Prime Factorization and Exponents Lesson #2:
Applications of Prime Factors

Review

Express the numbers 48 and 72 as products of prime factors.

48 = 2x2x2x2x3
=2^4x3

48
4 12
2 2 4 3
2 2

2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
1

72= 2x2x2x3x3 or 2^3x 3^2

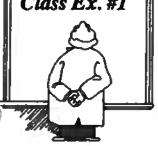
Greatest Common Factor

The **greatest common factor** (GCF) of a set of whole numbers is the largest whole number which divides exactly into each of the members of the set.

For example, the GCF of 8, 16, and 20 is 4.

*2 is a common factor but not greatest

Class Ex. #1



State the greatest common factor of

a) 15, 25, and 35 5 b) 18 and 20 2 c) 36 and 54 18

In the example above parts a) and b) were fairly simple to do, but part c) was more complicated because each number had a large number of factors.

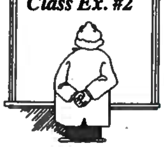
In cases like this we can use prime factorization to determine the GCF.

From the review 48 = 2 × 2 × 2 × 2 × 3 and 72 = 2 × 2 × 2 × 3 × 3.

To determine the GCF of 48 and 72 we find the product of each prime factor (including repeats) which is common to both prime factorizations.

GCF of 48 and 72 is 2 × 2 × 2 × 3 = 24.

Class Ex. #2



Use prime factorization to determine the greatest common factor of the given whole numbers.

a) 90 and 225 b) 154 and 198

5 | 90, 225
3 | 18, 45
3 | 6 15
2, 5

no more common factors

GCF
5x3x3
=45

154
14 11
7 2

198
11 18
9 2
3 3

GCF
11x2
=22

Class Ex. #3

Use prime factorization to determine the GCF of 245, 315, and 770.

5 | 245
7 | 49
7 | 7
1

5 | 315
7 | 63
3 | 9
3 | 3
1

7 | 770
5 | 110
11 | 22
2 | 2
1

GCF: 5x7 = 35

when I use GCF

$$\frac{245 \div 35}{315 \div 35} = \frac{7}{9}$$

reduce fraction

Lowest Common Multiple

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

Common multiples of 6 and 8 are 24, 48, ...

The **lowest common multiple** (LCM) of 6 and 8 is 24.

Class Ex. #4

Determine the lowest common multiple of the following:

a) 5 and 7

5: 5, 10, 15, 20, 25...
7: 7, 14, 21, 28, 35
LCM: 35

b) 10, 15, and 20

LCM: 60

c) 10, 12, and 14

420

In the example above, parts a) and b) were fairly simple to do, but part c) was more complicated. Prime factors can be used to simplify the solution.

$10 = 2 \times 5$

$12 = \cancel{2} \times 2 \times 3$

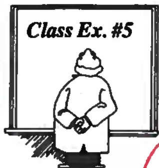
$14 = \cancel{2} \times 7$

To determine the LCM, take all the prime factors of one of the numbers and multiply by any additional factors in the other numbers.

Take **2** and **5** from 10, another **2** and **3** from 12, and **7** from 14.

$2 \times 5 \times 2 \times 3 \times 7 =$ 420, the LCM of 10, 12, and 14.

Class Ex. #5



Use prime factorization to determine the LCM of

a) 15 and 35 **b)** 126 and 441 **c)** 22, 154, and 198

a) 5 | 15, 35 → 3, 7

LCM: 5×3×7 = 105

b) 126 = 2 × 63 = 2 × 7 × 9 = 2 × 7 × 3 × 3

441 = 7 × 63 = 7 × 7 × 9 = 7 × 7 × 3 × 3

LCM: 7×3×3×7×2 = 882

c) 2 | 22, 154, 198 → 11 | 11, 77, 99 → 1, 7, 9

LCM: 2×11×7×9 = 1386

when do I use LCM?

$\frac{1}{22} \times 154 + \frac{1}{154} \times 154 =$

$\frac{7}{154} + \frac{1}{154}$

Complete Assignment Questions #1 - #7

Prime Factorization of a Perfect Square

3 | A=9 | 3

Perfect squares of whole numbers include 1, 4, 9, 16, 25, 36, 49, etc. (1×1, 2×2, 3×3, 4×4)

Every perfect square has two **square roots**: one positive and one negative. The square root which is positive is called the **principal square root**. 4×4 also −4×−4

The principal square root of each number above is 1, 2, 3, 4, 5, 6, 7, etc.

In this lesson, where we are dealing only with whole numbers, we will use the term square root to mean the principal square root.

Complete the prime factorization of the following perfect squares: 9, 36, 49, and 64.

9 → 3, 3

36 → 4, 9 → 2, 2, 3, 3

49 → 7, 7

64 → 2, 32 → 2, 16 → 2, 8 → 2, 4 → 2, 2

9 = 3 × 3 The square root of 9 is 3

36 = 2 × 2×3×3 The square root of 36 is 2 × 3 = 6

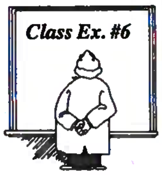
49 = 7×7 The square root of 49 is 7

64 = 2×2×2×2×2×2 The square root of 64 is 2×2×2 = 8

The prime factorization of a perfect square will involve sets of factors which each occur two times (or a multiple of two times).

If the prime factorization of a number does not result in sets of factors which each occur two times (or a multiple of two times), then we can say that the number is **not** a perfect square.

↳ each factor needs a "buddy" to be a perfect square (pair)



Class Ex. #6

Consider the number 44 100.

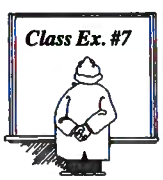
a) Use a calculator to find the square root of 44 100.

b) Explain how we can use the prime factorization of 44 100 to show that 44 100 is a perfect square. Verify your calculator answer by this method.

a) $\sqrt{44100} = 210$

44100
4410 10
441 10 5 2
3 147 5 2
49 3
7 7

Root:
$2 \times 5 \times 3 \times 7$
$= 210$



Class Ex. #7

In each case use prime factorization to determine if the number is a perfect square. If the number is a perfect square, state the square root of the number.

a) 1 225

5 245
5 49
7 7

yes, perfect square
root $5 \times 7 = 35$

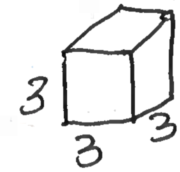
b) 3 042

2 1521
3 507
3 169
13 13

not a perfect square!

[redacted]

Prime Factorization of a Perfect Cube



volume $3^3 = 27$

Perfect cubes of whole numbers include 1, 8, 27, 64, 125, etc.
The **cube root** of each number above is 1, 2, 3, 4, 5, etc.

Complete the prime factorization of the following perfect cubes: 27, 216, and 3 375.

27 → 3, 9 → 3 3

216 → 2, 108 → 2, 54 → 2, 27 → 3, 9 → 3 3

$2 \times 3 = 6$

3375 → 25, 135 → 5 5, 5, 27 → 3, 9 → 3 3

$5 \times 3 = 15$

root: 3

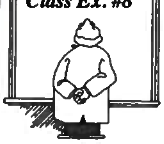
* cube needs groups of 3

The prime factorization of a perfect cube will involve sets of factors which each occur three times (or a multiple of three times).

If the prime factorization of a number does not result in factors which each occur three times (or a multiple of three times), then we can say that the number is not a perfect cube.

* has to make groups of 3 to be a cube, no singles or pairs

Class Ex. #8



In each case use prime factorization (division table) to determine whether the number is a perfect cube. If the number is a perfect cube, state the cube root of the number.

a) 6 912 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

6 912 → 12, 576 → 6, 2, 6, 96 → 3, 2, 3, 2, 12, 8 → 6, 2, 4, 2 → 3, 2, 2, 2

not a cube

b) 3 | 970 299
3 | 323 433
3 | 107 811
3 | 35 937
3 | 11 979
3 | 3 993
11 | 1 331
11 | 121
11 | 11
1

Root $3 \times 3 \times 11 = 99$

perfect cube ✓

Complete Assignment Questions #8 - #15

Assignment

1. State the greatest common factor of

a) 14 and 21 **b)** 30 and 40 **c)** 12, 30, and 54

2. Use prime factorization to determine the GCF of

a) 150 and 420 **b)** 126 and 189 **c)** 294 and 385

3. Use prime factorization to determine the greatest common factor of

a) 483 and 575 **b)** 180 and 504 **c)** 1 700 and 1 938

d) 663 and 910 **e)** 84 and 231 **f)** 525 and 850