				
$\sqrt{7} - x^5$				
$\pi x^2 - 7 - 3x$				
$-\frac{1}{10}$				
$9x + 12$				

6. Give an example of

a) a trinomial of degree 2 in one variable.

b) a binomial in four variables with a constant term of 6.

c) a monomial of degree 3 in two variables with a negative numerical coefficient.

d) a monomial with a degree of 0.

7. Arrange the following in descending powers of the variable.

a) $6w^2 - 9w + 5 + 2w^3$

b) $\frac{1}{4}a^2 - \frac{2}{3}a^3 - 1 - a$

c) $z - 3 - 4z^6 + z^3$

8. Arrange the following in ascending powers of the variable.

a) $6w^2 - 9w + 5 - 2w^3$

b) $3x^2 - 4x^5 - 2x^4 - 4x^3 + 9x - 7$

c) $8x^3 - 8x + 8$

9. State which of the following are true and which are false.

a) -54 is a polynomial.

b) The degree of the polynomial $3x^3y^3$ is 9.

c) The numerical coefficient of $\frac{6x}{5}$ is 6.

d) A polynomial may have 1000 terms.

e) $\frac{2}{a^3} - 1$ is a binomial.

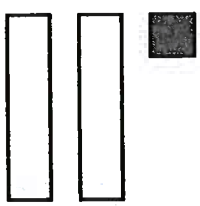
f) The degree of the polynomial 0 is 0.

g) The polynomial $x^3 + 2x^2 + 3x + 4$ is written in ascending powers of x.

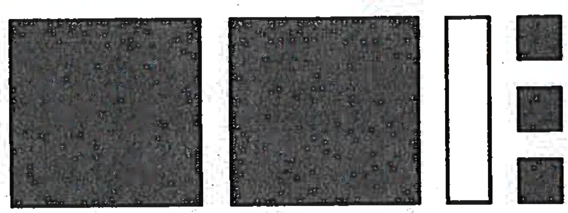
h) The polynomials $3x^2 - 9x + 1$ and $1 - 9x + 3x^2$ are equivalent.

10. State the polynomial expression which describes each diagram.

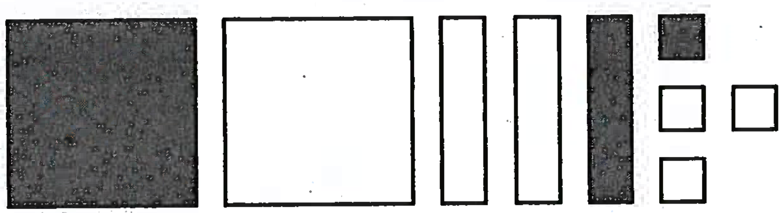
a)



b)



c)



11. Use algebra tiles to determine the result of the addition of :

a) $(x^2 - x - 3) + (x^2 - 2x - 3)$

b) $(3x + 1) + (2x^2 - 3x - 2)$

12. Use algebra tiles to determine the result of the subtraction of:

a) $(x^2 - 3) - (2x^2 + 4x + 1)$ **b)** $(2 - x - x^2) - (1 - 2x + x^2)$

13. Simplify

a) $6p - 7q - 3q - 2p$ **b)** $5x - 3x^2 + 2x - 8x^2$ **c)** $\frac{1}{2}x - 3 + \frac{3}{2}x + 18$

d) $4a^3 + 7a - 2a^2 - 6a - 4a^3 - a^2$ **e)** $3 - 2x + 7y + 4y - 2x + 8z - 9$

14. Simplify the following polynomial expressions by collecting like terms.

a) $(5a - 9b - 2c) + (c - 7b - 3a)$ **b)** $(3 - a - 2a^2) + (9 - 4a + 5a^2)$

c) $(2x^2 + 5x - 1) + (3x - 6 - 6x^2) + (4 - 5x + x^2)$ **d)** $(4a - 6b) - (5a - 2b)$

e)
$$\begin{array}{r} (5x^2 - 8x + 3) \\ -\ \underline{(3x^2 - 3x - 1)} \end{array}$$

f)
$$\begin{array}{r} (7x^2 + 2x - 1) \\ -\ \underline{(-5x^2 - 3x - 1)} \end{array}$$

g)
$$\begin{array}{r} (-4x^2 + 2x - 6) \\ \underline{-(3x + 6 - 2x^2)} \end{array}$$

15. **a)** Subtract $3x^2 - 2x + 7$ from $6x^2 - 5x - 2$.

b) Subtract the sum of $2x^3 - 7x^2 - 6x + 1$ and $8 - 3x + 5x^2 - 4x^3$ from $2x^3 - 7x + 9$.

16. A triangle has a perimeter of $(6m + n)$ cm. One side measures $(2m - 3n)$ cm and another side measures $(3n + 2m)$ cm.

a) Write and simplify an expression for the length of the third side of the triangle.

b) Determine the measure of each side when $m = 4$ and $n = -1$.

Multiple Choice

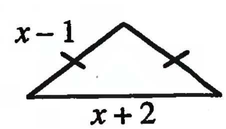
17. Which of the following is a polynomial expression of degree 4?

A. $4x^4 - 4x^7$
B. $5x^4 - 3x^3 + 2x^{-2} + x - 1$
C. $\dfrac{4x^4 - 3x}{x}$
D. $9 + 3x - \frac{1}{3}x^2 - x^3 + \frac{2}{5}x^4$

18. Which of the following polynomial expressions, when simplified, is equal to $5x$?

A. $(3x^2 - 3x) - (2x + 3x^2)$
B. $5x - (2x^2 - 2x) + (2x^2 + 2x)$
C. $8 + (4 - 2x) - (12 - 7x)$
D. $(2x^2 - 2x + 6) - (2x^2 - 2x) + (9x - 6)$

19. The perimeter of the isosceles triangle shown can be represented by



A. a monomial
B. a binomial
C. a trinomial
D. none of the above

Numerical Response

20. If the polynomial $4 - 7x + 2x^2 - 5x^3$ has degree a, leading coefficient b, and constant term c, then the value of $3a - 2b - c$ is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) binomial b) monomial c) trinomial **2.** a) 1 b) 4 c) 0 d) 4 e) 6

3. a) yes b) no, negative exponent c) yes

d) no, $\frac{7}{x^3} = 7x^{-3}$, which is a negative exponent. e) yes

f) no, the exponent 1.5 is not a positive integer.

4.

Polynomial expression	*# variables*	*# terms*	*Classification by Number of Terms*	*degree*
$2y^3 + y^4 - y + 13$	1	4	polynomial	4
$9ab - 4x + 11c$	4	3	trinomial	2
25	0	1	monomial	0
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$	3	2	binomial	9

5.

Polynomial expression	*leading coefficient*	*constant term*	*degree*	*Classification by Degree*
$y^4 - y + 13$	1	13	4	Quartic
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$	0.2	-0.5	3	Cubic
$\sqrt{7} - x^5$	-1	$\sqrt{7}$	5	Quintic
$\pi x^2 - 7 - 3x$	π	-7	2	Quadratic
$-\frac{1}{10}$	$-\frac{1}{10}$	0	0	Constant
$9x + 12$	9	12	1	Linear

6. answers may vary a) $x^2 - x + 30$ b) $abcd + 6$ c) $-2xy^2$ d) 10

7. a) $2w^3 + 6w^2 - 9w + 5$ b) $-\frac{2}{3}a^3 + \frac{1}{4}a^2 - a - 1$ c) $-4z^6 + z^3 + z - 3$

8. a) $5 - 9w + 6w^2 - 2w^3$ b) $-7 + 9x + 3x^2 - 4x^3 - 2x^4 - 4x^5$ c) $8 - 8x + 8x^3$

9. a) true b) false c) false d) true e) false f) true g) false h) true

10. a) $-2x + 1$ b) $2x^2 - x + 3$ c) $-x - 2$ **11.** a) $2x^2 - 3x - 6$ b) $2x^2 - 1$

12. a) $-x^2 - 4x - 4$ b) $1 + x - 2x^2$

13. a) $4p - 10q$ b) $-11x^2 + 7x$ c) $2x + 15$ d) $-3a^2 + a$ e) $-4x + 11y + 8z - 6$

14. a) $2a - 16b - c$ b) $3a^2 - 5a + 12$ c) $-3x^2 + 3x - 3$ d) $-a - 4b$
e) $2x^2 - 5x + 4$ f) $12x^2 + 5x$ g) $-2x^2 - x - 12$

15. a) $3x^2 - 3x - 9$ b) $4x^3 + 2x^2 + 2x$

16. a) $(2m + n)$ cm b) 11 cm, 5cm, and 7 cm

17. D **18.** C **19.** A **20.**

1	5		

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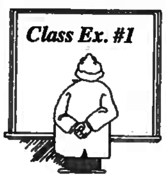
Polynomial Operations Lesson #2:
Multiplying a Polynomial by a Monomial

Using Algebra Tiles

In previous math courses, we learned how to multiply
i) two monomials, and ii) a monomial and a binomial or trinomial.

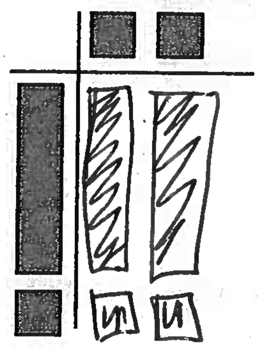
We can use algebra tiles to illustrate the process of multiplying a monomial by a polynomial.

Shaded tiles represent positive quantities and unshaded tiles represent negative quantities.



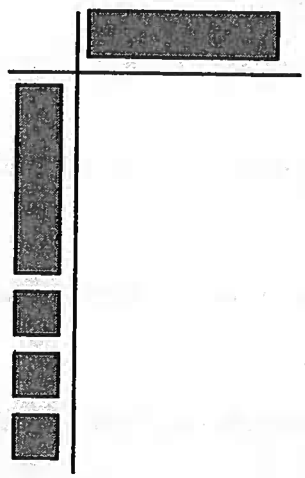
Complete the diagram to determine the product.

a) $2(x+1) =$

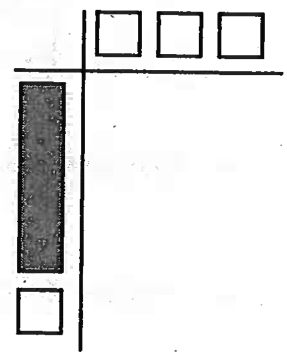


2x+2

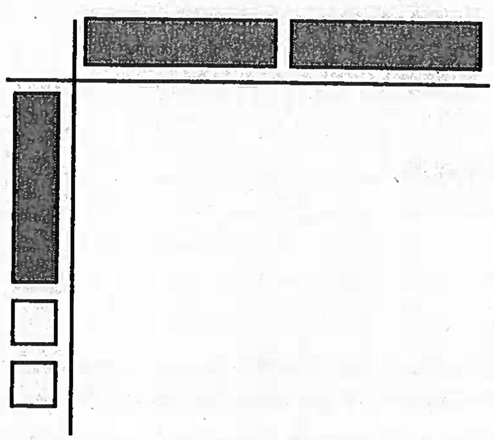
b) $x(x+3) =$



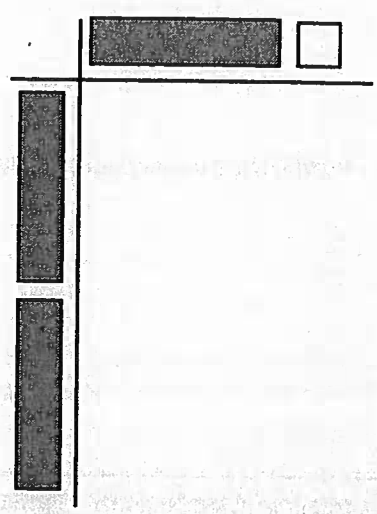
c) $-3(x-1) =$

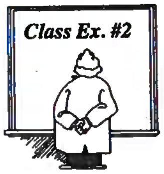


d) $2x(x-2) =$



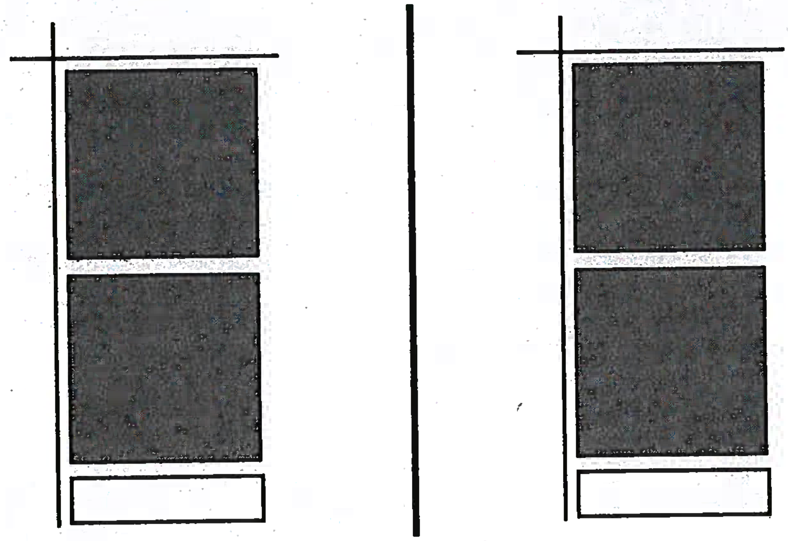
e) $(x-1)(2x) =$





Each diagram below illustrates the result of the product of a monomial and a binomial.

Diagram 1 Diagram 2



a) State the polynomial represented in each of the diagrams.

b) Complete the left side and the top of Diagram 1 and write the polynomial product.

c) Complete Diagram 2 to illustrate and write a different polynomial product than in b).

d) Write each product as a sum or difference of terms.

e) Verify the polynomial products in d) when $x = 3$.

Complete Assignment Questions #1 - #3

The Distributive Property

In Class Example #1 we have shown that:

$2(x+1) = 2x+2$, $x(x+3) = x^2+3x$, $2x(x-2) = 2x^2-4x$,

$-3(x-1) = -3x+3$, and $(x-1)(2x) = 2x^2-2x$.

These above are examples of the **distributive property**

$a(b+c) = ab + ac$ or $(b+c)(a) = ba + ca \Rightarrow ab + ac$.

The distributive property can be extended to any number of terms.

Using Numerical Values to Verify the Distributive Property

Consider the expression $-2(3-5)$.

i) Evaluate $-2(3-5)$ by calculating the value inside the brackets first and then multiplying by -2.

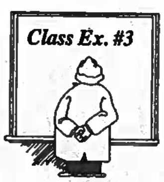
$= -2(-2)$

$= 4$

ii) Evaluate $-2(3-5)$ by using the distributive property.

$-6+10 = 4$

iii) Comment on your results from i) and ii).



Use the distributive property to determine the following products.

a) $4(3x+1)$

$12x+4$

b) $-5(2x^2+x-6)$

$-10x^2-5x+30$

c) $(x^3-2)x^2$

$= x^5-2x^2$

d) $-3x(7x-2y+z)$

$-21x^2+6xy-3xz$

In the example above we have written a <u>product of polynomials</u> as a <u>sum or difference of terms</u>.

In this process we **expanded** the polynomial expressions by using the distributive property, $a(b+c) = ab + ac$ and the exponent rule, $x^a \times x^b = x^{a+b}$.

[redacted]

Class Ex. #4

Expand and simplify.

a) $6 - 4(8x + 1)$ *distribute (multiply before subtraction!)

$6 - 32x - 4$

$= -32x + 2$ *simplify

b) $4(2x - 3) - 2(x - 6)$ *distribute all

$8x - 12 - 2x + 12$

$= 6x$

*simplify grouping like terms

c) $5x(3x^2 - 7x + 1) - (4x + 3x^2)$

$= 15x^3 - 35x^2 + 5x - 4x - 3x^2$

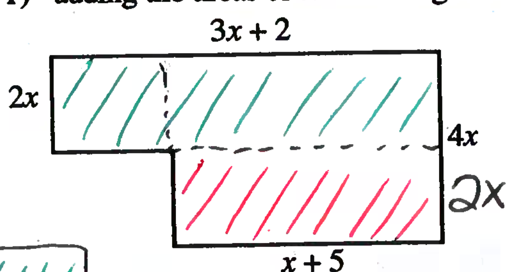
$= 15x^3 - 38x^2 + x$

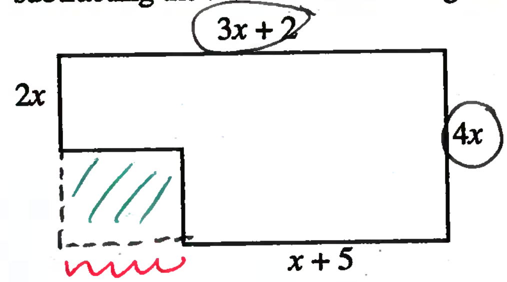
Class Ex. #5

Determine a simplified expression for the area of the given shape by (lw)

i) adding the areas of two rectangles.

ii) subtracting the areas of two rectangles.





$3x + 2 - (x + 5)$

$3x + 2 - x - 5$

$2x(3x + 2) + 2x(x + 5)$

$= 6x^2 + 4x + 2x^2 + 10x$

$= 8x^2 + 14x$

the area of the shape

$4x(3x + 2) - (2x - 3)2x$

$= 12x^2 + 8x - (4x^2 - 6x)$

$= 12x^2 + 8x - 4x^2 + 6x$

$= 8x^2 + 14x$

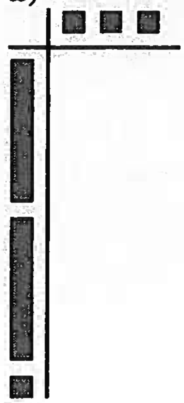
Complete Assignment Questions #4 - #11

[redacted]

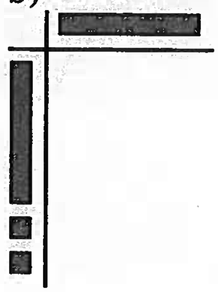
Assignment (4,5,7)acegi (,8b)

1. In each case complete the diagram, state the polynomial product in x, and express the product as a sum or difference of terms.

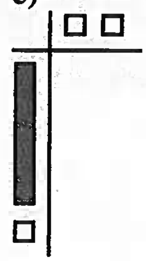
a)



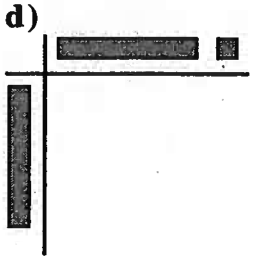
b)



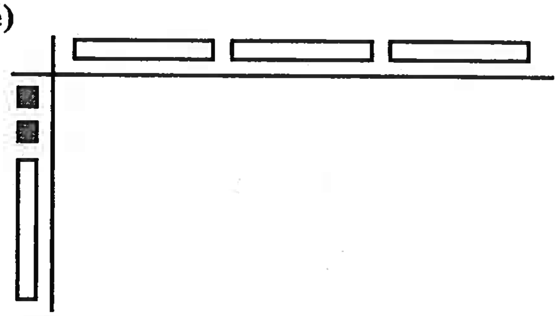
c)



d)

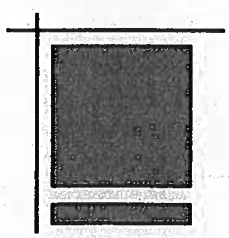


e)

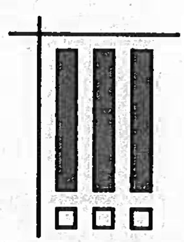


2. In each case state the polynomial product in x which is indicated by the algebra tile diagram. Express the product as a sum or difference of terms.

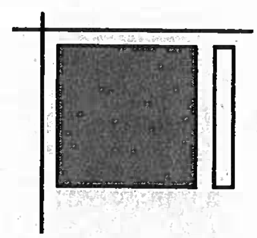
a)



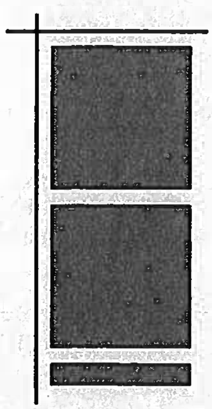
b)



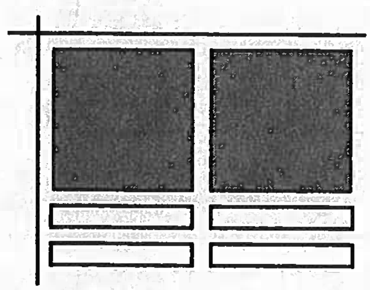
c)



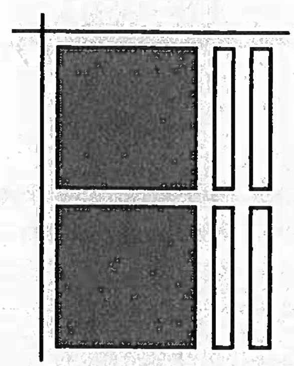
d)



e)



f)



3. For each of the following:

i) Draw an algebra tile diagram to model the product.
ii) Express the product as a sum or difference of terms.
iii) Verify the polynomial product when $x = 4$.

a) $2x(2x - 1)$ **b)** $-3x(2 - x)$

4. Expand.

a) $6(7x - 3)$ **b)** $-4(4x + 9)$ **c)** $4x(2y + 8z)$ **d)** $-x(x - 5y)$

e) $3(x - 2y + 3z)$ **f)** $-2a(b - c + 5d)$ **g)** $(x + 3)3x$

h) $2x(x - 5y + 4z)$ **i)** $x(x - 2x^2 + 3x^3)$ **j)** $(2x^2 + x - 6)(-4x)$

5. Expand and simplify.

a) $3(x+5)-7$ **b)** $8-2(5x+11)$ **c)** $6(x-2)+x$

d) $2(x+3)+4(2x-1)$ **e)** $2(x+3)-4(2x-1)$ **f)** $-2(x+1)+7(3x-2)$

g) $5(-x+12)+5(x-8)$ **h)** $(2-x)-2(2x-10)$ **i)** $6(-x+4)-(x-15)$

6. Identify the errors in the following and provide the correct simplification.

a) $3x(2x+y)=6x+3xy$ **b)** $x^2(x^3-2x+7)=x^6-2x^3+7x^2$

c) $4(x-2)-2(x-3)$
$=4x-8-2x-6$
$=2x-14$

d) $2(2t-3)-4(t+5)$
$=4t-3-4t-5$
$=-8$

e) $5(a+b)-(a+b)$
$=5a+5b-a+b$
$=4a+6b$

7. Expand and simplify.

a) $2a(a+3) - 4a(2a-1)$ **b)** $4(x^2+3) - (2x^2-1)$ **c)** $2(x+3) - x - 1$

d) $z(z^3+3) - (3z+7)$ **e)** $5(8x-3y) + 2(4y+x)$

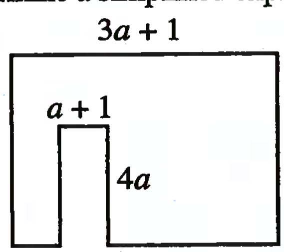
f) $-2x(x^4+3x^3) - 7x(2x^4-x^3)$ **g)** $3a(2a^2b - ab + b^2) - 6b(a^3 + 3ab - 5b^2)$

h) $3x(x-3) - 2x(x-1) + x(2x-2)$ **i)** $(p^2-3p)(4p) - (3+5p)(-2p^2)$

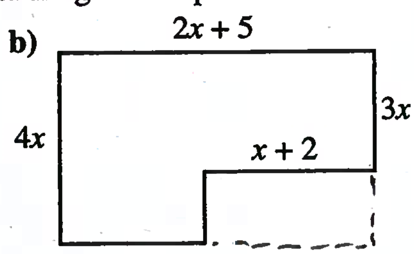
j) $a(b-c) + b(c-a) + c(a-b)$ **k)** $20x^3y^3 - 4x^3y^2(3x+5y-xy)$

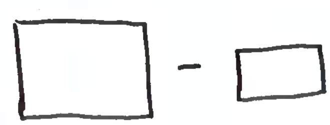
8. Determine a simplified expression for the area of the given shape.

a)

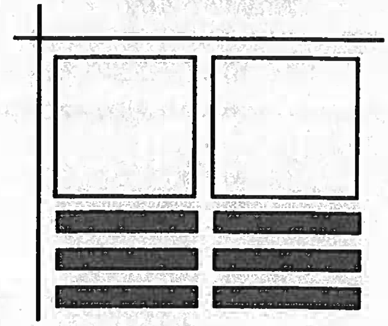


b)





Multiple Choice **9.** The algebra tile diagram represents the expansion of:



A. $2x(x+3)$
B. $-2x(x+3)$
C. $2x(x-3)$
D. $-2x(x-3)$

10. Which of the following expansions is incorrect?

A. $-2x^2(3x+2) = -6x^3 - 4x^2$
B. $-4x(2-x) = -8x + 4x^2$
C. $-5x(x^2-3) = -5x^3 - 15x$
D. $7x^2(x^2+3) = 7x^4 + 21x^2$

Numerical Response **11.** The expression $2x(4-3x) + 5x(2x-1) - 3(4x+2)$ can be written in the form $ax^2 + bx + c$. The value of $a+b-c$ is ______.

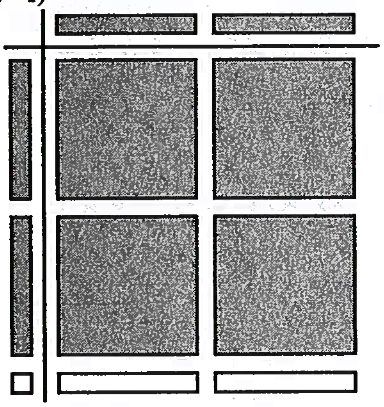
(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $3(2x+1) = 6x+3$ b) $x(x+2) = x^2+2x$ c) $-2(x-1) = -2x+2$
 d) $(x+1)(x) = x^2+x$ e) $-3x(2-x) = -6x+3x^2$

2. a) $x(x+1) = x^2+x$ b) $3(x-1) = 3x-3$ c) $(x-1)(x) = x^2-x$
 d) $x(2x+1) = 2x^2+x$ e) $2x(x-2) = 2x^2-4x$ f) $(x-2)(2x) = 2x^2-4x$

3. a) i)

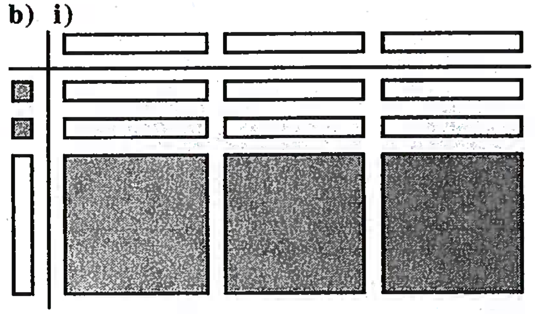


ii) $2x(2x-1) = 4x^2 - 2x$

iii)

Left Side	Right Side
(2 x 4) ((2 x 4) – 1)	4 (4²) – 2(4)
= (8) ((7)	= 64 – 8
= 56	= 56

b) i)



ii) $-3x(2-x) = -6x + 3x^2$

iii)

Left Side	Right Side
(–3 x 4) (2 – 4)	(–6 x 4) + 3(4²)
= (–12) ((–2)	= –24 + 48
= 24	= 24

4. a) $42x-18$ b) $-16x-36$ c) $8xy+32xz$
 d) $-x^2+5xy$ e) $3x-6y+9z$ f) $-2ab+2ac-10ad$
 g) $3x^2+9x$ h) $2x^2-10xy+8xz$ i) $x^2-2x^3+3x^4$ j) $-8x^3-4x^2+24x$

5. a) $3x+8$ b) $-10x-14$ c) $7x-12$
 d) $10x+2$ e) $-6x+10$ f) $19x-16$
 g) 20 h) $-5x+22$ i) $-7x+39$

6. a) $3x(2x) = 6x^2$, not $6x$. $3x(2x+y) = 6x^2+3xy$
 b) $x^2(x^3) = x^5$ not x^6. $x^2(x^3-2x+7) = x^5-2x^3+7x^2$
 c) $-2(-3) = 6$, not -6. $4(x-2)-2(x-3) = 4x-8-2x+6 = 2x-2$
 d) The monomials 2 and –4 multiply both terms in the binomials.
 $2(2t-3)-4(t+5) = 4t-6-4t-20 = -26$.
 e) The negative multiplies both a and b. $5(a+b)-(a+b) = 5a+5b-a-b = 4a+4b$.

7. a) $-6a^2+10a$ b) $2x^2+13$ c) $x+5$ d) z^4-7
 e) $42x-7y$ f) $-16x^5+x^4$ g) $-3a^2b-15ab^2+30b^3$ h) $3x^2-9x$
 i) $14p^3-6p^2$ j) 0 k) $-12x^4y^2+4x^4y^3$

8. a) $14a^2+2a$ b) $7x^2+18x$ 9. D 10. C 11.

1			

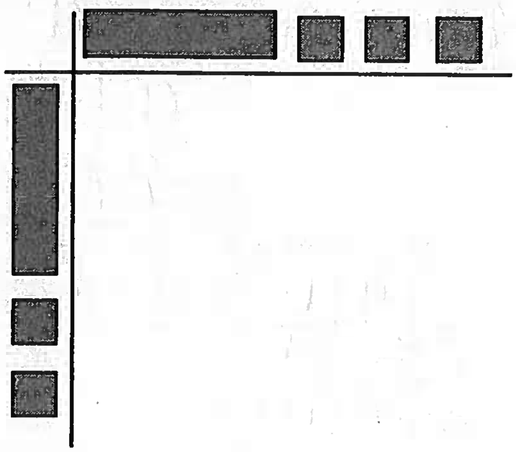
Polynomial Operations Lesson #3:
Multiplication of Two Binomials

Multiplying Two Binomials using Area Diagrams

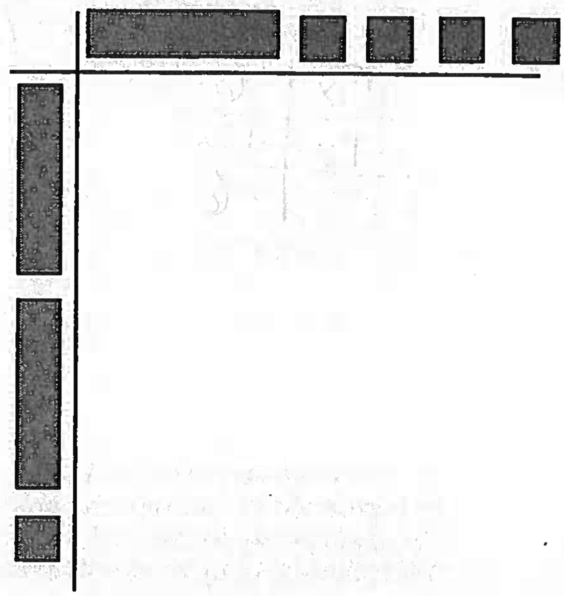
In the last lesson, we multiplied a monomial by a polynomial. In this lesson, we extend the process to the product of two binomials.

Complete the algebra tile diagrams and determine the binomial products.

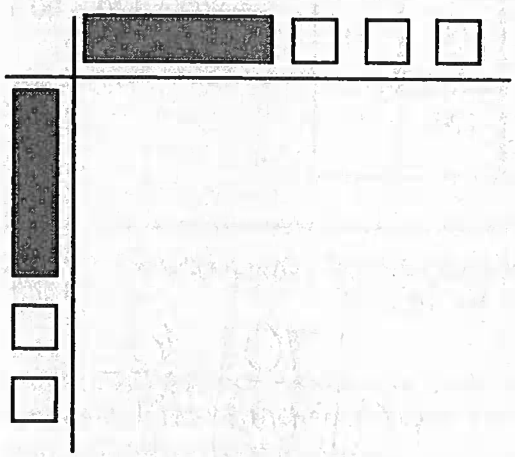
a) $(x+3)(x+2)$ =



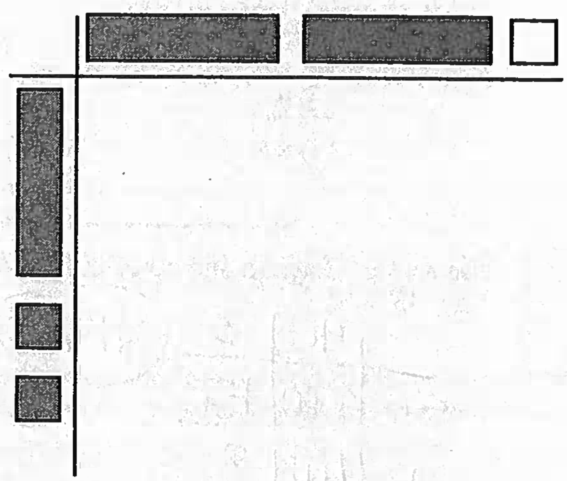
b) (   )(   ) =



c) (   )(   ) =



d) (   )(   ) =

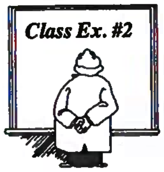


In class example 1a), we used an algebra tile diagram to show that the product $(x+3)(x+2)$ could be expressed in simplified expanded form as x^2+5x+6.

The algebra tile diagram used to model $(x+3)(x+2)$ can be modified into the following area diagram which shows that the product of two binomials is equivalent to four monomial products.

	x	3
x	x^2	$3x$
2	$2x$	6

$(x+3)(x+2)$
$=x^2+5x+6$



Use an area diagram like the one above to determine the product of each of the following binomials.

a) $(5x-6)(2x+1)$ $=10x^2-7x-6$

	$5x$	-6
$2x$	$10x^2$	$-12x$
1	$5x$	-6

b) $(a^2-5)(a^2-8)$

	a^2	-5
a^2	a^4	$-5a^2$
-8	$-8a^2$	40

$=a^4-13a^2+40$

c) $(3p+2q)(p+9q)$

d) $(a+b)(c+d)$

	a	b
c	ac	cb
d	ad	bd

An area diagram can be used to show that the multiplication of two, two-digit numbers can be performed as four separate products.
For example the product 32×34 can be determined without a calculator, by long multiplication or by an area diagram as follows:

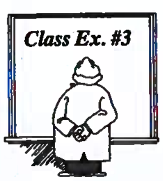
Long Multiplication

```
   32
 × 34
  128
  96
 1088
```

Area Diagram

	30	2
30	900	60
4	120	8

32 x 34
= 900 + 120 + 60 + 8
= 1088



Use an area diagram and no calculator to determine the following products.

a) 43×51 = 2193

	40	3
50	2000	150
1	40	3

2000+150+40+3

b) 76×82

	70	6
80	5600	480
2	140	12

5600 + 480 + 140 + 12 = 6232

Complete Assignment Questions #1 - #3

Multiplying Two Binomials using the Distributive Property

In the area diagram modelling $(x+3)(x+2)$, we noted that there were four separate monomial products involved in the expansion.
These products are simply the extension of the distributive property to binomial products.

Distributive property for binomials $(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd$

Class Ex. #4

Use the distributive property to determine the following products.

a) $(x+3)(x+2)$

$= x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

b) $(a-7)(2a-1)$

$= 2a^2 - a - 14a + 7$

$= 2a^2 - 15a + 7$

c) $(p-8)(q-8)$

$= pq - 8p - 8q + 64$

d) $(x+4y)(x-5y)$

$x^2 - 5xy + 4xy - 20y^2$

$= x^2 - xy - 20y^2$

e) $(9a^2-1)(5a^3+6)$

$= 45a^5 + 54a^2 - 5a^3 - 6$

The method used in the distributive property can be simplified by noticing that the four monomial products $(a+b)(c+d) = ac + ad + bc + bd$ can be memorized using the acronym FOIL.

F	– first term in each bracket	ie ac
O	– outside terms	ie ad
I	– inside terms	ie bc
L	– last term in each bracket	ie bd

Class Ex. #5

Use FOIL to determine each product.

a) $(x+6)(x+4)$

F x^2 I $6x$

O $4x$ L 24

$= x^2 + 10x + 24$

b) $(y-7)(y+2)$ Firsts, outsides, insides, lasts

$= y^2 + 2y - 7y - 14$

$= y^2 - 5y - 14$

c) $(3x+1)(x-5)$

$= 3x^2 - 15x + x - 5$

$= 3x^2 - 14x - 5$

d) $(6a-5b)^2$

$(6a-5b)(6a-5b)$

$= 36a^2 - 30ab - 30ab + 25b$

$= 36a^2 - 60ab + 25b$

$4^2 = 4 \cdot 4$

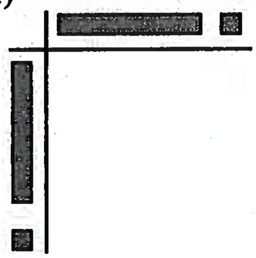
$x^2 = x \cdot x$

Complete Assignment Questions #4 - #9

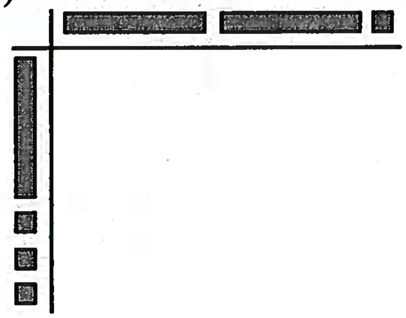
Assignment 2ace,(4,5,7)acegi

1. Complete the algebra tile diagrams and determine the binomial products.

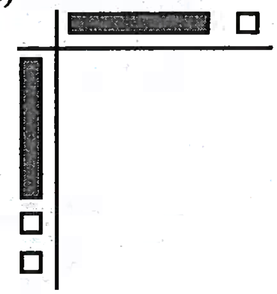
a)



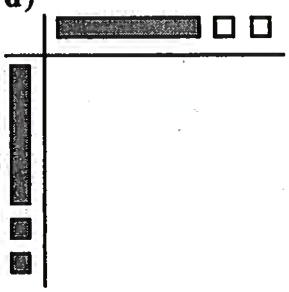
b)



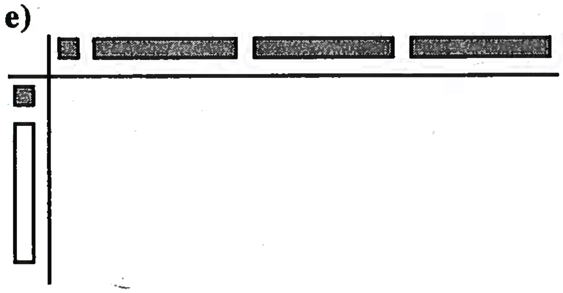
c)



d)



e)



2. Use an area diagram to determine the product of each of the following binomials.

a) $(x+6)(x-2)$

b) $(2x+3)(2x+7)$

c) $(y-3)(4y+1)$

d) $(3d-5)(6d-9)$

e) $(2x-y)(4x+y)$

f) $(3p-8q)(p-5q)$

g) $(a^2 + 8)(a^2 - 8)$ **h)** $(t^3 + 2s)(t^3 + 2s)$ **i)** $(a + b)(a + c)$

3. Without a calculator, use an area diagram to determine the following products.

a) 23×21 **b)** 34×12 **c)** 74×32

d) 65×73 **e)** 49×55 **f)** 86×86

4. Use the distributive property to determine the following products.

a) $(x + 4)(x + 7)$ **b)** $(a + 7)(3a - 5)$ **c)** $(p - 2)(p - 8)$

d) $(x + 6y)(x - 2y)$ **e)** $(4a + 9b)(2a + 3b)$ **f)** $(6 - y)(1 + 4y)$

g) $(2a - 1)(6b - 1)$ **h)** $(7x^2 - 3)(7x^2 - 3)$ **i)** $(2y^2 - 3)(5y^5 + 1)$

5. Use FOIL to determine each product.

a) $(x+3)(x+6)$ **b)** $(y+4)(y+9)$ **c)** $(x+1)(x-8)$ **d)** $(a-7)^2$

e) $(x+2)(5x+4)$ **f)** $(3y-5)(2y+9)$ **g)** $(6x+1)(x-6)$ **h)** $(6-5b)(6-5b)$

i) $(x+3y)(x+4y)$ **j)** $(a-7b)(3a+4b)$ **k)** $(5x+z)(5x-z)$ **l)** $(9-a^2)(5-a^2)$

6. A rectangle has length $(2a+5)$ cm and width $(a+4)$ cm.
Determine the area of the rectangle (in cm^2) by completing each of the following solutions.

Area = length × width = (   )(   )

(i) *use a diagram*

(ii) *use the distributive property*
$(2a+5)(a+4)$
$= 2a(a+\quad) +$

(iii) *use FOIL*
$(2a+5)(a+4)$

7. Expand and simplify where possible.

a) $(7x-2)(3x+5)$ **b)** $(2h-3)(2h-1)$ **c)** $(3z+4)(3z+5)$

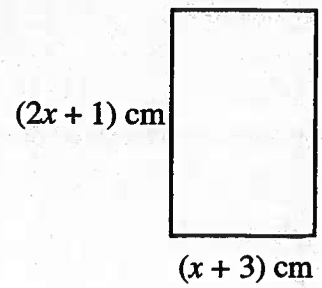
d) $(4x-3)(3x-4)$ **e)** $(8x-3y)(2x+y)$ **f)** $(1+3b)^2$

g) $(x-2)(6y-1)$ **h)** $(1+3y^2)(1-3y^2)$ **i)** $(x^2+7y^2)(2x^2-5y^2)$

Numerical Response

8. The area of the rectangle shown can be written in the form $px^2 + qx + r$, where p, q, and r are natural numbers.

Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.



(Record your answer in the numerical response box from left to right)

9. The expansion of $(3x - c)(x - 3)$, where c is a whole number, results in a polynomial in x with a leading coefficient of 3 and a constant term of 12. The value of c is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $(x+1)(x+1) = x^2 + 2x + 1$ **b)** $(2x+1)(x+3) = 2x^2 + 7x + 3$
c) $(x-1)(x-2) = x^2 - 3x + 2$ **d)** $(x-2)(x+2) = x^2 - 4$
e) $(1+3x)(1-x) = 1 + 2x - 3x^2$

2. **a)** $x^2 + 4x - 12$ **b)** $4x^2 + 20x + 21$ **c)** $4y^2 - 11y - 3$
d) $18d^2 - 57d + 45$ **e)** $8x^2 - 2xy - y^2$ **f)** $3p^2 - 23pq + 40q^2$
g) $a^4 - 64$ **h)** $t^6 + 4st^3 + 4s^2$ **i)** $a^2 + ab + ac + bc$

3. **a)** 483 **b)** 408 **c)** 2368 **d)** 4745 **e)** 2695 **f)** 7396

4. **a)** $x^2 + 11x + 28$ **b)** $3a^2 + 16a - 35$ **c)** $p^2 - 10p + 16$
d) $x^2 + 4xy - 12y^2$ **e)** $8a^2 + 30ab + 27b^2$ **f)** $6 + 23y - 4y^2$
g) $12ab - 2a - 6b + 1$ **h)** $49x^4 - 42x^2 + 9$ **i)** $10y^7 - 15y^5 + 2y^2 - 3$

5. **a)** $x^2 + 9x + 18$ **b)** $y^2 + 13y + 36$ **c)** $x^2 - 7x - 8$
d) $a^2 - 14a + 49$ **e)** $5x^2 + 14x + 8$ **f)** $6y^2 + 17y - 45$
g) $6x^2 - 35x - 6$ **h)** $36 - 60b + 25b^2$ **i)** $x^2 + 7xy + 12y^2$
j) $3a^2 - 17ab - 28b^2$ **k)** $25x^2 - z^2$ **l)** $45 - 14a^2 + a^4$

6. Area $= (2a+5)(a+4) = 2a^2 + 13a + 20$

7. **a)** $21x^2 + 29x - 10$ **b)** $4h^2 - 8h + 3$ **c)** $9z^2 + 27z + 20$
d) $12x^2 - 25x + 12$ **e)** $16x^2 + 2xy - 3y^2$ **f)** $1 + 6b + 9b^2$
g) $6xy - x - 12y + 2$ **h)** $1 - 9y^4$ **i)** $2x^4 + 9x^2y^2 - 35y^4$

8. | 2 | 7 | 3 | | **9.** | 4 | | | |

[redacted]