Lesson 5: Solving Problems Using Tables and Diagrams

Probability Lesson #5:
Solving Problems Using Tables and Diagrams

Review

Odds in favour of A = # of outcomes for A : # of outcomes against A.
Odds against A = # of outcomes against A : # of outcomes for A.

If the events A, B are **mutually exclusive**, then
$$P(A \cup B) = P(A) + P(B)$$

If the events A, B are **NOT mutually exclusive**, then
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If the events A, B are **independent**, then
$$P(A \cap B) = P(A) \cdot P(B)$$

If the events A, B are **dependent**, then
$$P(A \cap B) = P(A) \cdot P(B|A)$$

Using a Table

Class Ex. #1

The table shows how the students in a large high school generally travel to school.

if male, what is the probability of driving a car

	Bus B	Car C	Other O	Total
Male, M	350	200	75	625
Female, F	300	175	100	575
Total	650	375	175	1200 (total students)

a) Complete the totals in the chart.

b) How many students attend the high school? 1200

c) Use the numbers in the table to determine

i) $P(M)$ $\frac{625}{1200} = \frac{25}{48}$ **ii)** $P(C|M)$ $\frac{200}{625} = \frac{8}{25}$ **iii)** $P(M|C)$ $\frac{200}{375} = \frac{8}{15}$ **iv)** $P(M \cap C)$ $\frac{200}{1200} = \frac{1}{6}$

d) If a student is selected at random, determine the probability that the student

i) is female $\frac{575}{1200} = \frac{23}{48}$ **ii)** travels by bus $\frac{650}{1200} = \frac{13}{24}$ **iii)** is female and travels by bus $\frac{300}{1200} = \frac{1}{4}$

e) Identify the following events and determine the probability that

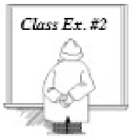
i) a female student travels by bus $P(B|F) = \frac{300}{575} = \frac{12}{23}$

ii) a student who drives to school is male $P(M|C) = \frac{200}{375} = \frac{8}{15}$

f) Are the events "the student is female" and "the student travels by bus" independent events? Explain.

Complete Assignment Question #1

Using a Probability Tree Diagram



Students were given the following problem to solve:

"The odds that Ava will pass Math this semester are 4:1 in favour and the odds that she will pass English this semester are 1:9 against. If these events are independent, determine the probability (to the nearest hundredth) that she will pass

i) Math and English **ii)** Math and not English **iii)** Math or English"

a) Express the odds in terms of probabilities.

$P(M) = \frac{4}{5}$ $P(E) = \frac{9}{10}$

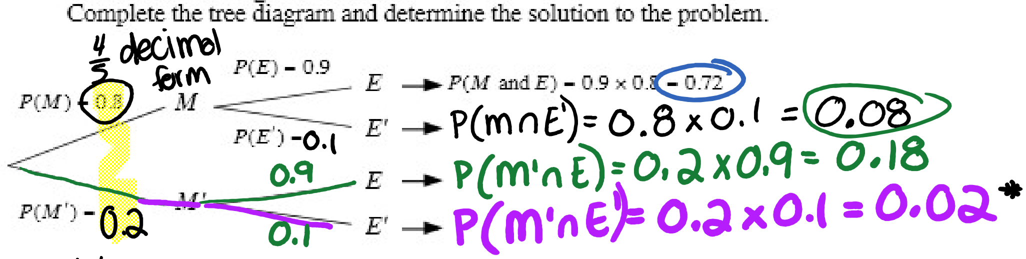
b) Some of the students chose to use probability formulas to solve the problem. Show this method of solution below.

i) $P(M \cap E) = \frac{4}{5} \cdot \frac{9}{10} = \frac{36}{50} = \frac{18}{25}$

ii) $P(M \cap E') = \frac{4}{5} \cdot \frac{1}{10} = \frac{4}{50} = \frac{2}{25}$

iii) $P(M \cup E) = P(M) + P(E) - P(M \cap E) = \frac{4}{5} + \frac{9}{10} - \frac{18}{25} = \frac{40}{50} + \frac{45}{50} - \frac{36}{50} = \frac{49}{50}$

c) Other students chose to use a probability tree diagram to solve the problem. Part of the tree diagram is shown. Complete the tree diagram and determine the solution to the problem.

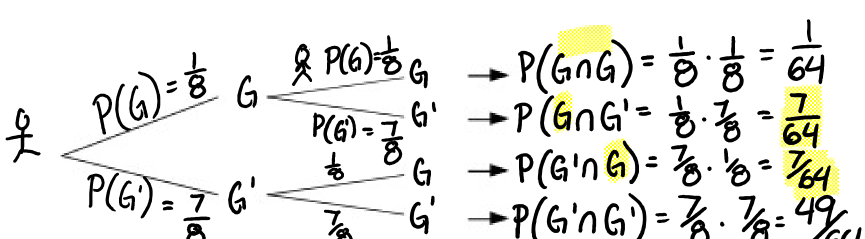


i) 0.72 ii) 0.08 iii) 0.72 + 0.08 + 0.18 = 0.98

In a game of field hockey, the probability that the Dinos score a goal from any short corner is $\frac{1}{8}$. During the first half of a match, they are awarded two penalty corners.

Use a probability tree diagram to determine the probability that, from these two short corners, the Dinos will score

a) two goals **b)** one goal **c)** at least one goal **d)** no goals



a) $\frac{1}{64}$

b) $\frac{7}{64} + \frac{7}{64} = \frac{14}{64} = \frac{7}{32}$

c) $\frac{1}{64} + \frac{7}{64} + \frac{7}{64} = \frac{15}{64}$

d) $\frac{49}{64}$

Notice that, at each branch of the tree, the sum of the probabilities is equal to 1.

In the above examples the two events were independent, but a probability tree diagram can also be used for dependent events, as in the next example.

Using a Probability Tree Diagram For Dependent Events

Cheryl is trying to show Jon how to solve problems based on the following information.

" Two machines, A_1 and A_2, produce all the glass bottles made in a factory. The percentages of broken bottles produced by these machines are 5% and 8% respectively. Machine A_1 produces 60% of the output. "

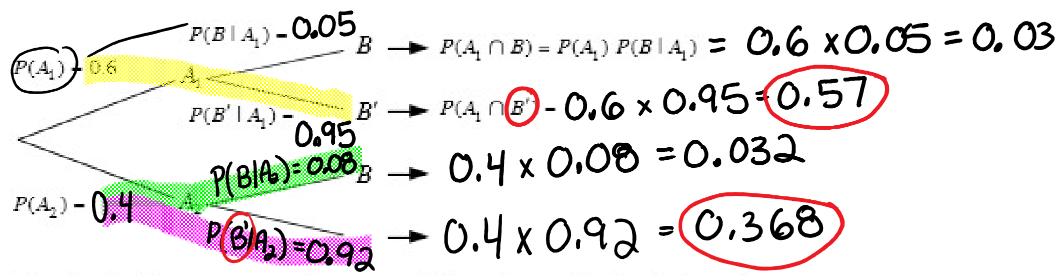
Cheryl suggests the following strategy.

First: Introduce symbols to represent the information.
Second: Write the given probabilities in terms of the symbols.
Third: Set up a probability tree diagram.

a) Complete ~~Jon~~ Cheryl's work, which is started below.

- A_1 - bottle is from machine A_1. A_2 - bottle is from machine A_2. B - bottle is broken.
- $P(A_1)$ = 0.6 $P(A_2)$ = 0.4 $P(B|A_1)$ = 0.05 $P(B|A_2)$ = 0.08

b) Jon sets up a probability tree diagram, with the first branches leading to the machines and the second set of branches leading to the broken/not broken items.
Complete the diagram.



c) Cheryl asked Jon some questions to see if he understands what he has drawn.

i) If a bottle is chosen at random, determine the probability that it is a broken bottle produced by machine A_1.

$P(A_1 \cap B) = 0.03$

ii) Which of the final outcomes in the diagram relate to the event "a bottle is not broken"?

$A_1 \cap B'$, $A_2 \cap B'$ = 0.57 + 0.368 = 0.938

iii) If a bottle is chosen at random, determine the probability that the bottle is not broken.

As part of an experiment into the learning process, a mouse is put into a maze and rewarded with food every time it turns right. If the mouse turns right, the probability that it turns right the next time is increased by 20%. If the mouse turns left, the probability that it turns left the next time is decreased by 20%. Assume that there is an equal probability that the first turn will be to the left or to the right.

Let R_1 and L_1 represent the events "the first turn is to the right" and "the first turn is to the left" respectively.

a) What is meant by the event $R_2 | R_1$?

The mouse turns right the 2nd time after they turned right the first time

b) Determine, as an exact decimal, the probability of each event.

i) The first turn is to the right.

0.5

ii) The first turn is to the left.

0.5

iii) The second turn is to the right, given that the first turn is to the right.

0.5 + 20% of 0.5 = 0.5 + 0.1 = 0.6

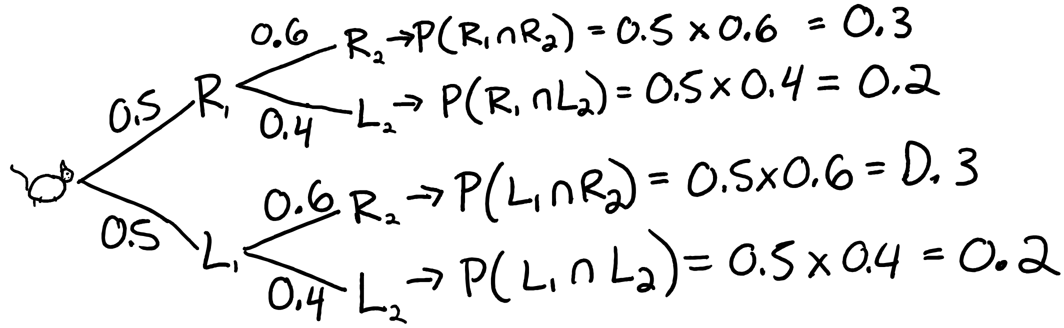
(20 ÷ 100 × 0.5)

iv) The second turn is to the left, given that the first turn is to the left.

0.5 − 20% of 0.5 = 0.5 − 0.1 = 0.4

c) Put the probabilities in b) onto a probability tree diagram.
Complete the diagram and use it to determine the probability that the first two turns are

i) both to the right **ii)** both to the left **iii)** different



i) 0.3 ii) 0.2 iii) 0.2 + 0.3 = 0.5

Complete Assignment Questions #2 - #11

#1-9 omit 1EG

Assignment

In this assignment, use a probability tree diagram or other means to determine the answers.

1. The table shows the distribution of blood types for students in the first year at a local college.

	O	*A*	*B*	*AB*	**Total**
Male, *M*	210	174	74	42	
Female, *F*	315	261	111	63	
Total					

a) Complete the totals in the chart.

b) How many students are in first year at the college?

c) Use the numbers in the table to determine, in simplest fraction form,

i) $P(O)$ **ii)** $P(B|F)$ **iii)** $P(F|B)$ **iv)** $P(F \cap B)$

d) If a student is selected at random, determine the probability, to four decimal places, that the student

i) is male **ii)** has blood type *A* **iii)** is male and has blood type *A*

e) Are the events "the student is male" and "the student has blood type *A*" independent events? Explain.

f) Identify the following events and determine the probability, as exact decimals, that

i) a female student has blood type *A* **ii)** a female student does not have blood type *O*

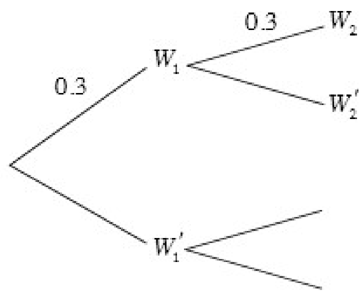
iii) a student with blood type *A* is male **iv)** a student with blood type *AB* is female.

g) Explain why the following statement is true.

"*P*(a student with blood type *A* is male) + *P*(a student with blood type *A* is female) = 1

but *P*(a male student has blood type *A*) + *P*(a female student has blood type *A*) ≠ 1."

2. Johann is planning a weekend trip. He is considering cancelling the trip if the weather is wet. The probability of a wet day is 0.3 and the weather on each day is independent of the weather on the other days.



a) Complete the tree diagram to determine the probability that
i) both days will be wet **ii)** there will be only 1 wet day

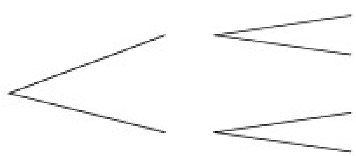
b) He decides to cancel the trip unless the odds of two dry days are better than evens. Should he cancel the trip? Explain.

3. The probability of being caught in a traffic jam in the morning rush hour is 0.6 on any particular week day. The numbers of traffic jams on different days are assumed to be independent of each other .

a) Determine the probability that

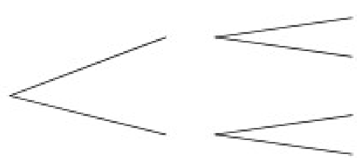
i) the journey is free from traffic jams for two consecutive week days

ii) there is a traffic jam on at least one out of two consecutive week days



b) Explain why the probabilities above add up to 1.

4. Two dice are rolled. Use a probability tree diagram to determine the probability that neither die shows a 6.

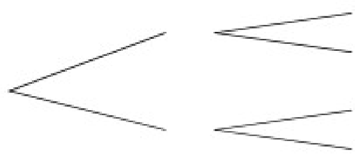


5. Two hockey players, Sid and Alex, each independently take a penalty shot.
The odds of Sid scoring are 7:3 in favour. The odds of Alex scoring are 2:3 against.

a) Determine the probability that **i)** Sid scores **ii)** Alex scores

b) Draw a probability tree diagram and determine the probability that

i) both score **ii)** both miss **iii)** only one scores **iv)** at least one scores



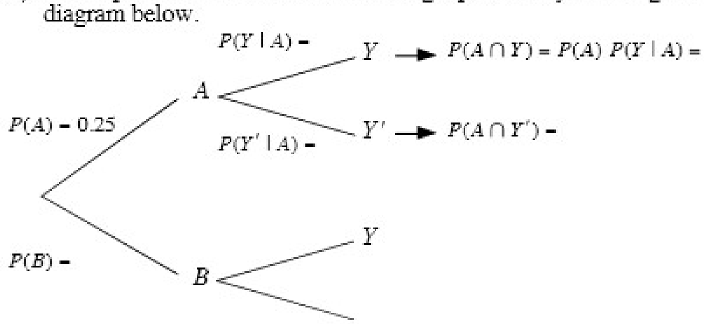
6. An experiment is performed in which there are two bags of marbles.
Bag *A* contains 5 yellow and 5 green marbles.
Bag *B* contains 7 yellow and 3 green marbles.

One of the bags is chosen by selecting one card at random from a deck of cards.
If a heart is selected, then a marble is taken from Bag *A*. Otherwise, a marble is taken from Bag *B*. Once the bag has been determined, a marble is chosen at random from that bag.

a) Determine the following:

$P(A) =$ $P(B) =$ $P(Y|A) =$ $P(Y|B) =$

b) The experiment can be modelled using a probability tree diagram. Complete the tree diagram below.



c) Which of the events at the end of the above tree diagram are subsets of the event "a yellow marble is chosen"?

d) Determine the probability that a yellow marble is chosen.

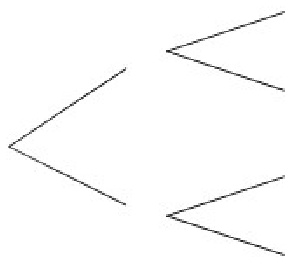
7. Consider the following two experiments:

Experiment 1: Two cards are drawn **with** replacement from a standard deck of 52 cards.

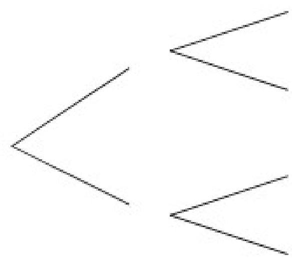
Experiment 2: Two cards are drawn **without** replacement from a standard deck of 52 cards.

In each case, complete the tree diagram and determine the probability that both cards are diamonds.

Experiment 1



Experiment 2



8. Pauline has heard on the news that there will be a comet passing near the Earth. The weatherman on the news reported that there is a 70% chance that the comet will be visible if the night sky is clear. If the night sky is not clear, there is only a 15% chance of being able to see the comet. He has also said that there is a 40% chance that the night sky will not be clear.

a) If V is the event "the comet is visible" and C is the event "the night sky is clear", express the following probabilities as reduced fractions.

i) $P(C)$ **ii)** $P(C')$ **iii)** $P(V|C)$ **iv)** $P(V|C')$

b) Using a tree diagram, determine the odds in favour of Pauline being able to see the comet.

9. A golfer is practicing putting. The probability that he holes the first putt is 0.4 .
Each time he holes a putt, the probability that he holes the next putt increases by 25%.
Each time he misses a putt, the probability that he misses the next putt increases by 20%.

a) Determine, as an exact decimal, the probability that

i) he holes the first putt **ii)** he misses the first putt

iii) he holes the second putt, given that he holes the first putt

iv) he misses the second putt, given that he holes the first putt

v) he misses the second putt, given that he misses the first putt

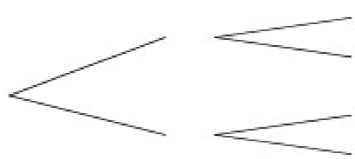
vi) he holes the second putt, given that he misses the first putt

b) Using a tree diagram, determine the probability that

i) he holes the first two putts

ii) he misses the first two putts

iii) he misses at least one of the first two putts



Use the following information to answer the next two questions.

Luigi has two faulty alarm clocks that he uses to wake him up each morning. The odds that the first alarm will go off are 3:1 in favour and the odds that the second alarm will go off are 3:17 against.

Multiple Choice **10.** The probability that both alarms go off is

A. $\frac{9}{80}$ **B.** $\frac{2}{5}$

C. $\frac{51}{80}$ **D.** $\frac{17}{20}$

Numerical Response **11.** The probability, to the nearest hundredth, that he is awakened by the alarm(s) is _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

i) P(A)=

1. b) 1250 **c) i)** $\frac{21}{50}$ **ii)** $\frac{37}{250}$ **iii)** $\frac{3}{5}$ **iv)** $\frac{111}{1250}$

d) i) 0.4000 **ii)** 0.3480 **iii)** 0.1392 **e)** yes, since $P(M \cap A) = P(M) \cdot P(A)$

f) i) $P(A \mid F) = 0.348$ **ii)** $P(O' \mid F) = 0.58$ **iii)** $P(M \mid A) = 0.4$ **iv)** $P(F \mid AB) = 0.6$

g) If a student has blood type A, then the student is either male or female. The events "a student with blood type A is male" and "a student with blood type A is female" are complementary events whose probabilities sum to 1. The events "a male student has blood type A" and "a female student has blood type A" are not complementary events.

2. a) i) 0.09 **ii)** 0.42
b) P(two dry days) = 0.49. Since this is not greater than 0.5, he should cancel.

3. a) i) 0.16 **ii)** 0.84 **b)** They are complementary events. **4.** $\frac{25}{36}$

5. a) i) 0.7 **ii)** 0.6 **b) i)** 0.42 **ii)** 0.12 **iii)** 0.46 **iv)** 0.88

6. a) $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{4}$, $P(Y \mid A) = \frac{1}{2}$, $P(Y \mid B) = \frac{7}{10}$ **c)** $A \cap Y$, $B \cap Y$ **d)** 0.65

7. Experiment 1 is $\frac{1}{16}$. Experiment 2 is $\frac{1}{17}$.

8. a) i) $\frac{3}{5}$ **ii)** $\frac{2}{5}$ **iii)** $\frac{7}{10}$ **iv)** $\frac{3}{20}$ **b)** 12:13

9. a) i) 0.4 **ii)** 0.6 **iii)** 0.5 **iv)** 0.5 **v)** 0.72 **vi)** 0.28
b) i) 0.2 **ii)** 0.432 **iii)** 0.8

10. C **11.**

0	.	9	6