

7.4

Quadratic Functions Lesson #4: Analyzing the Graph of $y = a(x - m)(x - n)$

Recall that a quadratic function may be written in two different forms.

General Form: $f(x) = ax^2 + bx + c$, where $a \neq 0$, or $y = ax^2 + bx + c$, where $a \neq 0$.

Standard Form: $f(x) = a(x - h)^2 + k$, where $a \neq 0$, or $y = a(x - h)^2 + k$, where $a \neq 0$.
(or **Vertex Form**)

In this lesson, we introduce a third form, called **Factored Form**.

Factored Form: $f(x) = a(x - m)(x - n)$, where $a \neq 0$, or $y = a(x - m)(x - n)$, where $a \neq 0$.

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

Investigation #1 Investigating the Factored Form $y = (x - m)(x - n)$

Consider the following quadratic functions written in factored form $y = (x - m)(x - n)$.

a) Using a graphing calculator, sketch the graphs of the functions and complete the table.

Function $y = (x - m)(x - n)$	m	n	x -intercepts of Graph	y -intercept of Graph	Axis of Symmetry
$y = (x - 2)(x - 8)$	2	8	2, 8	$m \cdot n = 16$	$x = 5$
$y = (x + 2)(x - 6)$	-2	6	-2, 6	-12	$x = 2$
$y = (x + 2)(x + 6)$	-2	-6	-2, -6	12	$x = -4$
$y = x(x - 4)$	0	4	0, 4	0	$x = 2$
$y = (x + 8)(x - 10)$					

$$\rightarrow \frac{2+8}{2} = \frac{m+n}{2}$$

b) What do you notice?

x -ints are $m + n$
 y -ints are $m \cdot n$
 axis of sym $\frac{m+n}{2}$

* axis of sym is
 x value of vertex

Summary of Investigation #1

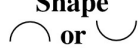
The following observations about the graph of $y = (x - m)(x - n)$ can be made from the previous investigation:

- The x -intercepts are m and n .
- The y -intercept is mn .
- The equation of the axis of symmetry is $x = \frac{m + n}{2}$.

Investigation #2 Investigating the Factored Form $y = a(x - m)(x - n)$

Consider the following quadratic functions written in factored form $y = a(x - m)(x - n)$.

a) Using a graphing calculator, sketch the graphs of the functions and complete the table.

Function $y = \underline{\hspace{1cm}}(x - m)(x - n)$	a	m	n	x -intercepts of Graph	y -intercept of Graph	Shape or 
$y = \underline{2}(x - 5)(x - 2)$	2	5	2	5, 2	20	U
$y = 4(x + 3)(x - 2)$						
→ $y = 5(x - 10)(x + 3)$	5	10	-3	10, -3	-150	U
$y = -3x(x + 2)$	-3	0	-2	0, -2	0	n
$y = -10(x + 1)(x - 3)$						
$y = -(x - 10)(x + 10)$						

b) Complete the following:

- The x -intercepts are m and n .
- The y -intercept is $a \cdot m \cdot n$
- If a is positive, the graph opens up and has a shape like U.
- If a is negative, the graph opens down and has a shape like n.

Features of the Graph of $y = a(x - m)(x - n)$

The following observations about the graph of $y = a(x - m)(x - n)$ can be made from the previous investigations:

- The x -intercepts are m and n .
- The y -intercept is amn .
- The equation of the axis of symmetry is $x = \frac{m + n}{2}$.
- If a is positive, the graph opens up and has a minimum point.
- If a is negative, the graph opens down and has a maximum point.



Class Ex. #1

Without graphing, state the x -intercepts, the y -intercept, the equation of the axis of symmetry, and the direction of opening of the graphs of the following functions.

a) $f(x) = (x + 4)(x - 10)$ b) $g(x) = -2(x + 6)(x + 9)$ c) $h(x) = -x(x - 20)$

x -int: $-4, 10$
 y -int: -40
 axis of sym: $\frac{-4+10}{2} = 3$
 $x = 3$ UP \uparrow

x -int: $-6, -9$
 y -int: -108
 axis of sym: $\frac{-6+(-9)}{2}$
 $x = -7.5$
 down

x -int: $0, 20$
 y -int: 0
 axis of sym: $x = 10$
 down

A Disguised "a" Value

Graph $y_1 = 2(x - 3)(x - 7)$ and $y_2 = -2(3 - x)(x - 7)$ on your graphing calculator.

What do you notice about the graphs?

Explain why the graph of $y = -2(3 - x)(x - 7)$ opens up when it appears that the "a" value is negative.



Class Ex. #2

Write each function in the form $y = a(x - m)(x - n)$ and state the x - and y -intercepts of the graph of the function.

a) $y = (1 - x)(x - 10)$

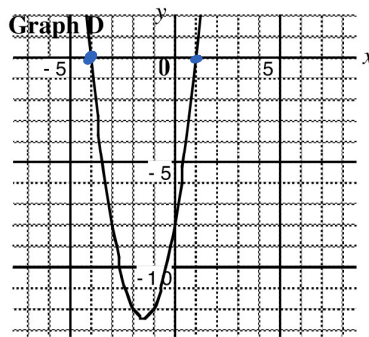
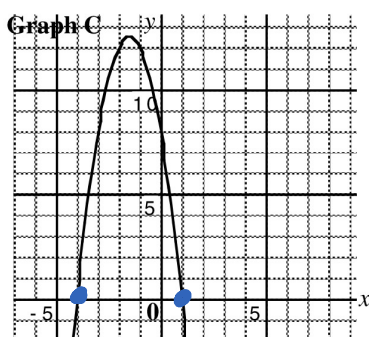
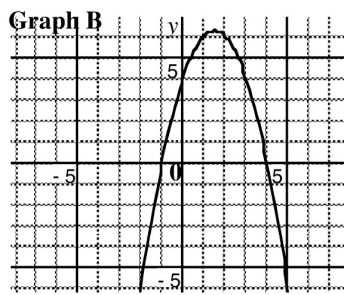
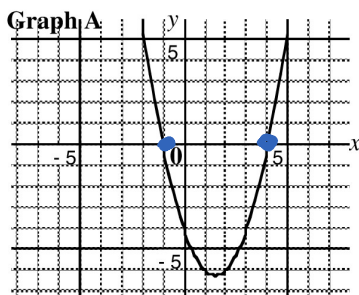
b) $y = -3(4 - x)(12 - x)$

$-x + 1$
 $-(x - 1)(x - 10)$

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Each of the graphs has an equation of the form $y = a(x - m)(x - n)$ where a , m , and n are integers. Match each equation with its corresponding graph.



Equation 1 :

$y = 2(x - 1)(x + 4)$

graph D

Equation 2 :

$y = (1 + x)(4 - x)$

Graph B

Equation 3 :

$y = (x + 1)(x - 4)$

graph A

Equation 4 :

$y = -2(x - 1)(x + 4)$

Graph C

Complete Assignment Questions #1 - #8

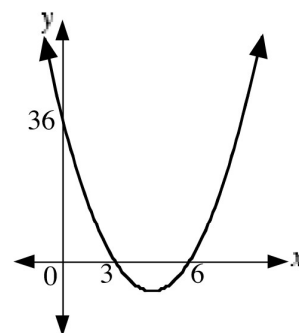
Determining the Equation from the Graph

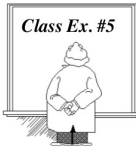
#1,3,4,5
Skill quizzes Tues.



The graph represents a quadratic function with equation $y = a(x - m)(x - n)$. The x - and y -intercepts are shown on the graph.

- State the values of m and n .
- Explain how to determine the value of a .
- Write the equation of the graph in the form $y = a(x - m)(x - n)$.
- Determine the coordinates of the vertex.
- Write the equation of the graph in standard form $y = a(x - h)^2 + k$.





Determine the coordinates of the vertex of the graph of a quadratic function if the graph passes through the points $(-8, 0)$, $(2, 0)$, and $(0, 48)$.

Complete Assignment Question #9

Assignment

1. Without graphing the functions, complete the table below.

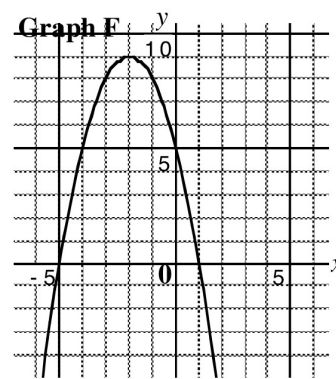
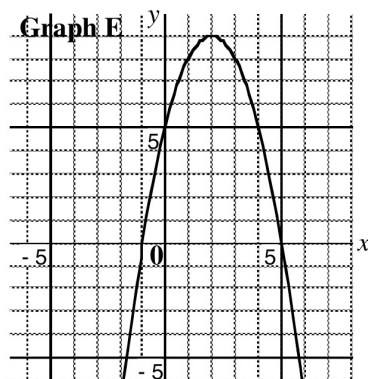
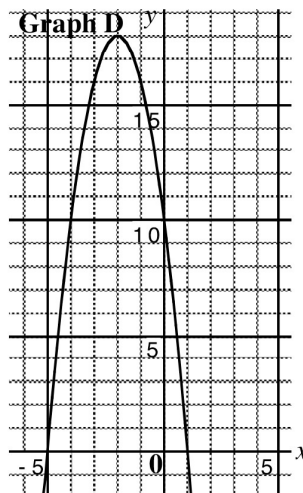
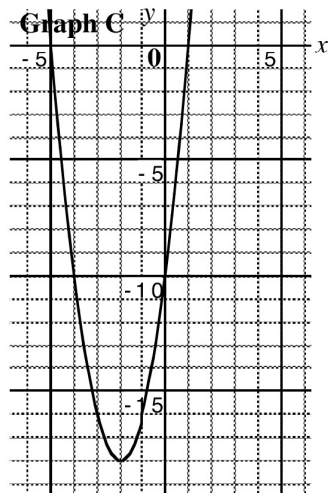
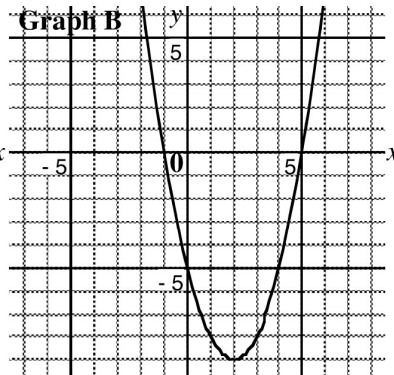
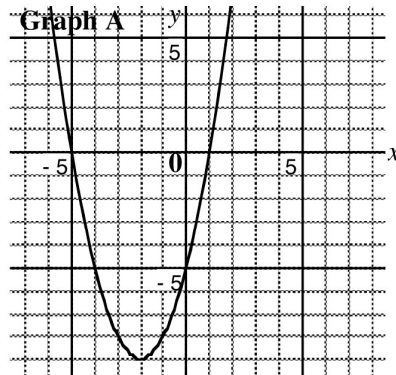
Function $y = (x - m)(x - n)$	x -intercepts of Graph	y -intercept of Graph	Axis of Symmetry	Opening Up or Down
$y = (x - 6)(x + 2)$				
$y = 5(x + 4)(x - 6)$				
$y = -2(x + 10)(x + 4)$				
$y = -3(x - 7)(x - 2)$				
$y = -x(x + 9)$				
$y = (x + 5)(x + 5)$				

2. Without graphing the functions, complete the table below.

Function $y = (x - m)(x - n)$	x -intercepts of Graph	y -intercept of Graph	Axis of Symmetry	Opening Up or Down
$y = (2 - x)(x - 2)$				
$y = 3(3 - x)(6 + x)$				
$y = 2(4 - x)(3 - x)$				
$y = -3(5 + x)(15 + x)$				

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3. Each of the graphs has an equation of the form $y = a(x - m)(x - n)$ where a , m , and n are integers. Match each equation with its corresponding graph.



Equation 1:

$$y = (x - 5)(x + 1)$$

Equation 2:

$$y = -(x - 5)(x + 1)$$

Equation 3:

$$y = (x + 5)(x - 1)$$

Equation 4:

$$y = -(x + 5)(x - 1)$$

Equation 5:

$$y = (5 - x)(x + 1)$$

Equation 6:

$$y = 2(x + 5)(x - 1)$$

Equation 7:

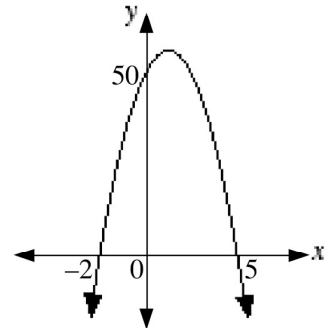
$$y = -2(x + 5)(x - 1)$$

Equation 8:

$$y = 2(5 + x)(1 - x)$$

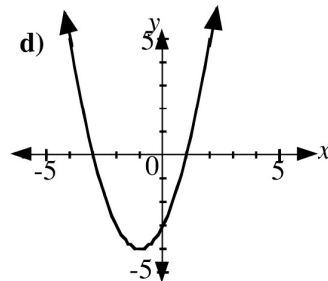
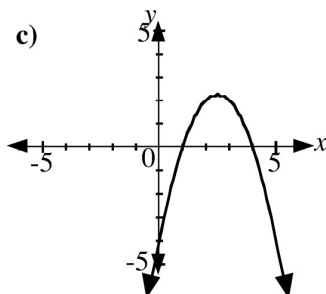
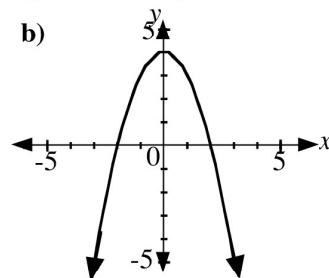
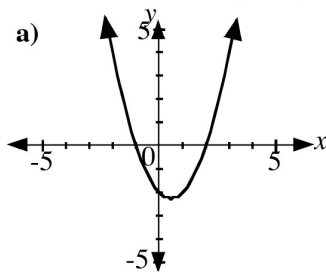
4. The graph represents a quadratic function with equation $y = a(x - m)(x - n)$. The x - and y -intercepts are shown on the graph.

- State the values of m and n .
- Determine the value of a .
- Write the equation of the graph in the form $y = a(x - m)(x - n)$.
- Determine the coordinates of the vertex.
- Write the equation of the graph in standard form $y = a(x - h)^2 + k$.



5. In each case, determine the equation of the quadratic function whose graphs are shown. Note that all intercepts are integers.

Answer in the form $y = a(x - m)(x - n)$ and in general form $y = ax^2 + bx + c$.



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6. Determine the equation of the graph of a quadratic function which opens up and has a vertex on the x -axis. Give your answer in the form $y = a(x - m)(x - n)$, in standard form $y = a(x - h)^2 + k$, and in general form $y = ax^2 + bx + c$.

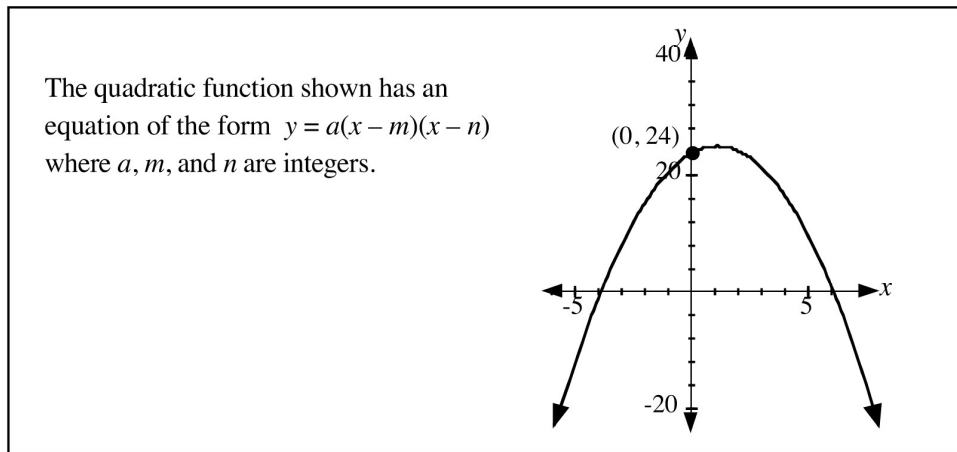
7. The graph of a quadratic function has x -intercepts of -7 and -1 , and passes through the point $(-4, 36)$.

Determine the equation of the quadratic function in the form $f(x) = a(x - m)(x - n)$.

- Multiple Choice** 8. The graph of a quadratic function has x -intercepts of -7 and 3 , and a y -intercept of 84 . The coordinates of the vertex of the graph are

- A. $(2, -36)$
- B. $(-2, 25)$
- C. $(-2, -100)$
- D. $(-2, 100)$

Use the following information to answer the next question.



Numerical Response 9. The value of $a + m + n$ is _____ .

(Record your answer in the numerical response box from left to right.)

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Answer Key

1. See chart below.

Function $y = (x - m)(x - n)$	x-intercepts of Graph	y-intercept of Graph	Axis of Symmetry	Opening Up or Down
$y = (x - 6)(x + 2)$	6 and -2	-12	$x = 2$	up
$y = 5(x + 4)(x - 6)$	-4 and 6	-120	$x = 1$	up
$y = -2(x + 10)(x + 4)$	-10 and -4	-80	$x = -7$	down
$y = -3(x - 7)(x - 2)$	7 and 2	-42	$x = 4.5$	down
$y = -x(x + 9)$	0 and -9	0	$x = -4.5$	down
$y = (x + 5)(x + 5)$	-5	25	$x = -5$	up

2. See chart below.

Function $y = (x - m)(x - n)$	x-intercepts of Graph	y-intercept of Graph	Axis of Symmetry	Opening Up or Down
$y = (2 - x)(x - 2)$	2	-4	$x = 2$	down
$y = 3(3 - x)(6 + x)$	3 and -6	54	$x = -1.5$	down
$y = 2(4 - x)(3 - x)$	4 and 3	24	$x = 3.5$	up
$y = -3(5 + x)(15 + x)$	-5 and -15	-225	$x = -10$	down

3. Equation 1: Graph B Equation 2: Graph E Equation 3: Graph A Equation 4: Graph F
Equation 5: Graph E Equation 6: Graph C Equation 7: Graph D Equation 8: Graph D

4. a) $m = -2$ and $n = 5$ OR $m = 5$ and $n = -2$ b) -5
c) $y = -5(x + 2)(x - 5)$ OR $y = -5(x - 5)(x + 2)$ d) vertex (1.5, 61.25)
e) $y = -5(x - 1.5)^2 + 61.25$

5. a) $y = (x + 1)(x - 2)$, $y = x^2 - x - 2$ b) $y = -(x + 2)(x - 2)$, $y = -x^2 + 4$
c) $y = -(x - 1)(x - 4)$, $y = -x^2 + 5x - 4$ d) $y = (x + 3)(x - 1)$, $y = x^2 + 2x - 3$

6. A possible answer is $y = (x - 8)(x - 8)$, i.e. m and n must be equal.
 $y = (x - 8)^2$,
 $y = x^2 - 16x + 64$

7. $f(x) = -4(x + 7)(x + 1)$

8. D

9.

1			
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