Lesson 7: Problem Solving with Permutations and Combinations

Permutations and Combinations Lesson #7:

Problem Solving with Permutations and Combinations

Review

Recall the following formulas from earlier lessons.

- Fundamental counting principle, number of ways = $a \times b \times c \times \ldots$ $\underline{\quad}\cdot\underline{\quad}\cdot\underline{\quad}$
- Factorial notation, $n!$ = $n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$, where $n \in W$
- Number of permutations of r from n, ${}_nP_r = \dfrac{n!}{(n-r)!}$ } order matters!
- Permutations with repetitions, number of ways = $\dfrac{n!}{a!\,b!\,c!}$ } PARALLEL $\dfrac{8!}{3!2!}$
- Number of combinations of r from n, ${}_nC_r = \binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

↳ order does not matter "n choose r"

In problem solving, it is essential to determine whether order is important or not.

How many arrangements of the word **POPPIES** can be made under each of the following conditions?

a) without restrictions

P with repetition

$\dfrac{7!}{3!} = 840$

b) if each arrangement begins with a **P**

P|_____ $\dfrac{6!}{2!} = 360$

c) if the first two letters are **P**

PP|⑤④③②①

$5! = 120$

d) if all the **P**'s are to be together

PPP, O, I, E, S

$5! = 120$

e) if the first letter is **P** and the next one is not **P**

P|④|5 4 3 2 1

$4 \cdot \dfrac{5!}{2!} = 240$

P repeated

A class consists of 5 girls and 7 boys. A committee is to be formed consisting of 2 girls and 3 boys. Determine the number of ways a teacher can choose the committee if

*does order matter? NO! the nCr

a) there are no further restrictions

$5C_2 \cdot 7C_3 = 10 \cdot 35$

girls boys

$= 350$

b) Johnny, the Principal's son, has to be on the committee ← one less boy.

$5C_2 \cdot 6C_2$

$= 10 \cdot 15$ $= 150$

c) the twins, Peter and Paul, cannot both be on the committee

Peter only or Paul only or neither | all possible − #(both Peter+Paul)

$5C_2 \cdot 6C_2 + 5C_2 \cdot 6C_2 + 5C_2 \cdot 5C_3$ | $350 - 5C_2 \cdot 5C_1 =$

$100 + 100 + 100$ | $350 - 10 \cdot 5 =$

$= 300$ | $350 - 50 = 300$

Use the following information to answer the next class example.

David, Steven, and Helen were trying to answer the following homework question.

> " The students in a school band have practiced 5 popular and 6 classical music compositions. For the school concert they will choose a program consisting of 3 popular and 2 classical music compositions. If the order of the compositions matters, determine the number of different programs which can be presented."

The students' answers are shown below.

David: ${}_{11}P_5$ Steven: ${}_5P_3 \times {}_6P_2$ Helen: ${}_5C_3 \times {}_6C_2$

Each student was convinced their answer was correct, and asked their teacher to check their work. The teacher asked the students to write their answers on the board and asked the class to discuss the merits of each answer.

a) If possible, discuss the merits of each answer with other students and indicate errors in any of David's, Steven's, or Helen's reasoning.

* David mixed all the songs together instead of 3 popular & 2 classical
* Steven popular then classic, each type plays consecutively.
* Helen correctly picks the # of selections possible but did not arrange them

b) The teacher indicated that all three students had given an incorrect answer. Determine the correct solution to the problem.

$5C_3 \cdot 6C_2 \cdot 5! = 18000$ programs

select popular songs; select classic; arrange them in order

Complete Assignment Questions #1 - #17

#1-5, 7 → Quiz Friday

Assignment

1. A basketball squad of 11 players is to be chosen from 17 available players. In how many ways can this be done if:

a) Jeff and Brent must be selected?

b) Jeff and Brent cannot both be selected?

2. Fifteen rugby players line up for a team picture, with seven players in the front row and eight players in the back row. Expressing your answer in factorial notation, determine the number of arrangements of the fifteen players

a) without restrictions

b) if the captain is in the middle of the front row

3. Determine the number of arrangements of the letters of the word **TATTOO** which

a) begin with a **T**

b) begin with two **T**'s

c) begin with three **T**'s

d) have all the **T**'s together

4. Ten students have been elected to serve on students' council.

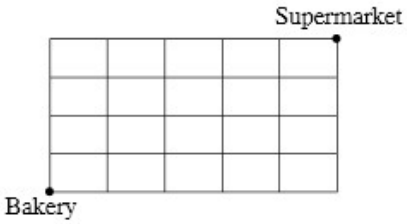
a) In how many ways can 4 of these students be chosen to represent the school as ambassadors at the provincial education conference?

b) In how many ways can a president, vice-president, secretary, and treasurer be chosen from the ten students?

c) Six of the ten students elected to students' council are girls. In how many different orders can four of the students line up for a photograph if there must be an equal number of boys and girls in the photograph?

5. The bakery is four blocks south and five blocks west of the supermarket. The bakery driver, bored with travelling the same route, decides to use a different route for each delivery.

Assuming that he always travels closer to the supermarket, how many deliveries are possible before he has to repeat a route?



6. A town has 6 streets running from north to south and 4 avenues running from west to east. A man wishes to drive from the extreme south-west intersection to the extreme north-east intersection, moving either north or east along one of the streets or avenues.

Tania used the expression $\frac{10!}{6!4!}$ to determine the number of different ways to drive from the extreme south-west intersection to the extreme north-east intersection.
Draw a diagram to illustrate the situation and explain why Tania was incorrect in her reasoning.

7. How many arrangements are there of the letters of the word **MONOTONOUS** under each condition?

a) without restrictions

b) if each arrangement begins with a **T**

c) if each arrangement begins with an **O**

d) if the four **O**'s are to be together

Use the following information to answer the next question.

A sports store has jerseys representing the seven Canadian NHL teams and the eight Canadian CFL teams. Five of these jerseys have to be chosen for display in a store window. The store owner decides to choose three NHL and two CFL jerseys. These jerseys will be arranged in a row in the store window.

Multiple Choice **8.** The number of displays that can be made by choosing the jerseys and then arranging them in the window is

A. 4900

B. 11 760

C. 117 600

D. 1 411 200

9. A researcher has collected data on families with 3 children, families with 4 children, and families with 5 children. Each family is given a code, reflecting the number, gender, and birth order of the children. For example, the code MMMF is given to a family with 4 children where the first 3 children born are boys and the youngest child is a girl. How many different codes are required in this study?

A. 32

B. 56

C. 150

D. 4096

10. The number of arrangements of the letters of the word **PARALLEL** in which all the **L**'s are together at the end of the arrangement is

A. 60

B. 120

C. 180

D. 360

Use the following information to answer the next question.

A student enrolled in a General Studies program at a particular university must take four courses in the first semester.
The student must take English, either Mathematics or Psychology, and either two courses from Group A or two courses from Group B below.

Group A	Group B
French	Biology
Spanish	Chemistry
German	Computing
History	Geology
Geography	

11. The number of four-course programs available to the student is

A. ${}_2C_1 \times ({}_5C_2 \times {}_4C_2)$

B. ${}_2C_1 \times ({}_5C_2 + {}_4C_2)$

C. ${}_2P_1 \times ({}_5P_2 \times {}_4P_2)$

D. ${}_2P_1 \times ({}_5P_2 + {}_4P_2)$

12. Customer service representatives at a men's designer store must wear company attire. They have a choice of three different shirts, four different ties, and two different pairs of pants. The number of different outfits that a customer service representative could create using these items can be represented by the expression

A. ${}_9C_3$

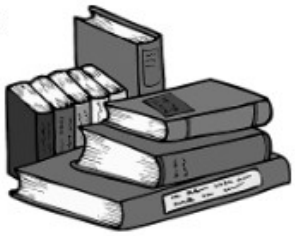
B. ${}_9P_3$

C. $\frac{9!}{3!\,4!\,2!}$

D. $3 \times 4 \times 2$

Use the following information to answer the next two questions.

On the floor of her study, a student has 5 different English books, 2 different Science books, and 2 different Mathematics books.



13. The number of ways in which three of these books can be arranged on a bookshelf is

A. ${}_5P_1 \times {}_2P_1 \times {}_2P_1$

B. ${}_9P_3$

C. ${}_9C_3$

D. 3!

Numerical Response

14. The number of ways of arranging two English books, two Science books, and a Mathematics book on a bookshelf is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

15. The number of arrangements of the letters of the word **STUDENT** in which the two **T**'s are not adjacent is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

16. 35 different quadrilaterals can be formed by connecting points on the circumference of a circle. The number of points on the circle is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

17. A coach must choose the 5 starters for a basketball team from 6 males and 5 females. If there must be at least two of each gender in the starting line-up, the number of different groups of players that can be chosen is _____ .

(Record your answer in the numerical response box from left to right.) | | | | |

Answer Key

1. **a)** 5005 **b)** 7371

2. **a)** 15! **b)** 14!

3. **a)** 30 **b)** 12 **c)** 3 **d)** 12

4. **a)** 210 **b)** 5040 **c)** 2160

5. 126

6. He has to drive 5 blocks east and 3 blocks north in any order. So the answer is $\frac{8!}{5!3!} = 56$.

7. **a)** 75 600 **b)** 7560 **c)** 30 240 **d)** 2520

8. C **9.** B **10.** A **11.** B **12.** D **13.** B

14. | 2 | 4 | 0 | 0 |

15. | 1 | 8 | 0 | 0 |

16. | 7 | | | |

17. | 3 | 5 | 0 | |