Prime Factorization and Exponents Lesson #5:
Integral Exponents

The Negative Exponent

a) Complete the patterns below.

$10^3 = 1000$ $\quad 3^3 = 27$

$10^2 = 100$ $\quad 3^2 = 9$

$10^1 = 10$ $\quad 3^1 = 3$

$10^0 = 1$ $\quad 3^0 = 1$ $\quad a^0 = 1$

$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$ 0.1 $\quad 3^{-1} = \frac{1}{3}$ $\quad a^{-1} = \frac{1}{a}$

$10^{-2} = \frac{1}{100} = \frac{1}{10^2} = 0.01$ $\quad 3^{-2} = \frac{1}{9} = \frac{1}{3^2}$ $\quad a^{-2} = \frac{1}{a^2}$

$10^{-3} = \frac{1}{1000} = \frac{1}{10^3} = 0.001$ $\quad 3^{-3} = \frac{1}{27} = \frac{1}{3^3}$ $\quad a^{-3} = \frac{1}{a^3}$

b) Write the following with positive exponents.

i) $10^{-7} = \frac{1}{10^7}$ $\quad$ ii) 3^{-5} $\frac{1}{3^5}$ $\quad$ iii) a^{-n} $\frac{1}{a^n}$

Using the Exponent Laws to Define the Negative Exponent

Consider the expression $5^4 \div 5^7$.

a) Evaluate the expression as an exact value using a calculator. $= 0.008$

b) Complete the following to evaluate the expression.

$$5^4 \div 5^7 = \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{5^{3}} = \frac{1}{125} = 0.008$$

c) Use the quotient law to complete the following.

$5^4 \div 5^7 = 5^{4-7} = 5^{-3}$

d) The results in a) to c) are examples of a general rule when a base is raised to a negative exponent. Complete: $a^{-p} = \frac{1}{a^p}$

e) Write the following with positive exponents and evaluate.

i) $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ $\quad$ ii) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $\quad$ iii) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

The Negative Exponent in the Denominator

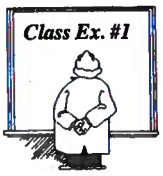
Use the rule for division of fractions to show that $\frac{1}{4^{-3}} = 4^3$. Use a calculator to confirm.

*negative exponent changes to positive when you move it to the top or bottom (opposite of where it was)

Negative Exponent Law

A base (not including zero) raised to a negative exponent has the following properties:

$$a^{-n} = \frac{1}{a^n},\ a \neq 0 \quad \text{and} \quad \frac{1}{a^{-n}} = a^n,\ a \neq 0$$

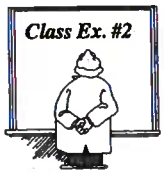


Simplify, express with positive exponents, and evaluate without using a calculator. $\frac{1}{2^3} = \frac{1}{8}$

a) $4^5 \times 4^{-3}$ b) $3^2 \times 3^{-5}$ c) $\frac{1}{2^{-5}}$ d) $\frac{6^{-7}}{6^{-5}}$ e) $(2^3)^{-1}$

$= 4^{5+-3}$ $= 3^{2+-5}$ $= 2^5$ $\frac{6^5}{6^7} = 6^{5-7} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

$= 4^2 = 16$ $= 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ $= 32$

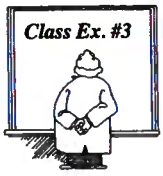


Identify the following as true or false.

a) $\frac{8^3}{8^{-1}} = 8^4$ True $8^{3--1} = 8^4$

b) $\frac{8^3}{4^{-1}} = 2^4$ False $\frac{(2^3)^3}{(2^2)^{-1}} = \frac{2^9}{2^{-2}}$

c) $a^{-3} = \frac{1}{a^3}$ TRUE



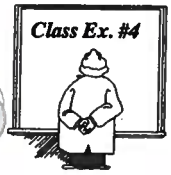
Explain why $2p^{-3} \neq \frac{1}{2p^3}$:

base

coefficient

$2p^{-3} = \frac{2}{p^3}$

only the base moves



a) $a^{-4} \times a^{-3}$

$= a^{-4+-3}$

$= a^{-7} = \frac{1}{a^7}$

b) $6x^2 \div 2x^7$

$3x^{2-7} = 3x^{-5}$

$= \frac{3}{x^5}$

c) $\frac{y^6}{2y^{-5}}$

$\frac{1}{2} y^{6--5} = \frac{y^{11}}{2}$

d) $(-2x)^{-3}$

$= \frac{1}{(-2x)^3} = \frac{1}{-8x^3}$

e) $\frac{8a^{-5}}{4b^{-3}}$

$= \frac{2b^3}{a^5}$

*simplify #'s
* get rid of neg exponents

f) $\frac{(5p)^{-2}}{5q^4}$

$= \frac{1}{5q^4(5p)^2}$

$= \frac{1}{5q^4 \cdot 25p^2}$

$= \frac{1}{125q^4p^2}$

Simplifying a Fractional Base with a Negative Exponent

Consider the expression $\left(\frac{2}{3}\right)^{-4}$.

a) Complete the following $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^{\square}} = \frac{1}{\quad} = 1 \times \quad =$

b) Evaluate $\left(\frac{3}{2}\right)^4$.

c) Classify the following statement as true or false. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$

d) Suggest a quick method for evaluating $\left(\frac{5}{2}\right)^{-3}$ without using a calculator.

In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $a, b \neq 0$.

$\frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$

Complete Assignment Questions #1 - #15

Quiz @ 8:20 tomorrow.

Assignment

1. Write the following with positive exponents.

a) x^{-3} **b)** y^{-9} **c)** 4^{-1} **d)** $\dfrac{1}{a^{-5}}$ **e)** $\dfrac{1}{6^{-2}}$

2. Without using a calculator show that $\dfrac{3}{5^{-2}} = 75$.

3. Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^3 \times 4^{-4}$ **b)** $3^0 \times 3^{-3}$ **c)** $\dfrac{1}{7^{-2}}$ **d)** $\dfrac{10^{-3}}{10}$ **e)** $(3^2)^{-2}$

4. Express with positive exponents.

a) n^2m^{-5} **b)** $c^{-2}x^{-5}$ **c)** $16h^{-1}$ **d)** $\dfrac{2}{3}b^{-8}$ **e)** $(y^{-4})^{-2}$

f) $\dfrac{t^{-5}}{4}$ **g)** $\dfrac{1}{4x^{-9}}$ **h)** $\dfrac{4}{x^{-9}}$ **i)** $\dfrac{a^2}{b^{-7}}$ **j)** $\dfrac{a^{-2}}{b^7}$

5. Evaluate the following without using a calculator.

a) -3^{-2} **b)** $(-3)^{-2}$ **c)** $-7^2 \cdot 8^{-2}$ **d)** $(-8.3)^0$ **e)** $[-(3.9)^0]^{-2}$

6. Use a calculator to find the exact value of the following.

a) -4^{-4} **b)** $(-7)^{-3}$ **c)** $(0.75)^{-3}$ **d)** $(-0.025)^{-2}$ **e)** $\left(\dfrac{4}{7}\right)^{-3}$