7.5

Quadratic Functions Lesson #5:
Applications of Quadratic Functions

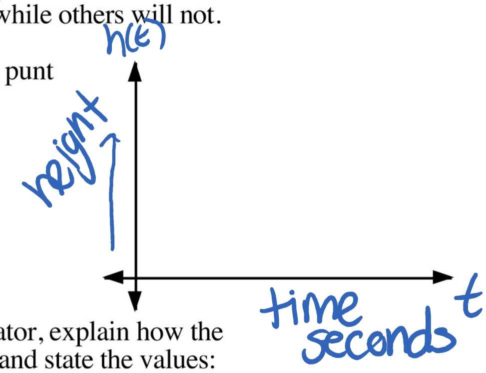
In this lesson we explore practical applications of quadratic functions.
Some questions will require a graphing calculator, while others will not.

During a high school football game, the height of a punt can be modelled as a quadratic function of time as

$$h(t) = -5(t - 1.5)^2 + 12.25$$

(1.5, 12.25)

where
t is the number of seconds which have elapsed since the football was punted, and
$h(t)$ is the number of metres above the ground after t seconds.



a) Without using the graphing features of a calculator, explain how the following can be determined from the equation and state the values:

vertex (h, k)

i) the maximum height of the football

k

k = 12.25 so max height is 12.25m

ii) the time it takes for the football to reach its maximum height

h

1.5 seconds

iii) the height of the football above the ground as the punter makes contact with it

$t=0$

$h(0) = -5(0-1.5)^2 + 12.25 = 1\text{m}$

iv) the height of the football above the ground 0.5 seconds after contact

$t=0.5$

$h(0.5) = -5(0.5-1.5)^2 + 12.25 = 7.25\text{m}$

b) Use the features of a graphing calculator to

i) sketch the graph of $h(t) = -5(t - 1.5)^2 + 12.25$, and verify the answers to a).

ii) determine after how many seconds, to the nearest tenth, the football is 10 metres above the ground.

0.8 , 2.2

c) The punt is not caught by the opposing team, and the football hits the ground. Use the features of a graphing calculator to determine how many seconds, to the nearest hundredth, it took for the football to hit the ground.

It took 3.07 seconds for the football to hit the ground.

Complete Assignment Questions #1 - #8

#1-4

Assignment

1. An aerial flare is a type of pyrotechnic which, when fired into the air, produces a brilliant light without causing fire or an explosion.
A stranded camper fires a flare to signal his location to the other campers in his group.
The flare follows a path defined by the formula $h(t) = -4.9(t - 4.3)^2 + 92$, where $h(t)$ represents the height of the flare above the ground after t seconds.

a) Without using the graphing features of a calculator, determine

i) the maximum height of the flare

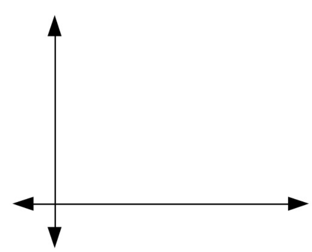
ii) the time it takes for the flare to reach its maximum height

iii) the height of the flare above the ground 6.3 seconds after it is fired

iv) the height of the flare above the ground, to the nearest tenth of a metre, at its firing point.

b) Use the features of a graphing calculator to

i) sketch the graph on the grid and verify the answers in a)

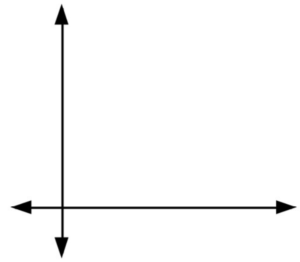


ii) determine how long, to the nearest tenth of a second, it took for the flare to hit the ground

iii) determine after how many seconds, to the nearest tenth of a second, the height of the flare was 45 metres above the ground.

2. The height, h, in metres above the ground, of a projectile at any time, t, in seconds, after the launch is defined by the function $h(t) = -4t^2 + 48t + 3$.

Use a graphing calculator to answer the following:



a) Sketch the relevant part of the parabola on the grid.

b) Find the height of the projectile 3 seconds after the launch.

c) Find the maximum height reached by the projectile.

d) How many seconds after the launch is the maximum height reached?

e) What was the height of the projectile at the launch?

f) Determine when the projectile hit the ground to the nearest tenth of a second.

3. A stone is thrown vertically upward at a speed of 22 m/s. Its height, h metres, after t seconds, is given approximately by the function $h(t) = 22t - 5t^2$.

a) Write an appropriate window for graphing the function on a graphing calculator.

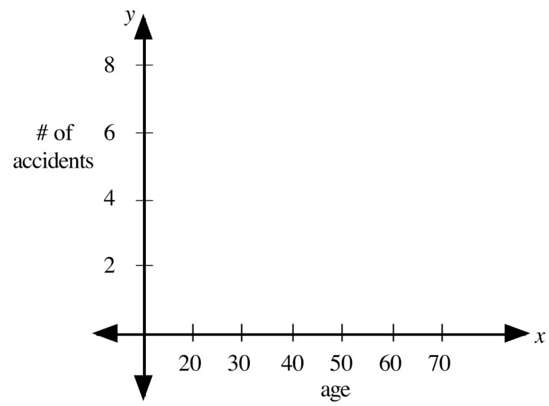
b) Determine the maximum height of the stone.

c) Calculate, to the nearest tenth of a second, when the stone is 15 metres up and explain the double answer.

4. The cost of car insurance depends on many factors, one of which is the age of the driver. Insurance companies know that younger drivers under the age of 25 and older drivers over the age of 70 are statistically more likely to have accidents than drivers between the ages of 25 and 70. The following data shows the number of accidents, per million kilometres driven, by drivers of a particular age.

Age (x)	18	30	45	60	75
Number of Accidents (y)	5.2	3.1	2.2	2.8	4.7

a) If x represents the age of drivers and y represents the number of accidents per million kilometres driven, plot the data on a Cartesian plane, and join the points with a smooth curve.



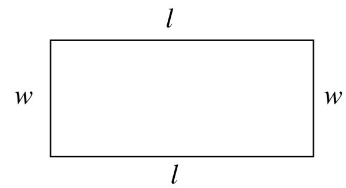
b) The data looks like it could be modelled by a quadratic function with equation $y = ax^2 + bx + c$. Using the technique of quadratic regression (which is taught in a higher level math course), a teacher determines that the equation which best models the data is $y = 0.0034x^2 - 0.3232x + 9.8505$.

Use the above model to determine what age, to the nearest year, results in the lowest number of accidents per million kilometres.

c) Determine the lowest number of accidents per million kilometres. Answer to the nearest tenth.

d) Based on this model, who is more likely to have an accident - a 17 year old student or a 78 year old senior?

5. A rancher has 500 metres of fencing with which to enclose a rectangular corral.



a) If he uses l metres of fencing for the length of the rectangle, and w metres of fencing for the width of the rectangle, explain why

i) $2l + 2w = 500$

ii) $w = 250 - l$

iii) area, $A = 250l - l^2$

b) As shown above, the area of the rectangle can be expressed using the equation $A = 250l - l^2$. Determine the length and width of the rectangle which will result in the maximum area.

Use the following information to answer questions #6, 7, and #8.

Researchers predict that the world population will peak sometime during the 21st century before starting to decline. In January 2010, the world population was approximately 6 900 000 000 (or 6.9 billion).

The following model has been suggested as an approximate relationship (up to the year 2100), between the number of years, x, since the year 2010 and the world population, y.

The equation of the relationship is $y = -595\,000x^2 + 71\,100\,000x + 6\,900\,000\,000$.

Numerical Response

6. How many years after the year 2010 will it take for the world population to reach a maximum value? Answer to the nearest tenth of a year.

(Record your answer in the numerical response box from left to right.)

7. The world population is expected to peak in the year _____ .

(Record your answer in the numerical response box from left to right.)

8. The maximum population, to the nearest tenth of a billion, is expected to be _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) i) 92 metres **ii)** 4.3 seconds **iii)** 72.4 metres **iv)** 1.4 metres
b) ii) 8.6 seconds **iii)** 1.2 seconds and 7.4 seconds

2. b) 111 metres **c)** 147 metres **d)** 6 seconds **e)** 3 metres **f)** 12.1 seconds

3. a) x:[–1, 5, 1] y:[–5, 30, 5] **b)** 24.2 metres
c) 0.8 s and 3.6 s. The stone travels up 15 m above the ground, and then as it falls it also reaches 15 m above the ground.

4. b) 48 years **c)** 2.2 accidents per million km **d)** both are about equally likely (17 year old is slightly more likely).

5. a) i) 500 metres is the perimeter of the rectangle, so $2l + 2w = 500$.
ii) If $2l + 2w = 500$, then $2w = 500 - 2l$ and $w = 250 - l$.
iii) Area, $A = lw = l(250 - l) = 250l - l^2$.
b) Length and width both equal 125 metres.

6.

5	9	.	7

7.

2	0	6	9

8.

9	.	0	