Prime Factorization and Exponents Lesson #3:
Powers with Whole Number Exponents

In this lesson we review numbers written as powers, and the exponent laws applied to powers with numerical bases and whole number exponents.
We extend the work to consider bases which are variable.

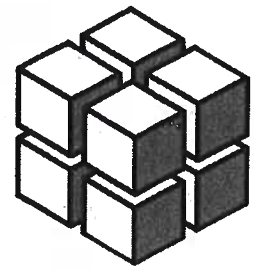
Exponents

x, y a

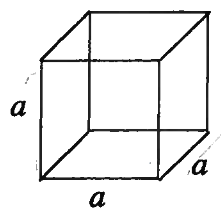
$3 \times 3 \times 3 \times 3 = 3^4$

In mathematics, exponents are used as a short way to write repeated multiplication.

The number of small cubes in the diagram can be calculated by the **repeated multiplication** $2 \times 2 \times 2$.
This can be written in **exponential form** as 2^3.



Exponents can also be used with variables.

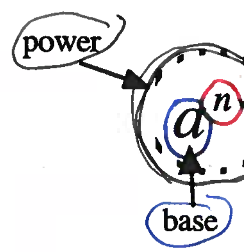


The volume of the cube to the left can be determined by repeated multiplication $a \times a \times a$ or in exponential form a^3.

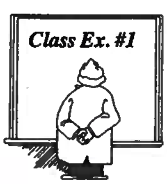
Powers

A **power** is a number written in exponential form.

It consists of a **base** and an **exponent**.



"little number"



State the base and the exponent in each of the following powers.

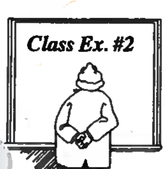
a) 4^5 base: 4 exponent: 5

b) $(-3)^6$ base: -3 exponent: 6

c) x^y base: x exponent: y

A number that **multiplies a variable is called a coefficient**.
In the expression $7p^3$ the coefficient is 7.

$x^3 \rightarrow$ coefficient is 1



State the coefficient in each of the of the following.

a) $8x^2$ coefficient: 8

b) $-3z^9$ -3

c) $\frac{a^8}{7}$ $\frac{1}{7}$

[redacted]

base p

Note that written as a repeated multiplication $7p^3 = 7 \times p \times p \times p$,
whereas $(7p)^3 = 7p \times 7p \times 7p = 7 \times p \times 7 \times p \times 7 \times p = 7 \times 7 \times 7 \times p \times p \times p = 343p^3$.

base 7p

Class Ex. #3

Write each of the following as a repeated multiplication.

a) $3a^4b$

$3 \times a \times a \times a \times a \times b$

b) $3ab^4$

$3 \times a \times b \times b \times b \times b$

c) $3(ab)^4$

$3 \times ab \times ab \times ab \times ab$

$3 \times a \times a \times a \times a \times b \times b \times b \times b$

d) $(3ab)^4$

$3ab \times 3ab \times 3ab \times 3ab$

$3 \times 3 \times 3 \times 3 \times a \times a \times a \times a \times b \times b \times b \times b$

Evaluating Powers

$10^3 = 10 \times 10 \times 10 = 1000$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$(-6)^2 = (-6) \times (-6) = +36$

$-6^2 = -(6 \times 6) = -36$

coefficient = −1

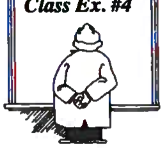
The Zero Exponent

Complete the patterns below by adding one more row.

$10^4 = 10000$	$3^4 = 81$
$10^3 = 1000$	$3^3 = 27$
$10^2 = 100$	$3^2 = 9$
$10^1 = 10$	$3^1 = 3$
$10^0 = 1$	$3^0 = 1$

The results above are examples of a general rule when a base is raised to the exponent zero.
Complete: $a^0 =$ 1 .

Class Ex. #4



Evaluate the following. **a)** 6^0 **b)** $(-9)^0$ **c)** -9^0 **d)** $2(6^2)^0$

a) 1 b) 1 c) −1 d) $2 \times 1 = 2$

Complete Assignment Questions #1 - #7

e) $(4^2 \times 51 + x)^0 = 1$

The Exponent Laws

The exponent laws with whole number exponents and numerical bases were covered in previous math courses.

The chart below extends the exponent laws to bases which are variables.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8\cdot 8\cdot 8)(8\cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a\cdot a\cdot a)(a\cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \dfrac{\cancel{8}\cdot\cancel{8}\cdot 8}{\cancel{8}\cdot\cancel{8}}$ $= 8^1$ or 8^{3-2}	$a^3 \div a^2 = \dfrac{\cancel{a}\cdot\cancel{a}\cdot a}{\cancel{a}\cdot\cancel{a}}$ $= a^1$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8\cdot 7)^3 = (8\cdot 7)(8\cdot 7)(8\cdot 7)$ $= (8\cdot 8\cdot 8)(7\cdot 7\cdot 7)$ $= 8^3\cdot 7^3$	$(a\cdot b)^3 = (a\cdot b)(a\cdot b)(a\cdot b)$ $= (a\cdot a\cdot a)(b\cdot b\cdot b)$ $= a^3 b^3$	**Power of a Product Law** Distribution $(ab)^m = a^m b^m$
$\left(\dfrac{8}{7}\right)^3 = \left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)\left(\dfrac{8}{7}\right)$ $= \dfrac{8^3}{7^3}$	$\left(\dfrac{a}{b}\right)^3 = \left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)$ $= \dfrac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (8\cdot 8\cdot 8)(8\cdot 8\cdot 8)$ $= 8^6$ or $8^{3\times 2}$	$(a^3)^2 = (a^3)(a^3)$ $= (a\cdot a\cdot a)(a\cdot a\cdot a)$ $= a^6$ or $a^{3\times 2}$	**Power of a Power Law** $(a^m)^n = a^{m\cdot n}$

[redacted]

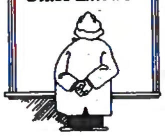
Class Ex. #5

Use the exponent laws to simplify and then evaluate. solve

a) $3^4 \cdot 3^2$ b) $\dfrac{(-2)^5}{(-2)^3}$ c) $(5^2)^3$

$3^{4+2} = 3^6 = 729$ $(-2)^{5-3} = (-2)^2 = 4$ $5^6 = 15625$

Class Ex. #6



Use the exponent laws to simplify.

a) $(a)^4(a)^3$ b) x^6x c) $b^7 \times b^0 \times b^4$ d) $\dfrac{x^8}{x^4}$ e) $\dfrac{y^{10}}{y^2}$ f) $(a^4)^3$

$a^{4+3} = a^7$ $x^{6+1} = x^7$ $b^{7+4} = b^{11}$ $x^{8-4} = x^4$ $y^{10-2} = y^8$ $a^{4 \cdot 3} = a^{12}$

g) $(y^5)^5$ h) $\left(\dfrac{x}{y}\right)^9$ i) $\left(\dfrac{c}{4}\right)^2$ j) $(st)^6$ k) $(2a)^5$ l) $(-3pq)^4$

$y^{5 \cdot 5} = y^{25}$ $\dfrac{x^9}{y^9}$ $\dfrac{c^2}{4^2} = \dfrac{c^2}{16}$ s^6t^6 $2^5a^5 = 32a^5$ $(-3)^4p^4q^4$ $81p^4q^4$

Complete Assignment Questions #8 - #22

Assignment

1. State the base and the exponent in each of the following powers.

a) 8^3 b) k^{15} c) 2^x d) $(-x)^4$ e) $\left(\dfrac{3}{4}\right)^6$

2. State the coefficient in each of the following.

a) $5x^7$ b) $-6z^2$ c) a^3 d) $\dfrac{y^3}{4}$ e) $\dfrac{5y^9}{8}$

3. Write each of the following as a repeated multiplication.

a) c^4 b) $5x^3$ c) $(ab)^2$ d) $(-5)^3$ e) s^2t

f) $2\left(\dfrac{5}{4}\right)^3$ g) $(4a)^3$ h) $3cd^2$ i) $3(cd)^2$ j) $(3cd)^2$

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