Factoring Polynomial Expressions Lesson #1:
Common Factors

Overview of Unit

In this unit, we introduce the process of factoring. This includes factoring by removing a common factor, factoring a trinomial, and factoring a difference of squares. These techniques are illustrated concretely, pictorially, and symbolically. We express a polynomial as a product of its factors and include, for enrichment, polynomial equation solving.

Expanding and Factoring

In the previous unit, we were concerned with multiplying polynomial expressions. In particular we multiplied

i) a monomial by a polynomial e.g. $2x(x+5)$ = $2x^2+10x$

ii) a binomial by a binomial to form a trinomial e.g. $(x+1)(x+3)$ = $x^2+3x+x+3$
x^2+4x+3

iii) a binomial by a binomial to form a binomial e.g. $(x-5)(x+5)$ = x^2-25

In these examples, we have **expanded** a product of polynomials to form a sum or difference of monomials.

In this unit, we are concerned with the opposite process. We want to write a sum or difference of monomials as a product of polynomials. This process is called **factoring**.

We will be studying the following three major types of factoring.

Complete the following using the results obtained above.

i) factoring by removing a common factor e.g. $2x^2+10x$ = $2x(x+5)$

ii) factoring a trinomial. e.g. x^2+4x+3 = $(x+1)(x+3)$

iii) factoring a difference of squares e.g. x^2-25 = $(x-5)(x+5)$

Greatest Common Factor

In the lesson "Applications of Prime Factors" page 9, we met the concept of the greatest common factor of whole numbers.

The GCF of 48 and 72 was found by using prime factorization.

$48 = 2 \times 2 \times 2 \times 2 \times 3$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

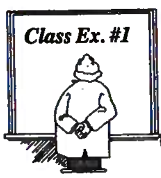
To determine the greatest common factor of 48 and 72, we found the product of each prime factor (including repeats) which is <u>common</u> to each prime factorization.

GCF of 48 and 72 is $2 \times 2 \times 2 \times 3 = \underline{24}$.

The same process can be used to determine the greatest common factor of two monomials like $6a^3$ and $9a^2b$.

$6a^3 = 2 \times 3 \times a \times a \times a$ and $9a^2b = 3 \times 3 \times a \times a \times b$

GCF of $6a^3$ and $9a^2b$ is $3 \times a \times a = \underline{3a^2}$.



Class Ex. #1 Write the prime factorization of $8x^2y^2$ and $20xy^3$ and determine the greatest common factor of $8x^2y^2$ and $20xy^3$.

$8x^2y^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$

$20xy^3 = 2 \cdot 2 \cdot 5 \cdot x \cdot y \cdot y \cdot y$

GCF: $2 \cdot 2 \cdot x \cdot y \cdot y$

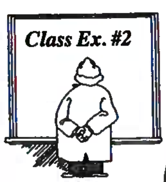
$= 4xy^2$

Note

- The greatest common factor of two simple monomials can be determined by <u>inspection</u>, by taking the GCF of any numerical coefficients and multiplying by each common variable to the lowest common exponent.

  The greatest common factor of $10p^3q$ and $15p^2q^2$ is determined by multiplying 5 by p^2 by q, i.e. $5p^2q$.

- If all the monomials are negative, the GCF is usually considered to be negative (see example d) below).



Class Ex. #2 In each case, state the greatest common factor of the following sets of monomials.

a) $12ab$, $15a^2b^3$

GCF: $3ab$

b) $18x^4y^2$, $-24x^3y^5$

GCF: $6x^3y^2$

c) a^3bc^2, $2ac^7$

GCF: ac^2

d) $-40a^3b$, $-20a^2b^3$, $-10a^2b^2$

~~$-10a$~~ $-10a^2b$

Complete Assignment Question #1 - #3

Factoring a Polynomial by Removing the Greatest Common Factor

Factoring is a process in which a sum or difference of terms is expressed as a product of factors.

A polynomial like $8x^2y^2 + 20xy^3$ can be factored by removing (or taking out, or dividing out) the greatest common factor from each term.

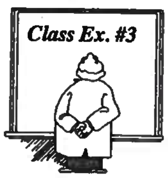
We know that

$4xy^2(2x + 5y)$ can be <u>expanded</u> to give $8x^2y^2 + 20xy^3$.

It follows that

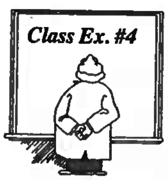
$8x^2y^2 + 20xy^3$ can be <u>factored</u> to give $4xy^2(2x + 5y)$.

In this case, the greatest common factor $4xy^2$ has been removed from each term.



In each case, complete the factoring.

a) $21x + 14y =$ 7 $(3x + 2y)$ **b)** $5x^4 + 15x^3 + 5x^2 =$ $5x^2$ $(x^2 + 3x + 1)$



In each case, the greatest common factor has been removed. Complete the factoring.

a) $5a^2 + 25a = \boxed{5a(a + 5)}$

5a 5a

b) $18p - 16q = 2(9p - 8q)$

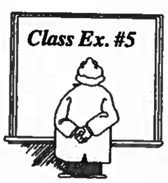
expanded factored

c) $-4mn - 6m^2 = -2m(2n + 3m)$

-2m -2m

d) $18x^2y^2 - 45xy + 9x = 9x(2xy^2 - 5y + 1)$

9x -9x 9x



Factor each polynomial by removing the greatest common factor.

a) $20x - 6$

$2(10x - 3)$

b) $16x^4 + 4x^2$

$4x^2(4x^2 + 1)$

c) $10a^3b^2 + 8ab^3 + 2ab^4$

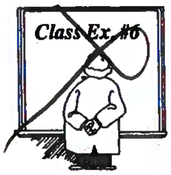
$2ab^2(5a^2 + 4b + b^2)$

d) $12p^3 - 6p^2 + 15p$

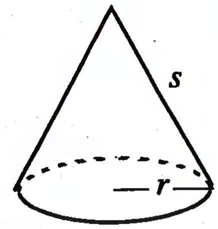
$3p(4p^2 - 2p + 5)$

e) $25xy^2z^3 - 20x^2y^4z^2 + 30x^4y^2z^5$

$5xy^2z^2(5z - 4xy^2 + 6x^3z^3)$



The surface area of a cone is given by the formula $A = \pi r^2 + \pi rs$, where r is the radius of the base of the cone and s is the slant height.



i) Determine the surface area of a cone, to the nearest $0.01\ \text{cm}^2$, which has slant height 7.40 cm and base radius 2.60 cm.

ii) Write the formula for A in factored form.

iii) Calculate the surface area of the cone, to the nearest $0.01\ \text{cm}^2$, using the factored form of A.

iv) Which method i) or iii) is simpler to use?

Complete Assignment Questions #4 - #13

Assignment 3aceigk, (4-7)ace, 8ac

1. Write the prime factorization of $12a^3$ and $30a^2$ and determine the greatest common factor of $12a^3$ and $30a^2$.

2. Write the prime factorization of $10xy^4$ and $25x^2y^3$ and determine the greatest common factor of $10xy^4$ and $25x^2y^3$.

3. In each case, state the greatest common factor of the following sets of monomials.

a) $7m,\ 14m$ b) $6x^2,\ 9x$ c) $bc^2,\ bc^7$

d) $ab,\ a^2b^2$ e) $4x^4,\ 8x^3$ f) $3xyz,\ 9rst,\ 12def$

g) $-8pq^3,\ 18p^2q$ **h)** $-10x^5z^6,\ -15x^5z^4$ **i)** $8ab^2,\ 9ab,\ 6a^2b$

j) $10xy,\ 16xz,\ 20xyz$ **k)** $-2x^3y,\ -4x^3y^4,\ -4x^2y^4$ **l)** $-28pqr^3,\ -56p^2q,\ -64q^2r$

4. Complete the factoring in each case.

a) $12a + 24b = \underline{\qquad}(a + 2b)$ **b)** $4p^2 - 7p = \underline{\qquad}(4p - 7)$

c) $2xy + 3xz = \underline{\qquad}(2y + 3z)$ **d)** $5x^2 + 10x + 15 = \underline{\qquad}(x^2 + 2x + 3)$

e) $6cde - 4cd = \underline{\qquad}(3e - 2)$ **f)** $3y^3 - 9y = \underline{\qquad}(y^2 - 3)$

5. In each case, the greatest common factor has been removed. Complete the factoring.

a) $3a^2 + 15a = 3a(a +$ **b)** $20p - 10q = 10(\quad - q)$

c) $6x^3 - 9x^2 = 3x^2($ **d)** $4a^2b + 8a^3b^2 = 4a^2b($

e) $-15x^2y - 10x^2y^2 = -5x^2y($ **f)** $16xm^2n^3 - 12mn^2 - 4mn = 4mn($

6. Factor the following polynomials by removing the greatest common factor.

a) $6m + 6n$ **b)** $7xy^2 + 49$ **c)** $15pq - 5$ **d)** $8c + 12d$

e) $xy + y$ **f)** $6x^2 - 9x$ **g)** $9ab - 12ac$ **h)** $48y^2 - 72y^5$

7. Factor the following polynomials

a) $12x - 8y + 16z$ **b)** $9pq + 6pr - 15p$ **c)** $t^3 + t^2 + t$

d) $5x^2 - 10xy - 20xz$ **e)** $4abc - 2abd + 8abe$ **f)** $14a^2b^2 + 21a^3b^2 - 35a^2b^3$

8. In each of the following:
i) simplify the expression by combining like terms.
ii) factor the resulting polynomial.

a) $5x^2 - 2x + 7 - 2x^2 + 8x - 7$

b) $6 - 2y + 5y^2 - 10y + 3y^2 - 12$

c) $xy^3 - 2x^3y + 6x^2y^2 - 5xy^3 + 8x^3y$

d) $2(x^3 - 3x) - 4x(x - 6) + 5x^2(x - 2) - 4x$

9. Factor the following polynomials. Expand your answer to verify the factoring.

a) $24x^3 - 60x^2$

b) $-8p^3 - 32p^2 - 8p$

10. An archer standing on the ground fires an arrow vertically upward into the air at a speed of 30 m/s.
The height (h metres) of the arrow above the ground after t seconds can be approximated by the formula $h = 30t - 5t^2$.

a) Write h in factored form.

b) Use the factored form of h to calculate the height for each of the times in the table. Record your answer in the table.

Time (t seconds)	0	1	2	3	4	5	6
Height (h metres)							

c) Explain why the height of the arrow after two seconds is the same as the height of the arrow after four seconds.

d) Calculate h when $t = 7$. Explain why this has no meaning in the context of the question.

Multiple Choice

11. One factor of $9x^4 - 6x^3 + 3x^2$ is

A. $9x^4$
B. $3x^2 - 2x$
C. $3x^2 - 6x + 3$
D. $3x^2 - 2x + 1$

Numerical Response

12. When $x^4y^3 - x^2y^3 + x^6y$ is factored, the greatest common factor has degree A and the remaining trinomial factor has degree B. The value of $A + 2B$ is _____.

(Record your answer in the numerical response box from left to right)

13. When the greatest common factor is removed from the binomial $98x^2 - 28x$, the binomial can be written in the form $ax(bx + c)$. The value of $a + b + c$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. $12a^3 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a$ $\quad 30a^2 = 2 \cdot 3 \cdot 5 \cdot a \cdot a$ $\quad GCF = 2 \cdot 3 \cdot a \cdot a = 6a^2$

2. $10xy^4 = 2 \cdot 5 \cdot x \cdot y \cdot y \cdot y \cdot y$ $\quad 25x^2y^3 = 5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$ $\quad GCF = 5 \cdot x \cdot y \cdot y \cdot y = 5xy^3$

3. a) $7m$ b) $3x$ c) bc^2 d) ab e) $4x^3$ f) 3 g) $2pq$ h) $-5x^5z^4$ i) ab j) $2x$ k) $-2x^2y$ l) $-4q$

4. a) 12 b) p c) x d) 5 e) $2cd$ f) $3y$

5. a) $a+5$ b) $2p-q$ c) $2x-3$ d) $1+2ab$ e) $3+2y$ f) $4xmn^2-3n-1$

6. a) $6(m+n)$ b) $7(xy^2+7)$ c) $5(3pq-1)$ d) $4(2c+3d)$
e) $y(x+1)$ f) $3x(2x-3)$ g) $3a(3b-4c)$ h) $24y^2(2-3y^3)$

7. a) $4(3x-2y+4z)$ b) $3p(3q+2r-5)$ c) $t(t^2+t+1)$ d) $5x(x-2y-4z)$
e) $2ab(2c-d+4e)$ f) $7a^2b^2(2+3a-5b)$

8. a) $3x^2+6x = 3x(x+2)$ b) $8y^2-12y-6 = 2(4y^2-6y-3)$
c) $6x^3y+6x^2y^2-4xy^3 = 2xy(3x^2+3xy-2y^2)$ d) $7x^3-14x^2+14x = 7x(x^2-2x+2)$

9. a) $12x^2(2x-5)$ b) $-8p(p^2+4p+1)$

10. a) $h = 5t(6-t)$ b) 0, 25, 40, 45, 40, 25, 0
c) At 2 sec. the arrow is on the way up and at 4 sec. the arrow is on the way down.
d) $h = -35$. The arrow has already hit the ground at $t = 6$. It does not travel 35m below the ground.

11. D

12.

1	1		

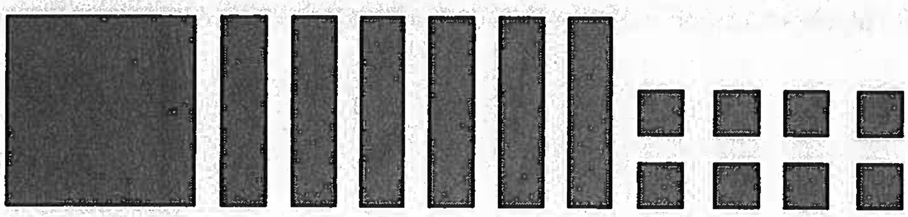
13.

1	9		

Factoring Polynomial Expressions Lesson #2:
Factoring Trinomials of the Form $x^2 + bx + c$ - Part One

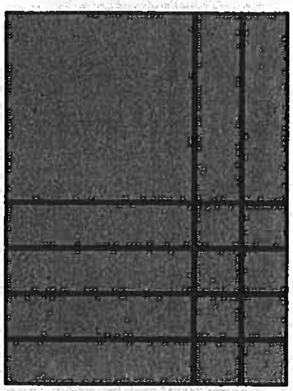
Factoring Trinomials using Algebra Tiles

Consider the algebra tile diagram shown.



- Write the polynomial expression which is represented by the algebra tiles.

- The algebra tiles can be rearranged into a rectangular form as shown below.



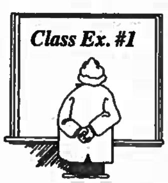
i) Write an expression for the length of the rectangle.

ii) Write an expression for the width of the rectangle.

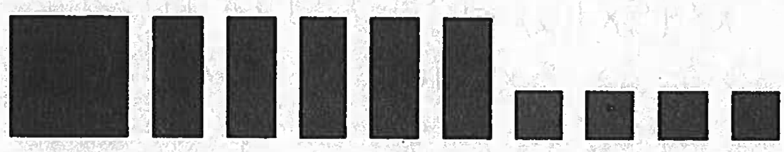
iii) Write the area of the rectangle as a product of two binomials.

iv) Write the area of the rectangle in expanded form.

- The work above provides a method for factoring the trinomial $x^2 + 6x + 8$ into the product of two binomials $(x+2)(x+4)$: i.e. $x^2 + 6x + 8 = (x+2)(x+4)$.



a) Write the polynomial expression which is represented by the algebra tiles.

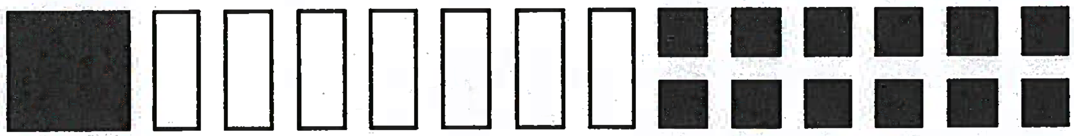


b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

c) Use the results above to express the polynomial in factored form.

Class Ex. #2

a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and express the polynomial in factored form.

Complete Assignment Questions #1 - #3

Investigation: Factoring Trinomials by Inspection

- Expand the following binomials as shown.

$(x+2)(x+4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

$(x+3)(x+3) = x^2+3x+3x+9 = x^2+6x+9$

$(x+1)(x+7) = x^2+7x+x+7 = x^2+8x+7$

$(x+5)(x+2) = x^2+5x+2x+10 = x^2+7x+10$

$(x-5)(x-2) = x^2-2x-5x+10 = x^2-7x+10$

$(x+8)(x-6) = x^2-6x+8x-48 = x^2+2x-48$

- Consider the expansion $(x+p)(x+q) = x^2 + bx + c$.

  In each of the examples above what is the connection between

  i) the value of b and the values of p and q? $b =$ $p+q$

  ii) the value of c and the values of p and q? $c =$ pq

[redacted]

Class Ex. #3

Use FOIL to show that $(x+p)(x+q)$ can be written in the form $x^2+(p+q)x+pq$.

$x^2 + xq + xp + pq$

$x^2 + (q+p)x + pq$

Factoring $x^2 + bx + c$ by Inspection

In order to factor $x^2 + bx + c$ by inspection we need to find two integers which have a product equal to c and a sum equal to b. If no two such integers exist, then the polynomial cannot be factored.

In order to factor $x^2 + 8x + 12$ we need to find two numbers which multiply to 12 and add to 8.

In order to factor $x^2 - 13x + 12$ we need to find two numbers which multiply to 12 and add to −13 .

The next example practices this skill.

Class Ex. #4

Complete the tables to find two numbers with the given sum and the given product.

Sum	Product	Integers
12	20	2, 10
9	20	4, 5
4	4	2, 2
–9	18	−3, −6

Sum	Product	Integers
–15	14	−1, −14
–1	–6	−3, 2
2	–15	−3, 5
–26	48	−2, −24

1·18, −1·−18
2·9 −2·−9
3·6 −3·−6

−3·2 −1
3·−2 1
1·−6 −5
−1·6 5

"multiply to ___ and add to ___"

Note Notice that:

- if the product is **positive**, then the two integers must be either **both positive** or **both negative**.
- if the product is **negative**, then one integer is **positive** and the other is **negative**.

Note **For the remainder of this lesson, we will only deal with examples where the product is positive. In the next lesson we will include examples where the product is negative.**

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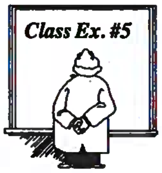


Factor the following trinomials where possible.

a) $x^2 + 8x + 12$ $(x+6)(x+2)$ x | + 12 | 8

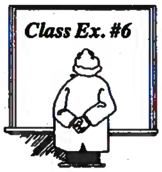
b) $x^2 + 13x + 12$ $(x+1)(x+12)$ x | + 12 | 13

c) $x^2 - 13x + 12$ $(x-1)(x-12)$ x | + 12 | −13

d) $a^2 - 11a + 10$ $(a-10)(a-1)$ x | + 10 | −11

e) $y^2 + 3y + 4$ x | + 4 | 3 ~~(y)(y)~~ not possible

f) $x^2 + 27x + 50$ $(x+25)(x+2)$ x | + 50 | 27



Factor the polynomial expressions by first removing a common factor.

a) $4x^2 - 32x + 48$

$4(x^2-8x+12)$ x | + 12 | −8

$4(x-6)(x-2)$

*fully factored ✓

b) $3x^3 + 21x^2 + 30x$

$3x(x^2+7x+10)$ x | + 10 | 7

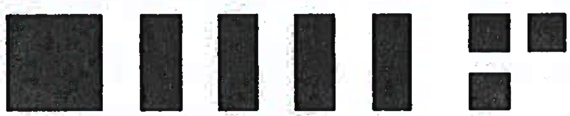
$3x(x+2)(x+5)$

In this example there were two steps in the factoring process - a common factor followed by a trinomial. If we are asked to factor a polynomial expression, it is understood this means to continue factoring until no further factoring is possible. This is sometimes written as "factor completely ...". The operation "factor" means "factor completely".

Complete Assignment Questions #4 - #15

Assignment 4, 6, 7, 10ace

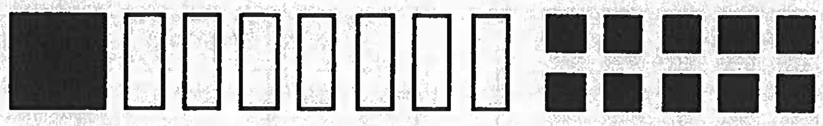
1. a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

c) Use the results above to express the polynomial in factored form.

2. a) Write a polynomial expression for the group of algebra tiles shown.



b) Arrange the algebra tiles into a rectangle.

c) State the length and width of the rectangle and hence express the polynomial in factored form.

3. Use algebra tiles to factor the following trinomials.

a) $x^2 + 5x + 6$ **b)** $x^2 + 6x + 5$ **c)** $x^2 - 6x + 8$

4. Complete the tables to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	5	6	
b)	8	7	
c)	11	30	
d)	–11	30	

	Sum	Product	Integers
e)	11	10	
f)	–8	15	
g)	–15	56	
h)	–18	56	

5. Complete the following.

a) $x^2 + 7x + 12 = (x + 3)(x + \quad)$ b) $x^2 + 9x + 8 = (x + \quad)(x + \quad)$

c) $x^2 - 7x + 10 = (x - 2)(x - \quad)$ d) $t^2 - 14t + 24 = (t - \quad)(t - \quad)$

e) $z^2 + 8z + 15 = (z + 5)(\qquad)$ f) $b^2 - 12b + 20 = (b - 2)(\qquad)$

6. Factor the following.

a) $x^2 + 3x + 2$ b) $x^2 - 3x + 2$ c) $x^2 + 9x + 18$

d) $x^2 + 8x + 12$ e) $x^2 - 10x + 21$ f) $x^2 - 11x + 24$

7. Factor where possible.

a) $x^2 + 11x + 10$ b) $x^2 + 10x + 11$ c) $n^2 + 12n + 32$

d) $y^2 - 11y + 28$ e) $y^2 + 17y + 42$ f) $f^2 - 10f + 21$

g) $p^2 - 16p + 28$ h) $x^2 + 24x + 80$ i) $c^2 - 32c + 60$

j) $a^2 - 12a + 24$ k) $d^2 + 18d + 45$ l) $p^2 - 29p + 100$

m) $m^2 + 22m + 121$ n) $n^2 - 23n + 102$ o) $q^2 - 28q + 115$

8. **a)** The expression $x^2 + bx + 12$ can be factored over the integers. Determine all possible values of b.

b) If the expression $x^2 + 6x + c$, where $c > 0$, can be factored over the integers, determine all possible values of c.

9. A volleyball court has an area of $x^2 + 15x + 36$ square metres.

a) Factor $x^2 + 15x + 36$ to find binomials that represent the length and width of the court.

b) If $x = 3$, determine the length and width of the court.

10. Factor.

a) $2x^2 + 6x + 4$ **b)** $4x^2 - 48x + 128$ **c)** $-2a^2 - 30a - 108$

d) $5x^2 - 20x + 15$ **e)** $ax^2 - 14ax + 45a$ **f)** $-10a^4 + 100a^3 - 240a^2$

$-10a^2(a^2-10a+24)$

$\cancel{-10a^2(a-12)(a+2)}$

$-10a^2(a-6)(a-4)$

11. Consider the following in which each letter represents a whole number.

$x^2 + 5x + 6 = (x + A)(x + B)$ $x^2 + 10x + 21 = (x + B)(x + G)$

$x^2 - 9x + 20 = (x - T)(x - L)$ $2x^2 - 16x + 32 = 2(x - T)^2$

$x^3 + 10x^2 + 9x = x(x + S)(x + E)$ $6x^2 - 54x + 48 = 6(x - I)(x - S)$

Determine the value of each letter and hence name the famous person represented by the following code.

(3) (8) (5) (5) (7) (2) (4) (9) (1)

_ _ _ _ _ _ _ _ _

Multiple Choice

12. Which of the following is **not** a factor of $3m^2 - 27m + 54$?

A. $m - 3$

B. $m - 6$

C. $m - 9$

D. 3

13. For which of the following trinomials is $a + 5$ **not** a factor?

A. $a^2 + 6a + 5$

B. $a^2 + 11a + 30$

C. $a^2 + 10a + 50$

D. $a^2 + 10a + 25$

14. The expression $t^2 + kt + 12$ **cannot** be factored if k has the value

A. -13 **B.** -8 **C.** 7 **D.** 11

Numerical Response

15. The largest value of b for which $x^2 + bx + 32$ can be factored over the integers is ____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x^2 + 4x + 3$ **b)** $x + 3,\ x + 1,$ **c)** $x^2 + 4x + 3 = (x + 3)(x + 1)$

2. a) $x^2 - 7x + 10$ **c)** $x - 2, x - 5,$ $x^2 - 7x + 10 = (x - 2)(x - 5)$

3. a) $(x + 2)(x + 3)$ **b)** $(x + 1)(x + 5)$ **c)** $(x - 4)(x - 2)$

4. a) 2, 3 **b)** 1, 7 **c)** 5, 6 **d)** –5, –6
e) 1, 10 **f)** –3, –5 **g)** –7, –8 **h)** –4, –14

5. a) $(x + 3)(x + 4)$ **b)** $(x + 1)(x + 8)$ **c)** $(x - 2)(x - 5)$
d) $(t - 2)(t - 12)$ **e)** $(z + 5)(z + 3)$ **f)** $(b - 2)(b - 10)$

6. a) $(x + 1)(x + 2)$ **b)** $(x - 1)(x - 2)$ **c)** $(x + 3)(x + 6)$
d) $(x + 2)(x + 6)$ **e)** $(x - 3)(x - 7)$ **f)** $(x - 3)(x - 8)$

7. a) $(x + 1)(x + 10)$ **b)** not possible **c)** $(n + 4)(n + 8)$
d) $(y - 4)(y - 7)$ **e)** $(y + 3)(y + 14)$ **f)** $(f - 3)(f - 7)$
g) $(p - 2)(p - 14)$ **h)** $(x + 4)(x + 20)$ **i)** $(c - 2)(c - 30)$
j) not possible **k)** $(d + 3)(d + 15)$ **l)** $(p - 4)(p - 25)$
m) $(m + 11)(m + 11)$ OR $(m + 11)^2$ **n)** $(n - 6)(n - 17)$ **o)** $(q - 5)(q - 23)$

8. a) 7, 8, 13,–7, –8, –13 **b)** 5, 8, 9 **9. a)** $(x + 12)(x + 3)$ **b)** 15m, 6m

10. a) $2(x + 1)(x + 2)$ **b)** $4(x - 4)(x - 8)$ **c)** $-2(a + 6)(a + 9)$
d) $5(x - 1)(x - 3)$ **e)** $a(x - 5)(x - 9)$ **f)** $-10a^2(a - 4)(a - 6)$

11. BILL GATES **12.** C **13.** C **14.** D **15.**

3	3		

Factoring Polynomial Expressions Lesson #3:
Factoring Trinomials of the Form $x^2 + bx + c$ - Part Two

Review of Factoring By Inspection

In order to factor $x^2 + bx + c$ by inspection, we need to find two integers which have a <u>product equal to c</u> and a <u>sum equal to b</u>. If no two such integers exist, then the polynomial cannot be factored.

In order to factor $x^2 + 6x + 9$, we need to find two numbers whose product is __9__ and whose sum is __6__.

In order to factor $x^2 + x - 12$, we need to find two numbers whose product is __-12__ and whose sum is __1__.

Recall the following points from the previous lesson.

- If the product is **positive**, then the two integers must be either **both positive** or **both negative.**
- If the product is **negative**, then one integer is **positive** and the other is **negative.**

Class Ex. #1

Factor the following trinomials by inspection.

a) $x^2 - x - 12$ 　 $\begin{array}{c|c} \times & + \\ \hline -12 & -1 \end{array}$

-3 4　-6 2
3 -4　6 -2
1 -12　12 -1

$=(x+3)(x-4)$

b) $x^2 + 3x - 18$ 　 $\begin{array}{c|c} \times & + \\ \hline -18 & 3 \end{array}$

$=(x+6)(x-3)$

c) $a^2 - 7a - 8$ 　 $\begin{array}{c|c} \times & + \\ \hline -8 & -7 \end{array}$

$=(a-8)(a+1)$

Class Ex. #2

Factor where possible.

a) $-a^2 - 6a + 27$ 　*factor out -1

$-(a^2+6a-27)$ 　 $\begin{array}{c|c} \times & + \\ \hline -27 & 6 \end{array}$

$-(a+9)(a-3)$

b) $2t^2 - 14t + 20$ 　 $\begin{array}{c|c} \times & + \\ \hline 10 & -7 \end{array}$

$2(t^2-7t+10)$

$2(t-5)(t-2)$

c) $x^2 - 3x - 6$ 　 $\begin{array}{c|c} \times & + \\ \hline -6 & -3 \end{array}$

-6 1
6 -1
3 -2
-3 2

not possible

d) $4x^4 - 16x^3 - 20x^2$ 　 $\begin{array}{c|c} \times & + \\ \hline -5 & -4 \end{array}$

$4x^2(x^2-4x-5)$

$4x^2(x-5)(x+1)$

Complete Assignment Questions #1 - #5

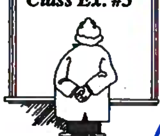
Factoring Trinomials of the form $x^2 + bxy + cy^2$

Complete the following statements:

i) $(x+2)(x+4)$ can be expanded to x^2+6x+8, Polynomial
so x^2+6x+8 can be factored to $(x+2)(x+4)$. Factoring

ii) $(x+2y)(x+4y)$ can be expanded to $x^2+4xy+2xy+8y^2 = x^2+6xy+8y^2$
so $x^2+6xy+8y^2$ can be factored to $(x+2y)(x+4y)$.

Class Ex. #3



Factor.

a) $x^2 + 13xy + 30y^2$ x | +   30 | 13

$(x+10y)(x+3y)$

b) $x^2 + 71xy - 72y^2$ x | + -72 | 71

$(x+72y)(x-1y)$

c) $3a^2 - 15ab - 252b^2$ x | + -84 | -5

$3(a^2-5ab-84b^2)$

$3(a-12b)(a+7b)$

Complete Assignment Questions #6 - #11

Assignment (1-4)ace, (6,7)ace

1. Complete the table to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	8	−20	
b)	−8	−20	
c)	−1	−20	

	Sum	Product	Integers
d)	3	−70	
e)	−11	28	
f)	0	−16	

2. Factor the following trinomials.

a) $x^2 - 2x - 15$ **b)** $x^2 - 2x - 24$ **c)** $x^2 + 2x - 24$

d) $x^2 + 2x - 3$ **e)** $x^2 + x - 30$ **f)** $x^2 - 3x - 10$

3. Factor where possible.

x | +
18 | -11

a) $x^2 + 10x + 16$ **b)** $x^2 - 11x + 18$ **c)** $x^2 - 2x - 8$ **d)** $x^2 + 3x - 18$

$(x-2)(x-9)$

e) $x^2 - 4x + 12$ **f)** $x^2 - 4x - 12$ **g)** $x^2 - 10x + 25$ **h)** $x^2 + x - 20$

i) $m^2 + 21m + 38$ **j)** $a^2 - 17a + 42$ **k)** $p^2 - 10p - 9$ **l)** $p^2 - 9p - 10$

4. Factor.

a) $-x^2 - 7x - 12$ **b)** $4x^2 - 28x - 32$ **c)** $5x^2 - 20x + 15$

x | +
-8 | -7

$4(x^2-7x-8)$

$4(x-8)(x+1)$

x | +
-110 | -1

d) $-2a^2 + 2a + 220$ **e)** $b^2x^2 - 4b^2x - 45b^2$ **f)** $2x^3 + 2x^2 - 40x$

$-2(a^2-a-110)$

$-2(a-11)(a+10)$

5. Consider the following in which the each letter represents a whole number.

$x^2 + 4x - 5 = (x + A)(x - O)$ $x^2 - 3x - 54 = (x - E)(x + I)$

$x^3 + 2x^2 - 8x = x(x - Y)(x + P)$ $3x^2 - 48x + 192 = T(x - R)^2$

$-5x^2 + 20x + 105 = -5(x + T)(x - H)$

Determine the value of each letter and hence name the fictional character represented by the following code.

(7) (5) (8) (8) (2) (4) (1) (3) (3) (9) (8)

_ _ _ _ _ _ _ _ _ _ _

6. Factor.

a) $x^2 + 18xy + 45y^2$ **b)** $x^2 + 10xy - 24y^2$ **c)** $a^2 - 12ab + 36b^2$

$\boxed{(a-6b)^2}$

$(a-6b)(a-6b)$

x | $+$

36 | -12

d) $p^2 - 12pq + 11q^2$ **e)** $x^2 + xy - 72y^2$ **f)** $x^2 - 54xy - 112y^2$

7. Factor completely.

a) $4x^2 - 80xy + 144y^2$ **b)** $3b^2 - 15bv - 72v^2$ **c)** $2c^2 + 66cd - 140d^2$

$3(b^2 - 5bv - 24v^2)$

$= 3(b-8v)(b+3v)$

x | $+$

-24 | -5

Multiple Choice

8. When factored, the trinomials $x^2 - 10x + 21$ and $x^2 - 4x - 21$ have one binomial factor in common. This factor is

A. $x - 7$ **B.** $x + 7$
C. $x - 3$ **D.** $x + 3$

9. One factor of $-m^3 - m^2 + 6m$ is

A. $m - 2$ **B.** $m + 2$
C. $m - 3$ **D.** $m - 6$

10. One factor of $3x^2 - 6xy - 9y^2$ is

A. $3x$ **B.** $x + 2y$
C. $x + 3y$ **D.** $x + y$

11. The expression $x^2 - 4x + c$ **cannot** be factored if c has the value

A. -5
B. 0
C. 4
D. 5

Answer Key

1. a) $-2, 10$ **b)** $-10, 2$ **c)** $-5, 4$ **d)** $-7, 10$ **e)** $-4, -7$ **f)** $-4, 4$

2. a) $(x-5)(x+3)$ **b)** $(x-6)(x+4)$ **c)** $(x+6)(x-4)$ **d)** $(x+3)(x-1)$
e) $(x+6)(x-5)$ **f)** $(x-5)(x+2)$

3. a) $(x+8)(x+2)$ **b)** $(x-9)(x-2)$ **c)** $(x+2)(x-4)$ **d)** $(x+6)(x-3)$
e) not possible **f)** $(x-6)(x+2)$ **g)** $(x-5)^2$ **h)** $(x+5)(x-4)$
i) $(m+2)(m+19)$ **j)** $(a-14)(a-3)$ **k)** not possible **l)** $(p-10)(p+1)$

4. a) $-(x+3)(x+4)$ **b)** $4(x-8)(x+1)$ **c)** $5(x-3)(x-1)$ **d)** $-2(a-11)(a+10)$
e) $b^2(x-9)(x+5)$ **f)** $2x(x+5)(x-4)$ **5.** HARRY POTTER

6. a) $(x+15y)(x+3y)$ **b)** $(x-2y)(x+12y)$ **c)** $(a-6b)^2$ **d)** $(p-q)(p-11q)$
e) $(x-8y)(x+9y)$ **f)** $(x+2y)(x-56y)$

7. a) $4(x-18y)(x-2y)$ **b)** $3(b-8v)(b+3v)$ **c)** $2(c+35d)(c-2d)$

8. A **9.** A **10.** D **11.** D

[redacted]

Factoring Polynomial Expressions Lesson #4:
Difference of Squares

Investigation

a) Complete the following using the trinomial factoring method from the previous lessons.

	Sum +	Product ×	Integers	Polynomial	Factored Form
i)	–6	–16	–8, 2	$x^2 - 6x - 16$	$(x-8)(x+2)$
ii)	–15	–16	–16, 1	$x^2 - 15x - 16$	$(x-16)(x+1)$
iii)	0	–16	–4, 4	$x^2 + 0x - 16 = \boxed{x^2 - 16}$	$(x-4)(x+4)$
iv)	0	–64	–8, 8	$x^2 - 64$	$(x-8)(x+8)$
v)	0	–25	–5, 5	$x^2 - 25$	$(x-5)(x+5)$

b) The third row in a) shows that the factored form of $x^2 - 16$ is $(x-4)(x+4)$.
Use the pattern from the last three rows to factor the following.

i) $x^2 - 9 =$
$(x-3)(x+3)$

ii) $x^2 - 49 =$
$(x-7)(x+7)$

iii) $x^2 - 36 =$
$(x-6)(x+6)$

iv) $x^2 - 1 =$
$(x-1)(x+1)$

v) $a^2 - 100 =$
$(a-10)(a+10)$

c) Extend the procedure from above to factor $m^2 - n^2$.
Verify your answer by expanding the factored form.

$(m-n)(m+n) = m^2 \cancel{+mn} \cancel{-mn} - n^2$
$= m^2 - n^2$ ✓

d) Consider the expansion $(x-y)(x+y) = x^2 + bx + c$.

0

i) Explain why the value of b is zero.

OI of FOIL are + and - of the same so they cancel each other out

ii) Express c in terms of y.

$c = -y \cdot y = -y^2$

Difference of Squares

The examples on the previous page are trinomials of the form $x^2 + bx + c$, where $b = 0$ and c is the negative of a square number.

subtraction

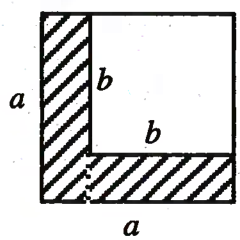
This results in a **difference of squares** such as $x^2 - 25$, $x^2 - 100$, etc.

"two squares being subtracted from each other"

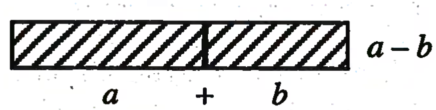
To factor a difference of squares we can use the identity:

$$\boxed{a^2 - b^2 = (a - b)(a + b)}$$

The identity $a^2 - b^2 = (a - b)(a + b)$ can be illustrated in the following diagram.



Shaded area = $a^2 - b^2$



Shaded area = $(a - b)(a + b)$

The shaded area on the left is cut along the dotted line and rearranged to form the diagram on the right.
The shaded area on the left is represented by $a^2 - b^2$ *and the shaded area on the right is represented by* $(a - b)(a + b)$.

Class Ex. #1

Factor the following polynomials using the difference of squares method.

a) $a^2 - 4$ $= a^2 - 2^2$
$= (a-2)(a+2)$

b) $t^2 - 144$ $= t^2 - 12^2$
$(t-12)(t+12)$

c) $x^2 - y^2$
$(x-y)(x+y)$

d) $p^2 - 7^2$
$(p-7)(p+7)$

Note

Note that it is not possible to factor a sum of squares like $x^2 + 4$, i.e. $x^2 + 0x + 4$.
It is not possible to find two integers whose product is positive and whose sum is zero.

In the identity $a^2 - b^2 = (a-b)(a+b)$ we can replace a and/or b by numbers, variables, monomials and even polynomials.

For example, $4x^2 - 25$ can be written as $(2x)^2 - (5)^2$ and can be factored using the above identity with $a = 2x$ and $b = 5$.

$$4x^2 - 25 = (2x-5)(2x+5)$$

$9m^2 - 4n^2$ can be written as $(3m)^2 - (2n)^2$, and can be factored using the above identity with $a = 3m$ and $b = 2n$.

$$9m^2 - 4n^2 = (3m-2n)(3m+2n)$$

The factoring above can be verified by expanding the product of the factors.

$9m^2 \cancel{+6mn} \cancel{-6mn} - 4n^2$ ✓

Factor, if possible, using the difference of squares method.

*two squares, − in the middle

a) $16t^2 - 49$

$(4t)^2 - 7^2$

$(4t-7)(4t+7)$

b) $81a^2 - 1$

$(9a)^2 - 1^2$

$(9a-1)(9a+1)$

c) $100 - y^2$

$10^2 - y^2$ $(10-y)(10+y)$

d) $36p^2 - 25q^2$

$= (6p)^2 - (5q)^2$

$= (6p-5q)(6p+5q)$

e) $4x^2 + 25$

not possible $+$

f) $64 - 9a^2b^2$

$8^2 - (3ab)^2$

$(8-3ab)(8+3ab)$

The floor of an international doubles squash court is rectangular with an area of $25a^2 - b^2$ square feet.

a) Write expressions for the length and width of the floor.

$(5a)^2 - b^2 = (5a-b)(5a+b)$

b) The perimeter of the floor is 140 feet. Determine the length and width of the floor if the length is 1.8 times the width.

Difference of Squares involving a Common Factor

The first step in factoring any polynomial expression should be to determine if we can remove a common factor.

Class Ex. #4

Factor the following polynomials by first removing the greatest common factor.

a) $2a^2 - 50$ $=2(a^2-25)$ $=2(a-5)(a+5)$

b) $3x^2 - 12y^2$ $3(x^2-4y^2)$ $3(x-2y)(x+2y)$

c) $144p^2q^2 - 4$ $4(36p^2q^2-1)$ $4(6pq-1)(6pq+1)$

$(12pq-2)(12pq+2)$ Fine but is better

d) $3x^3 - 27x$ $3x(x^2-9)$ $3x(x-3)(x+3$

Complete Assignment Questions #1 - #14

Assignment 1ac, (4,6,7) aceg

1. Complete the following by determining the missing factor.

a) $x^2 - 36 = (x-6)(\quad)$ **b)** $c^2 - 121 = (c+11)(\quad)$ **c)** $j^2 - k^2 = (j-k)(\quad)$

2. Factor the following polynomials using a difference of squares.

a) $x^2 - 49$ **b)** $x^2 - 1$ **c)** $x^2 - 15^2$ **d)** $x^2 - 400$

3. Explain how factoring a difference of squares in one variable can be regarded as a special case of factoring trinomials by inspection.

4. Factor where possible.

a) $m^2 - n^2$ **b)** $c^2 - 7^2$ **c)** $1 - k^2$ **d)** $g^2 - 64h^2$

e) $25x^2 - 144$ **f)** $16a^2 - 9b^2$ **g)** $4x^2 + z^2$ **h)** $121a^2 - 36b^2$

i) $49 - 4h$ **j)** $100 - 81b^2$ **k)** $1 - 25z^2$ **l)** $225a^2 - b^2$

m) $169z^2 - 4q^2$ **n)** $256 - y^2$ **o)** $t^2 + 36z^2$ **p)** $49a^2 - 400$

5. The floor of a classroom is rectangular with an area of $81m^2 - 4n^2$ square metres.

a) Write expressions in m and n for the length and width of the floor.

b) If the perimeter of the floor is 72 metres, form an equation in m and n and solve for m.

c) Determine the length and width of the floor if the length is 25% greater the width.

6. Factor.

a) $8x^2 - 32$

b) $4a^2 - 100y^2$

c) $3t^2 + 27s^2$

d) $7x^2 - 7y^2$

e) $9a^2b^2 - 36$

f) $8 - 50p^2q^2$

g) $xy^2 - x^3$

h) $20a^2b^2 - 5a^4b^4$

7. Factor.

a) $a^2b^2 - 9$

b) $c^2 - d^2e^2$

c) $100x^2 - y^2z^2$

d) $p^2q^2 - r^2s^2$

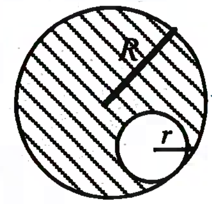
e) $25x^2y^2 - 1$

f) $c^2d^2 - 4f^2$

g) $4x^2a^2 - 49z^2t^2$

h) $16a^2c^2 - 225b^2d^2$

8. The diagram shows a circle of radius R with a circle of radius r removed.



a) Write an expression for the shaded area.

b) Write the expression in a) in factored form.

c) Determine the shaded area (as a multiple of π) if $R = 8.5$ and $r = 1.5$.
Do not use a calculator.

9. The expression $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ occurs in physics.

a) Write the expression in factored form.

b) Determine the value of the expression when $m = 10$, $v = 75$, and $u = 25$.
Do not use a calculator.

10. Consider the following in which each letter represents a whole number.

$64x^2 - y^2 = (Hx - y)(Hx + y)$ $\qquad$ $16x^2 - 4 = C(Ix + 1)(Ix - 1)$

$7x^2 - 252y^2 = P(x - Ey)(x + Ey)$ $\qquad$ $Lx^2 - Ny^2 = (3x - 5y)(Sx + Ay)$

Determine the value of each letter and hence name the country represented by the following code.

(4) (8) (2) (9) (6)

_ _ _ _ _

11. Susan was showing Rose how the difference of squares method can be used to multiply certain numbers without using a calculator. She showed Rose the following:

$$38 \times 42$$
$$= (40-2)(40+2) \quad = (40^2 - 2^2) \quad = (1600 - 4) \quad = 1596$$

a) Use the above process to evaluate:

i) 27×33

ii) 61×59

b) Explain why this process is more difficult to determine the product 66×72.

c) Make up your own multiplication question which can be answered using this process.

Multiple Choice

12. One factor of $16 - 4m^2$ is

A. $4 - m$
B. $8 - 2m$
C. $4 + m$
D. $2 + m$

13. Given that $x^2 - y^2 = 45$ and $x + y = 9$, the value of x is

A. 2
B. 5
C. 7
D. impossible to determine

Numerical Response

14. $3x + 2y$ is a factor of the binomial $a^2x^2 - b^2y^2$.
The value of $a^2 + b^2$ is _____ .
(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $(x+6)$ **b)** $(c-11)$ **c)** $(j+k)$

2. a) $(x-7)(x+7)$ **b)** $(x-1)(x+1)$ **c)** $(x-15)(x+15)$ **d)** $(x-20)(x+20)$

3. A difference of squares can be regarded as a trinomial of the form x^2+bx+c in which $b=0$ and c is negative. We need to find two numbers which multiply to c and add to zero.

4. a) $(m-n)(m+n)$ **b)** $(c-7)(c+7)$ **c)** $(1-k)(1+k)$
d) $(g-8h)(g+8h)$ **e)** $(5x-12)(5x+12)$ **f)** $(4a-3b)(4a+3b)$
g) not factorable **h)** $(11a-6b)(11a+6b)$ **i)** not factorable using whole number exponent.
j) $(10-9b)(10+9b)$ **k)** $(1+5z)(1-5z)$ **l)** $(15a+b)(15a-b)$
m) $(13z-2q)(13z+2q)$ **n)** $(16-y)(16+y)$ **o)** not factorable **p)** $(7a+20)(7a-20)$

5. a) $(9m+2n)$ metres, $(9m-2n)$ metres **b)** $2(9m+2n)+2(9m-2n)=72,\ m=2$
c) Length = 20 metres, Width = 16 metres.

6. a) $8(x-2)(x+2)$ **b)** $4(a-5y)(a+5y)$ **c)** $3(t^2+9s^2)$ **d)** $7(x-y)(x+y)$
e) $9(ab-2)(ab+2)$ **f)** $2(2-5pq)(2+5pq)$ **g)** $x(y-x)(y+x)$ **h)** $5a^2b^2(2-ab)(2+ab)$

7. a) $(ab-3)(ab+3)$ **b)** $(c-de)(c+de)$ **c)** $(10x-yz)(10x+yz)$
d) $(pq-rs)(pq+rs)$ **e)** $(5xy-1)(5xy+1)$ **f)** $(cd-2f)(cd+2f)$
g) $(2xa-7zt)(2xa+7zt)$ **h)** $(4ac-15bd)(4ac+15bd)$

8. a) $A=\pi R^2-\pi r^2$ **b)** $\pi(R-r)(R+r)$ **c)** 70π

9. a) $\frac{1}{2}m(v-u)(v+u)$ **b)** 25 000 **10.** CHILE

11. a) i) 891 **ii)** 3599
b) 66×72 expressed as a difference of squares (69^2-3^2) cannot easily be evaluated without a calculator or long multiplication.

12. D **13.** C **14.**

1	3		

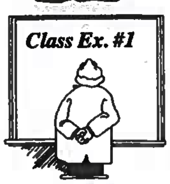
Factoring Polynomial Expressions Lesson #5:
Factoring Review

Guidelines for Factoring a Polynomial Expression

If we are asked to factor a polynomial expression, the following guidelines should help us to determine the best method.

1. Look for a common factor. If there is one, take out the common factor and look for further factoring.
2. If there is a binomial expression, look for a difference of squares.
3. If there is a trinomial expression of the form $x^2 + bx + c$, look for factoring by inspection.
4. After factoring, check to see if further factoring is possible.

Polynomial expressions of the form $ax^2 + bx + c$ will be discussed in the next math course.



*group like terms

Factor the following.

a) $9x^2 - 36$

$= 9(x^2 - 4)$ (1)

$= 9(x-2)(x+2)$ (2)

b) $x^2 - 16x - 36$

x | +
-36 | -16

$(x-18)(x+2)$ (3)

c) $-x^2 + 26x + 27$

x | +
-27 | -26

$-(x^2 - 26x - 27)$ (1)

$-(x-27)(x+1)$ (3)

d) $x^2 - 3x - 5x + 15$

$x^2 - 8x + 15$

$(x-5)(x-3)$

x | +
15 | -8

Complete Assignment Questions #1 - #9

Assignment 1-3

1. Factor.

a) $x^2 - 49$

b) $x^2 - 8x + 15$

c) $8x^2 + 32$

d) $-a^2 + 64$

e) $e^2 - 3e + 4$

f) $v^2 + 7v + 10$

g) $a^2 + 2ab - 35b^2$

h) $4 - 25t^2$

i) $x^2 + 16$

2. Factor.

a) $a^2 - 64b^2$ **b)** $108 - 3z^2$ **c)** $-x^2 - 5x - 4$

d) $625p^2 - 1$ **e)** $-3x^2 - 3x + 36$ **f)** $8v^2 - 32v - 96$

3. Factor.

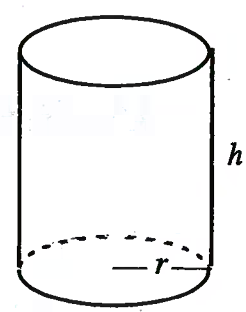
a) $b^2 - 16 - 6b + 24$ **b)** $x^3 - 81x$ **c)** $-256 + t^2$

d) $12 - 4x - x^2$ **e)** $x^2 - 8xy - 33y^2$

*rearrange, factor out −1

4. The surface area of a cylinder is given by the formula $A = 2\pi r^2 + 2\pi rh$, where r is the radius of the base and h is the height of the cylinder.

a) Calculate the surface area, to the nearest $0.01\ \text{cm}^2$, of a cylinder which has vertical height 14.5 cm and base diameter 11 cm.



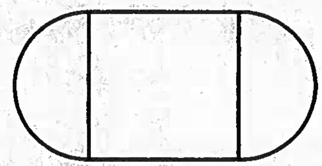
b) Write the formula for A in factored form.

c) Calculate, using the factored form of A, the surface area of the cylinder to the nearest $0.01\ \text{cm}^2$.

d) Which method a) or c) is simpler to use?

[redacted]

5. A square of side $2r$ cm has semicircles drawn externally on each of two opposite sides.



Find expressions in factored form for

a) the external perimeter of the shape

b) the area of the shape

Multiple Choice

Use the following information to answer the next two questions.

In questions #6 -#7 one or more of the four responses may be correct. Answer

A. if only 1 and 2 are correct
B. if only 1, 2, and 3 are correct
C. if only 3 and 4 are correct
D. if some other response or combination of responses is correct

6. The set of factors of $5x^2 - 10x - 15$ contains

1. $x - 1$ **2.** $x + 3$ **3.** $x + 1$ **4.** $x - 3$

7. $x + 4$ is a factor of

1. $-x^2 - 6x - 8$ **2.** $48 - 3x^2$ **3.** $3x^2 + 12x$ **4.** $x^2 + 16$

[redacted]

8. $\pi r^3 + 3\pi r$ is equivalent to

A. $3\pi^2 r^4$
B. $3\pi(r^2 + r)$
C. $\pi r(2r + 3)$
D. $\pi r(r^2 + 3)$

Numerical Response

9. Triangle PQR is right angled at P. The area of the triangle is $\frac{1}{2}x^2 + 10x + 18$ cm^2, where x is a positive integer.

If the length of PQ is 10 cm, then the length of PR, is _____ cm.
(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $(x-7)(x+7)$ b) $(x-5)(x-3)$ c) $8(x^2+4)$
d) $-(a+8)(x-8)$ e) not factorable f) $(v+5)(v+2)$
g) $(a+7b)(a-5b)$ h) $(2-5t)(2+5t)$ i) not factorable

2. a) $(a-8b)(a+8b)$ b) $3(6-z)(6+z)$ c) $-(x+4)(x+1)$
d) $(25p-1)(25p+1)$ e) $-3(x-3)(x+4)$ f) $8(v+2)(v-6)$

3. a) $(b-2)(b-4)$ b) $x(x-3)(x+3)$ c) $(t-4)(t+4)$
d) $-(x-2)(x+6)$ e) $(x-11y)(x+3y)$

4. a) 691.15 cm^2 b) $A = 2\pi r(r+h)$ c) 691.15 cm^2 d) c) is simpler

5. a) $2r(\pi + 2)$ cm. b) $r^2(\pi + 4)$ cm^2.

6. C 7. B 8. D 9.

2	6		