Prime Factorization and Exponents Lesson #4:
Combining the Exponent Laws

> ### *Using Factors To Combine the Exponent Laws*

Part One:

Three students are attempting to simplify the following expression:

$$3x^2 \times 5x^3$$

Their answers are shown below.

Harry $\Rightarrow$ $8x^5$ Janet $\Rightarrow$ $15x^6$ Laura $\Rightarrow$ $15x^5$
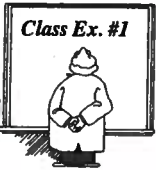
Explain using factors which student is correct.

Part Two:

Use factors to explain why $6a^6 \div 3a^2 = 2a^4$.

$6 \div 3 = 2$

$a^6 \div a^2 = a^{6-2} = a^4$

Class Ex. #1 State the simplified form of the following.

a) $(-7a^8)(6a^{12})$

$-7 \cdot 6 \, a^8 a^{12}$

$= -42a^{20}$

b) $3a^4 \times a^5 \times 6a^3$

$= 3 \cdot 1 \cdot 6 \, a^{4+5+3}$

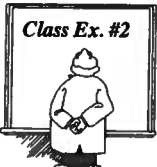$= 18a^{12}$

c) $-16n^5 \div (-2n)$

$8n^4$

d) $\dfrac{20y^5}{5y^5}$

$4y^0 = 4$

$4 \cdot 1$

e) $\dfrac{30b^{14}}{45b^{10}}$

$\dfrac{2}{3} b^{14-10}$ $\dfrac{2b^4}{3}$

$\dfrac{20y^6}{5y^5} = 4y$

~~$20y^6$~~

Class Ex. #2 Simplify the following.

a) $x^5 y^8 x^3 y^4$

$= x^8 y^{12}$

b) $\dfrac{x^5 y^8}{x^3 y}$

$x^{5-3} y^{8-1}$

$= x^2 y^7$

c) $(-3bc)(b^3 c^2)(-4b^2 c)$

$12b^{1+3+2} c^{1+2+1}$

$= 12b^6 c^4$

d) $\dfrac{10e^8 f^{12}}{4 e^4 f^7}$
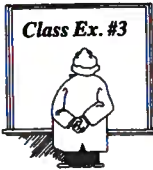
$\dfrac{5}{2} e^{8-4} f^{12-7}$

$= \dfrac{5}{2} e^4 f^5$

$= \dfrac{5e^4 f^5}{}$

> **Complete Assignment Questions #1 - #4**

Combining the Exponent Laws

The following examples use two or more of the exponent laws in their solution.

Class Ex. #3   Simplify.

a) $(3x^2)^3$

$$= 3^3 x^{2 \cdot 3}$$
$$= 27x^6$$

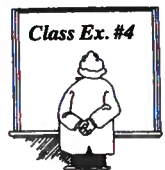**b)** $(-2a^2b^3)^2$

$$= 4a^4b^6$$

c) $\dfrac{x^3x^5}{x^2x}$

$$= \dfrac{x^8}{x^3}$$
$$= x^5$$

d) $\left(-\dfrac{2a}{y^3}\right)^3$

$$-\dfrac{2^3a^3}{y^9}$$
$$= -\dfrac{8a^3}{y^9}$$

$(-1)^3 = -1$
*odd exponent
$- \rightarrow -$
►even exponent
$- \rightarrow +$
$(-1)^4 = 1$

Class Ex. #4   Simplify the following.

a) $-(-n^2)^5$

$$-(-n^{10})$$
$$= n^{10}$$

b) $\left(\dfrac{4y^3 \times 3x^6}{6x^5}\right)^4$

*simplify in brackets first

$$= \left(\dfrac{12y^3x^6}{6x^5}\right)^4$$
$$= \left(2y^3x\right)^4$$
$$= 2^4y^{12}x^4$$
$$= 16y^{12}x^4$$

c) $\dfrac{16(x^3y^5)^2}{(2x^2)^3}$

$$\dfrac{16x^6y^{10}}{8x^6}$$
$$= 2y^{10}$$

d) $(5ab^6)^2\,(4a^2b)$

$$(25a^2b^{12})(4a^2b)$$
$$= 100a^4b^{13}$$

Class Ex. #5   Write in simplest form.

base a   base −a

a) $(-a)^6 \div (-a)^4$

$$= (-a)^{6-4}$$
$$= (-a)^2 = a^2$$

b) $-a^6 \div (-a)^4$

$$-a^6 \div a^4$$
$$= -a^{6-4}$$
$$= -a^2$$

c) $-a^7 \div (-a)^3$

$$-a^7 \div -a^3$$
$$= +a^{7-3}$$
$$= a^4$$

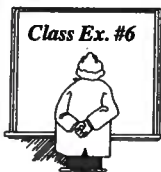**Complete Assignment Questions #5 - #12**

Extension

In higher level mathematics courses, you may meet variable bases and variable exponents including binomial exponents.

Use the exponent laws to simplify the following.

Class Ex. #6

a) $\dfrac{b^{4x+y}}{b^{x-2y}}$

$= b^{(4x+y)-(x-2y)}$

$= \boxed{b^{3x+3y}}$

b) $\dfrac{x^{5a+7b} \cdot x^{3a+b}}{x^a \cdot x^{2a-7b}} = \dfrac{x^{(5a+7b)+(3a+b)}}{x^{a+(2a-7b)}}$

$= \dfrac{x^{8a+8b}}{x^{3a-7b}} = x^{8a+8b-(3a-7b)}$

$= \boxed{x^{5a+15b}}$

> **Complete Assignment Question #13**

Assignment

1. Simplify the following.

   a) $3a^3 \times 3a^4$ **b)** $(10b^7)(3b^8)$ **c)** $3a^3 \cdot 5a^3$

   d) $(-2x^4)(12x^9)$ **e)** $\left(-\dfrac{1}{2}e^7\right)(-14e^8)$ **f)** $0.4c^3 \times 0.5c$

2. Simplify.

   a) $12x^4 \div 6x^2$ **b)** $(81e^9) \div (9e^8)$ **c)** $\dfrac{21d^6}{7d^2}$

   d) $\dfrac{-80d^{80}}{8d^8}$ **e)** $(-10e^{10}) \div (-5e^5)$ **f)** $\dfrac{12f^6}{12f^5}$

3. Write in simplest form.

   a) $(3a^2b^3)(5a^4b^8)$ **b)** $x^9y^0x^2y^4$ **c)** $\dfrac{6x^4y^7}{2x^3y^2}$

   d) $\dfrac{5x^4y^7}{x^3y^2}$ **e)** $\dfrac{4f^{12}d^3}{12f^4d}$ **f)** $(7b^4c)(bc^2)(-2b^2c^6)$

4. Simplify.

a) $\dfrac{10e^8 f^8}{15\,e^4 f^2}$

b) $(2p^3)(4p^7)(-2p)$

c) $(-2xy)(x^2 y^3)(-3xy)$

d) $(-8b^6 c) \div (2b^3 c)$

e) $(-10t^8 y^6) \div (-2t^7 y^3)$

f) $(4x^5 z^7) \div (-16xz^6)$

5. Write in simplest form.

a) $(-a^2 b^3)^4$

b) $(-a^2 b^3)^5$

c) $\left(\dfrac{b^4}{a^3}\right)^3$

d) $\dfrac{c^5 \times c^2}{c^4 \times c}$

6. Simplify.

a) $(3ab^2)^4$

b) $(-4a^5 c^2)^4$

c) $(-2m^3 n^4)^5 (m^2 n^3)$

d) $(-4x^2 y^3)^3 (8xy^8)$

e) $(a^3 b^4 c^5)(3abc^2)^3$

7. Write each expression in simplest form without brackets.

a) $\left(\dfrac{2d^5 \times d^4}{4d^3}\right)^3$

b) $\left(\dfrac{-16a^5 b^3 \cdot 2a^2 b^6}{8ab^7}\right)^3$

c) $\left(\dfrac{-5k^3 \cdot k^2}{k}\right)^2 \left(\dfrac{(-k)^5 \cdot k^2}{5k^2}\right)$