

Lesson 6: Probability Involving Permutations and Combinations

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Review

Factorial Notation $n! = n(n-1)(n-2)(n-3) \dots (3)(2)(1)$, where $n \in W$.

order matters
Permutations

$${}_n P_r = \frac{n!}{(n-r)!}$$

Permutations with Repetitions $\frac{n!}{a!b!c!}$

order doesn't matter

Combinations ${}_n C_r = \frac{n!}{(n-r)!r!} = \binom{n}{r}$

ex CHOOSE $\frac{6!}{2!}$



Class Ex. #1

Two cards are selected without replacement from a deck of 52 playing cards. Determine the probability that both cards are kings using

a) the multiplication law

$$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \left(\frac{1}{13}\right)\left(\frac{3}{51}\right) = \frac{3}{663}$$

$$= \frac{1}{221}$$

b) combinations

$$\frac{{}_4 C_2 - \text{outcome I want}}{{}_{52} C_2 - \text{all possibilities}} = \frac{6}{1326}$$

$$= \frac{1}{221}$$



Class Ex. #2

In a card game, you are dealt 5 cards from a pack of 52 shuffled cards. When you look at your 5 cards, what is the probability, expressed in combination notation, that you have

a) four aces? 4Ace 1other

$$\frac{{}_4 C_4 \cdot {}_{48} C_1}{{}_{52} C_5}$$

b) four tens and an ace?

$$\frac{{}_4 C_4 \cdot {}_4 C_1}{{}_{52} C_5}$$

c) 10, J, Q, K, and ace?

$$\frac{{}_4 C_1 \cdot {}_4 C_1 \cdot {}_4 C_1 \cdot {}_4 C_1 \cdot {}_4 C_1}{{}_{52} C_5}$$

d) at least one Jack? 134J', 233J', 3J23', 4J13'

$$1 - P(\text{no Jacks})$$

$$= 1 - \frac{{}_{40} C_5}{{}_{52} C_5}$$



Class Ex. #3

The word **COUNTED** has been spelled using Scrabble tiles. Two tiles are randomly chosen one at a time and placed in the order in which they were chosen. Determine the probability that the tiles are

a) CO

order matters

$$P(\text{CO}) = \frac{1}{{}_7 P_2} = \frac{1}{42}$$

$$\underline{\text{C}} \underline{\text{O}}$$

b) both vowels

$$P(\text{both vowels}) = \frac{{}_3 P_2}{{}_7 P_2} \leftarrow \text{what you want}$$

$$\frac{6}{42} = \frac{1}{7} \leftarrow \text{all possibilities}$$

$$\left(\text{or } \frac{\underline{\text{O}} \underline{\text{E}}}{\underline{\text{C}} \underline{\text{O}} \underline{\text{U}} \underline{\text{N}} \underline{\text{T}} \underline{\text{E}} \underline{\text{D}}} = \frac{6}{42} \right)$$



Mike's MP3 player contains 50 songs. If he listens to 10 songs on "random", determine the probability, as a percentage to the nearest hundredth, that the playlist contains

- a) Mike's two favourite songs *order does not matter*

$$\frac{2C_2 \cdot 48C_8}{50C_{10}} = 0.0367... \times 100 = \boxed{3.67\%}$$

- b) one of his two favourite songs at the beginning of the 10 songs and the other at the end *order matters!*

② (- any 8 - -) ① → $\frac{2P_2 \cdot 48P_8}{50P_{10}} = 0.000816... = \boxed{0.08\%}$



City Council consists of nine men and six women. Three representatives are chosen at random to form an environmental sub-committee.

- a) What is the probability that Mayor Jim Milonovich and two women are chosen?

order does not matter $\frac{1C_1 \cdot 6C_2}{15C_3} = \boxed{\frac{3}{91}}$

- b) What is the probability that two women are chosen if Mayor Jim Milonovich must be on the committee?

$$\frac{6C_2}{14C_2} = \frac{15}{91}$$



A hacker is attempting to break into a friend's security-protected file. The friend tells the hacker all the numbers that are in the 4-digit PIN but not the order or how many times each digit may be repeated in the PIN.

Determine the probability that the hacker correctly guesses the 4-digit PIN on the first attempt if the friend tells her that the PIN contains

- a) the numbers 4, 5, 6, and 7 $\frac{1}{4!} = \boxed{\frac{1}{24}}$

- b) only the numbers 4, 5, and 6
- $4456 \rightarrow \frac{4!}{2!} = 12$
 $4556 \rightarrow \frac{4!}{2!} = 12$
 $4566 \rightarrow \frac{4!}{2!} = 12$
- } 36 total possibilities $\boxed{\frac{1}{36}}$

- c) only the numbers 4 and 5
- $4445 \rightarrow \frac{4!}{3!} = 4$
 $4455 \rightarrow \frac{4!}{2!2!} = 6$
 $4555 \rightarrow \frac{4!}{3!} = 4$
- } 14 possibilities $\boxed{\frac{1}{14}}$

- d) only the number 4 $\boxed{1}$

Complete Assignment Questions #1 - #17

#1-11

*all homework due
Unit test
Friday
games due
Tuesday!*

Assignment

1. A marble is drawn at random from a box containing 10 red, 30 yellow, 20 blue, and 10 pink marbles. State, as an exact fraction, the probability that the marble drawn is:
a) yellow b) yellow or red c) not blue d) green
2. A bag contains 6 blue marbles and 10 yellow marbles. Two marbles are drawn from the bag without replacement. Determine the following using
i) the multiplication law ii) permutations or combinations
 - a) $P(\text{both are blue})$
 - b) $P(\text{the first is blue and the second is yellow})$
 - c) $P(\text{one is blue and one is yellow})$
3. Three prizes are rewarded in a raffle during a halftime show at a school basketball game. Ben, Janelle, Jamie, and 17 other students each have one ticket.
 - a) If the raffle has a first, second, and third prize, determine the probability, as an exact value, that Ben wins first prize, Janelle wins second prize, and Jamie wins third prize.
 - b) If the raffle has three identical prizes, determine the probability that Ben, Janelle, and Jamie win the prizes.
4. A bank card personal identification number consists of any four digits. Repeat digits are allowed and the code can start with zero. What is the probability that a code begins and ends with the digit 5?
5. A child randomly selects three toys from a box of nine toys, three of which are defective. Determine the probability that none of the toys selected is defective.

11. Elizabeth has 30 songs on her MP3 player: 15 in the “Pop” genre, 10 in the “Country” genre, and 5 in the “Alternative” genre. She generates a random playlist of 9 songs. Determine, as a percentage to the nearest tenth, the probability that the playlist
- a) contains 6 pop songs and 3 country songs b) contains exactly 6 pop songs

 - c) contains 3 songs from each genre d) starts and ends with a pop song
12. A company’s database requires a three-digit access code. Victor remembers the three digits in the code, but not the order in which they are arranged.
- a) Determine the probability Victor gains access to the database on the first attempt if
 - i) all three digits are different ii) exactly two of the digits are the same

 - b) An alarm is triggered if an incorrect access code is entered twice in a row. Determine the probability that Victor avoids triggering the alarm if
 - i) all three digits are different ii) exactly two of the digits are the same
13. The national government of a country is to form a committee on environmental regulation. There are 83 legislators to choose from: 52 from urban areas and 31 from rural areas. If 5 legislators are chosen at random, determine the probability, to 3 decimal places, that
- a) the Prime Minister and the Opposition Leader are both on the committee
 - b) exactly 4 urban legislators are on the committee
 - c) at least one rural legislator is on the committee
 - d) at least one rural legislator is on the committee, if the urban-based Minister of the Environment must be on the committee.

Multiple Choice

14. Taqana buys two of a total of thirty raffle tickets. There are two winning numbers. The probability that Taqana wins exactly one prize is
- A. $\frac{56}{435}$ B. $\frac{28}{435}$ C. $\frac{2}{435}$ D. $\frac{1}{435}$
15. Five DVDs and their corresponding five cases are sitting on a table. As part of a party game, a student is blindfolded and randomly puts a DVD in each case. The probability that all the DVDs are put in the correct cases is
- A. $\frac{1}{5}$ B. $\frac{1}{60}$ C. $\frac{1}{120}$ D. $\frac{1}{3125}$
16. Four cards are dealt from a deck of 52 cards. The probability that the cards contain exactly two aces and exactly one king can be represented by
- A. $\frac{{}_4C_2 \cdot {}_4C_1 \cdot {}_{47}C_1}{52C_4}$ B. $\frac{{}_4C_2 \cdot {}_4C_1}{52C_4}$
- C. $\frac{{}_4C_2 \cdot {}_4C_1 \cdot {}_{44}C_1}{52C_4}$ D. $\frac{{}_4P_2 \cdot {}_4P_1 \cdot {}_{44}P_1}{52P_4}$

Numerical Response

17. The letters of the word **OKOTOKS** are arranged. The probability, to the nearest hundredth, that an arrangement, chosen at random from all possible arrangements, begins and ends with a vowel is _____.

(Record your answer in the numerical response box from left to right.)

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Answer Key

1. a) $\frac{3}{7}$ b) $\frac{4}{7}$ c) $\frac{5}{7}$ d) 0 2. a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$
3. a) $\frac{1}{6840}$ b) $\frac{1}{1140}$ 4. $\frac{1}{100}$ 5. $\frac{5}{21}$
6. a) 0.05% b) 0.20% c) 2.53% d) 97.47% 7. $\frac{1}{720}$ 8. $\frac{56}{143}$
9. a) 0.02 b) 0.09 10. a) 1:3149 b) 1:69
11. a) 4.2% b) 15.9% c) 3.8% d) 24.1% 12. a) i) $\frac{1}{6}$ ii) $\frac{1}{6}$ b) i) $\frac{1}{3}$ ii) $\frac{1}{3}$
13. a) 0.003 b) 0.289 c) 0.910 d) 0.857
14. A 15. C 16. C 17.

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