

Prime Factorization and Exponents Lesson #1: Prime Factors

Name: _____

Overview

In this unit we look at prime factorization of whole numbers and applications including greatest common factor, least common multiple, square root, and cube root. We review powers with whole number exponents where the base is numerical. We also extend these concepts to powers where the base is variable and where the exponents are integers using inquiry, analyzation, and reasoning.

2³

↓ X³

Factors

The whole number 6 is exactly divisible by the whole numbers 1, 2, 3, and 6.

The numbers 1, 2, 3, and 6 are the **factors** of the whole number 6.

The number 6 has four factors.

↑ what a number can be divide by



In each case, state the number of factors of the given whole number.

a) 1
①

b) 2
②

c) 3
② = 1, 3

d) 4
③ = 1, 2, 4

e) 12
⑥
1, 2, 3, 4, 6, 12

Factor Pairs

We say 2 and 3 are a factor pair of 6 because $2 \times 3 = 6$.

A factor pair is a set of two whole numbers which when multiplied result in a specific product.

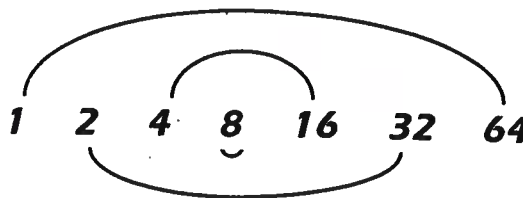
↑ multiply

i) State another factor pair of 6. 1×6

We can use a diagram listing all the factors of a number to help determine all possible factor pairs of that number.

The diagram opposite shows there are four factor pairs of 64.

8 and 8 are a factor pair because $8 \times 8 = 64$.



ii) List the other factor pairs.

$1 \times 64, 2 \times 32, 4 \times 16$



List the factors of the following and determine all the factor pairs for each.

a) 15
1, 3, 5, 15
 1×15 3×5

b) 16
1, 2, 4, 8, 16
 1×16 4×4
 2×8

Prime and Composite Numbers

A **prime number** is defined as a whole number which has exactly two distinct factors. The two factors are always **1 and the number itself**, e.g. 3, 7.

A **composite number** is a whole number which has more than two factors, e.g. 10, 18.

The number 1 has only one factor and is neither prime nor composite.

In this course the number 0 is defined to have no factors.

- Complete the list of the first ten prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...



Classify the following whole numbers as prime or composite.

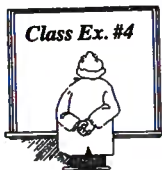
- a) 46 composite b) 37 prime c) 39 composite d) 101 prime e) 103 prime

Prime Factors

The **prime factors** of a whole number are the factors of the number which are prime.

For example The factors of 6 are 1, 2, 3, and 6.

The prime factors of 6 are 2 and 3.



- a) State the factors of 12.

1, 2, 3, 4, 6, 12

- b) State the prime factors of 12.

2, 3

- c) Express 12 as a product of prime factors.

$2 \times 3 \times 2$

Complete Assignment Questions #1 - #9



Prime Factorization

Every composite number can be expressed as a product of prime factors. Expressing a whole number as a product of prime factors is called the **prime factorization** of the number.

The prime factorization of small numbers like 12 can probably be done mentally. For larger numbers, a division table or a tree diagram can be used.

The diagrams below illustrate these techniques for the prime factorization of 48.

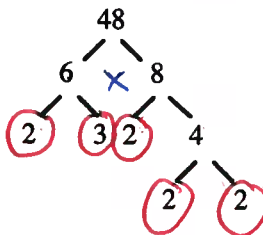


- Prime numbers must be used as divisors when using a division table.
- Any divisor may be used when using a tree diagram.

division table

2	48
2	24
2	12
2	6
3	3
	1

tree diagram



$48 = 2 \times 2 \times 2 \times 2 \times 3$

or using powers

$48 = 2^4 \times 3$

"2 to the power of 4"

Class Ex. #5

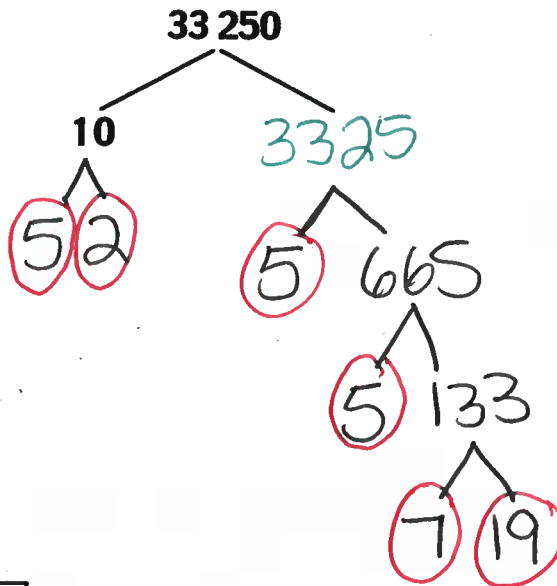


Two students, Tantoo and Craig, work together to confirm that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$. Tanto uses a division table and Craig uses a factor tree. Complete their work to verify that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$.

Tantoo Division Table

2	33 250
5	16 625
5	3 325
5	665
7	133
19	19
	1

Craig Tree Diagram



Complete Assignment Questions #10 - #17

Assignment

- In each case, determine the number of factors of the given whole number.
 - 8
 - 11
 - 17
 - 33
 - 45
- List the factors of the following numbers and determine all the factor pairs.
 - 21
 - 22
 - 25
 - 36
- In some of the parts of question #2, the number of factor pairs is exactly one half of the number of factors and in other parts it is not. Explain.
- State the numbers in question #1 which are
 - prime
 - composite
- Classify the following whole numbers as prime or composite.
 - 30
 - 41
 - 43
 - 57
 - 59
 - 121
 - 133
 - 169
- Twin primes are defined to be consecutive odd numbers that are both prime (e.g. 5 and 7). List the seven other twin primes less than 80.