

Lesson 1: Functions

Friday, August 31, 2018 2:33 AM

Functions Lesson #1: Functions

Review

We have considered six ways in which the relationship between two quantities can be represented.

- in words (\neq)
- a mapping (or arrow) diagram
- function notation (this unit)
- a table of values $\begin{array}{c|c} x & y \end{array}$
- an equation $y = 2x + 3$
- a set of ordered pairs $(1, 2), (2, 3)$
- a graph

In a **relation** each element of the **domain** (the **input**) is related to an element or elements of the **range** (the **output**).

In this lesson we will study a special type of relation called a **function**.

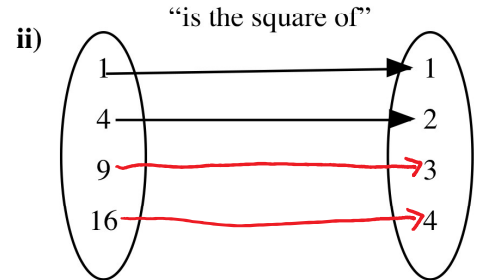
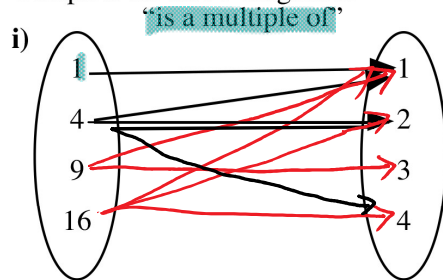
Exploration

To illustrate the concept of function, we will look at two relations described in words with domain $D = \{1, 4, 9, 16\}$ and range $R = \{1, 2, 3, 4\}$.

i) "is a multiple of"

ii) "is the square of"

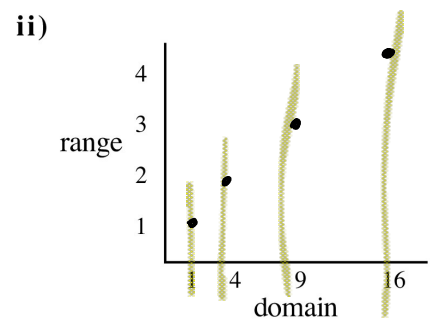
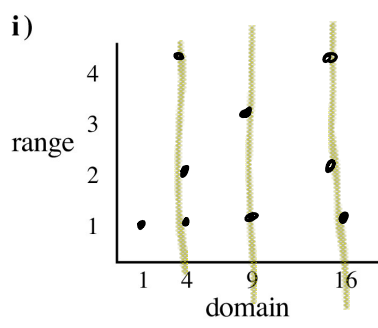
a) Complete the arrow diagrams.



b) Complete the set of ordered pairs.

- i) $(1, 1), (4, 1), (4, 2), (9, 1), (9, 3), (16, 1), (16, 2), (16, 4)$ ii) $(1, 1), (4, 2), (9, 3), (16, 4)$

c) Plot the ordered pairs on the grid.



Function

A functional relation, or **function**, is a special type of relation in which **each element of the domain is related to exactly one element of the range**. If any element of the domain is related to more than one element of the range, then the relation is not a function.



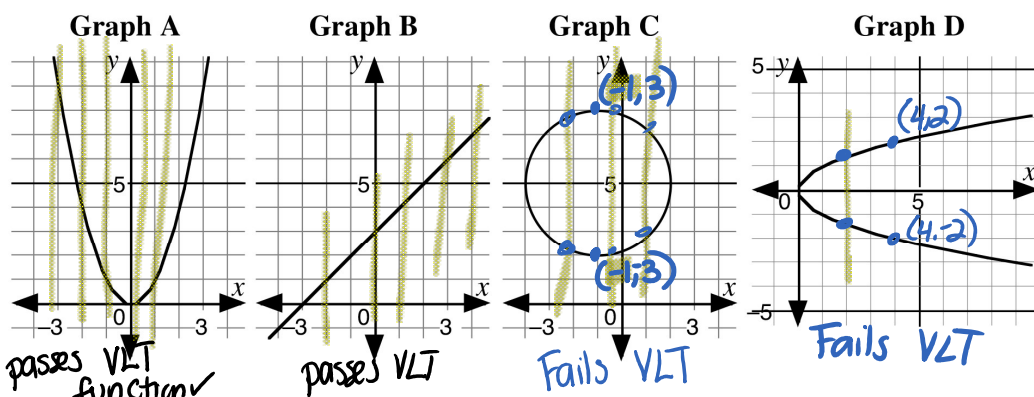
In the exploration on the previous page, one of the relations is a function, and the other relation is not a function.

Explain how we can determine which relation is a function by looking at the following:

- a) arrow diagrams
if only one arrow leaves every element of the domain then the relation is a function
- b) ordered pairs
if each x-coordinate has only one y-coordinate then the relation is a function
- c) graphs
Vertical line test



Each of the following is the graph of a relation.



a) Classify the following statements as true (T) or false (F).

- For each input value there is only one output.
- For each output value there is only one input.
- The relation is a function.

A	B	C	D

- b) From graph C, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.
- c) From graph D, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.
- d) On graphs A and B draw a series of vertical lines. Do any of these lines intersect the graph of the relation at more than one point?

Vertical Line Test

The vertical line test can be used on the graph of a relation to determine whether the relation is a function or not.

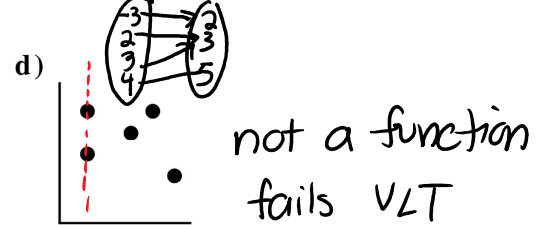
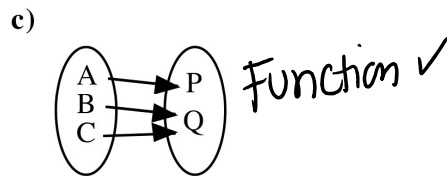
- If every vertical line, drawn on the domain of the relation, intersects the graph exactly once, then the relation is a function.
- If any vertical line intersects the graph more than once, then it is **not** a function.



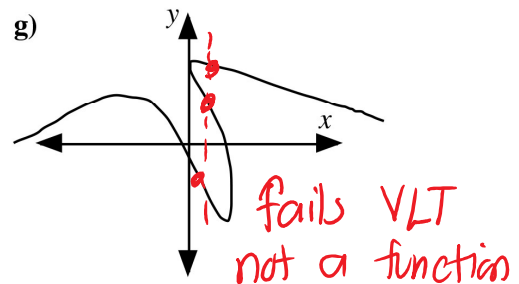
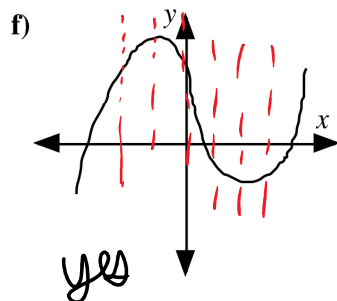
Determine which of the following are functions. Explain your answers.

a) (5, 8), (6, 7), (-5, 3), (2, 3), (6, 8)
 6 < 8 not a function

b) (3, 3), (2, 3), (4, 5), (-3, 2).
 no repeat x's function ✓



e) The relation connecting the provinces and territories of Canada with their capital cities.
 Yes, function. Each input only has one output



A Function as a Mapping

A function from a set D , the domain, to a set R , the range, is a relation in which each element of D is related to exactly one element of R .

If the function f maps an element x in the domain to an element y in the range, we write $f: x \rightarrow y$.

Complete the following for the function “is the square of” on the first page of this lesson.

$1 \rightarrow 1$ $4 \rightarrow 2$ $9 \rightarrow 3$ $16 \rightarrow 4$



function $f:$

Consider the function $f: x \rightarrow 3x + 1$, for domain $\{-1, 0, 1, 2\}$.

let $x = -1$
 $3(-1) + 1$
 $-3 + 1$
 $= -2$
 ↑
 output

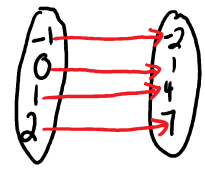
a) Complete $-1 \rightarrow -2$ $0 \rightarrow 1$ $1 \rightarrow 4$ $2 \rightarrow 7$

b) List the elements of the range of the function.

range: $\{-2, 1, 4, 7\}$

c) Show the function as:

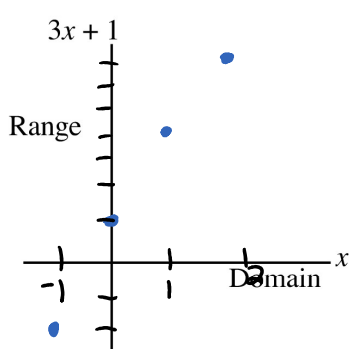
i) an arrow diagram



ii) a set of ordered pairs

- $(-1, -2)$
- $(0, 1)$
- $(1, 4)$
- $(2, 7)$

iii) a Cartesian graph.



1, 3, 5, 7

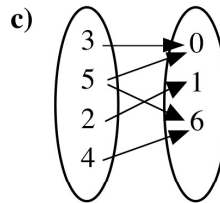
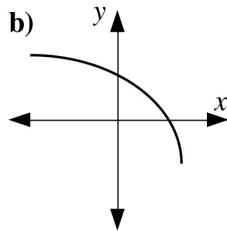
At this time we label the top of the vertical axis with $3x + 1$. In the next lesson we will learn function notation which is more commonly used.

Complete Assignment Questions #1 - #12

Assignment

1. Determine which of the following relations are functions. Give reasons for your answers.

- a) $(-1, 3), (-2, 1), (5, 2), (7, 3)$

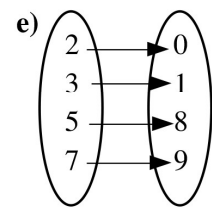
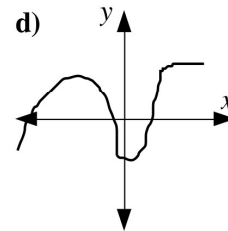
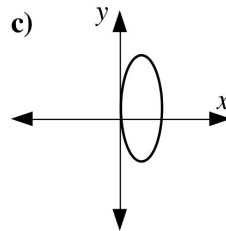
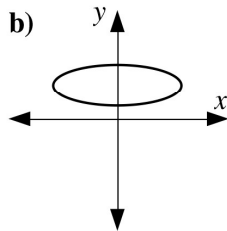


d)

Input (x)	Output (y)
2	3
0	4
-3	5
2	6

2. State which of the following relations are functions.

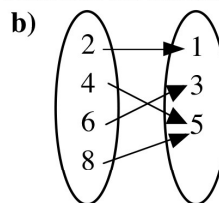
- a) $(0, 0), (1, 2), (2, 3), (3, 4), (4, 3)$



3. State which of the following relations are functions.

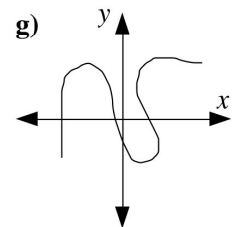
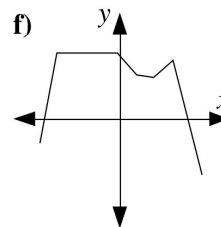
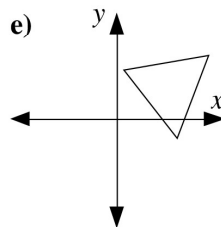
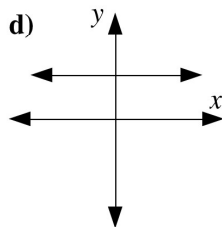
a)

Input (x)	Output (y)
0	3
2	4
4	5
6	3



c)

Input (x)	Output (y)
1	5
-1	5
3	5
7	5

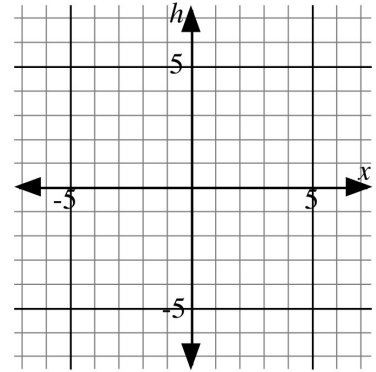


4. Mr. A has a son Jim and a daughter Kristen. Mr. B has three daughters, Lauren, Melanie, and Noreen.
- Draw an arrow diagram to illustrate the relation “is the father of” from the set of fathers to the set of children. Is the relation “is the father of” a function?
 - Draw an arrow diagram to illustrate the relation “is the child of” from the set of children to the set of fathers. Is the relation “is the child of” a function?
5. The function $f: x \rightarrow 2x + 5$ has domain $\{0, 1, 2, 3\}$.
- List the elements of the range of the function.
 - Show the function f in a Cartesian graph.
6. The function $g: x \rightarrow x^3$ has domain $\{-2, -1, 0, 1, 2\}$.
- List the elements of the range of the function.
 - Show the function g in a Cartesian graph.

7. Consider the function $f: x \rightarrow x^2 - 4$.
 a) Complete the following table of values.

Elements of Domain	3	2	1	-1	-2	-3
Elements of Range						

- b) Plot the ordered pairs on a Cartesian graph.
 c) Draw a smooth curve through the points to illustrate the function $f: x \rightarrow x^2 - 4, x \in R$.



8. The domain of the function $h: x \rightarrow 6$ is $\{0, 10, 20\}$.
 a) List the ordered pairs of the graph of the function.
 b) Show the function h in a Cartesian graph.

Multiple Choice 9.

The function $f: x \rightarrow 6 - 2x$ has domain $\{0, 2, 4, 6, 8\}$. Which of the following is **not** an element of the range of the function?

- A. -10
 - B. 2
 - C. 4
 - D. -6
10. Which of the following statements is not always true for a function?
- A. A function is a set of ordered pairs (x, y) in which for every x there is only one y .
 - B. A vertical line must not intersect the graph of a function in more than one point.
 - C. For every output there is only one input.
 - D. For every element in the domain, there is only one element in the range.

11. Which of the following represents a function?

1

“multiply the number by 3 and add 5.”

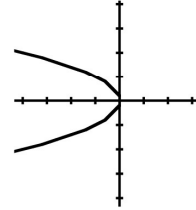
2

$$y = -x^2$$

3

$(9, 3), (4, 2), (1, 1), (0, 0), (1, -1), (4, -2), (9, -3)$

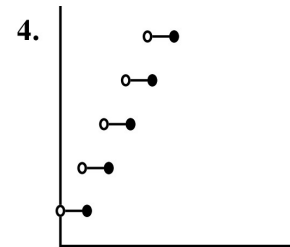
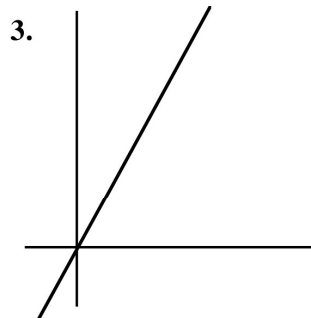
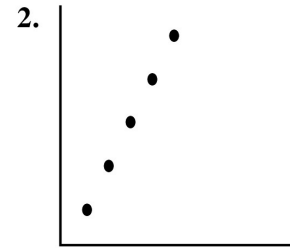
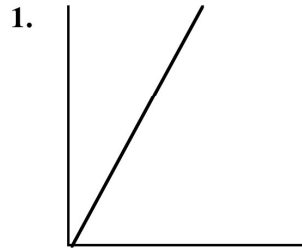
4



- A. 1 only
- B. 1 and 2 only
- C. 1 and 3 only
- D. some other combination of 1 - 4

Numerical Response

12. Partial graphs of four functions are shown.



The functions are described as follows:

- A:** Coffee costs \$8 per jar. Graph cost as a function of the number of jars purchased.
- B:** Distance cycled at a constant speed of 8 km/h. Graph distance as a function of time.
- C:** Parking costs \$8 per hour (or part of an hour). Graph cost as a function of time.
- D:** Set of ordered pairs which satisfy the equation $y = 8x, x \in R$. Graph y as a function of x .

Place the graph number for function A in the first box.
 Place the graph number for function B in the second box.
 Place the graph number for function C in the third box.
 Place the graph number for function D in the last box.

(Record your answer in the numerical response box from left to right)

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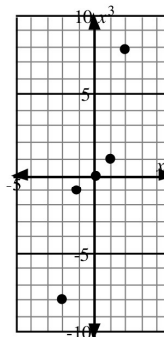
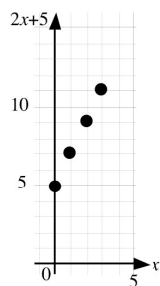
Answer Key

1. a) function: each first coordinate has only one second coordinate
 b) function: vertical lines intersects the graph exactly once
 c) not a function: the input 5 has two outputs
 d) not a function: the input 2 has two outputs
2. a) function b) not a function c) not a function d) not a function e) function
3. a) function b) function c) function d) function e) not a function
 f) function g) not a function

4. a) Neither mapping diagrams represents a function b) Both mapping diagrams represent functions

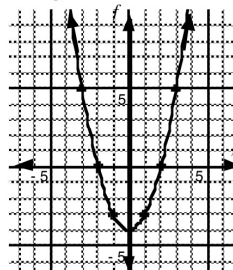


5. a) {5, 7, 9, 11} b) see graph below
6. a) {-8, -1, 0, 1, 8} b) see graph below

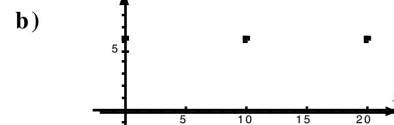


7. a) see table b) see grid c) see grid

Elements of Domain	3	2	1	-1	-2	-3
Elements of Range	5	0	-3	-3	0	5



8. a) {(0, 6), (10, 6), (20, 6)}



9. C 10. C 11. B 12.

2	1	4	3
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