7.3

Quadratic Functions Lesson #3:
Analyzing the Graph of $y = a(x - h)^2 + k$

Recall the following from the previous lesson.

- A **quadratic function** is a function of degree 2 which can be written in the form

  $f(x) = ax^2 + bx + c$ **or** $y = ax^2 + bx + c$, where $a, b, c \in R$, and $a \neq 0$.
- The graph of a quadratic function is a **parabola**.
- A parabola has a **maximum point** (opens down), if $a < 0$.
- A parabola has a **minimum point** (opens up) if $a > 0$.
- The **vertex** of a parabola is the **maximum <u>or</u> minimum point**.
- The maximum or minimum **value** of a quadratic function is the ***y*-coordinate** of the vertex.

the x-coordinate of the vertex is the axis of symmetry

Investigation

<u>Part 1</u>

Consider the functions $f(x) = x^2 - 6x + 11$ and $g(x) = (x-3)^2 + 2$.

a) Graph these functions on your calculator and comment on your observations.

they're the same

b) Expand $g(x)$. What do you notice?

$= (x-3)^2 + 2$

$= (x-3)(x-3) + 2$

$= x^2 - 3x - 3x + 9 + 2$

$= x^2 - 6x + 11$ $= f(x)$

$g(x) = f(x)$

<u>Part 2</u>

Consider the functions $y = -3x^2 - 30x - 77$ and $y = -3(x+5)^2 - 2$.

a) Graph these functions on your calculator and comment on your observations.

b) Show that both equations represent the same function.

General Form and Standard Form of a Quadratic Function

A quadratic function may be written in two different forms.

General Form: $f(x) = ax^2 + bx + c$, where $a \neq 0$, or $y = ax^2 + bx + c$, where $a \neq 0$.

Standard Form: $f(x) = a(x - h)^2 + k$, where $a \neq 0$, or $y = a(x - h)^2 + k$, where $a \neq 0$.

Standard form is sometimes called **vertex form** because, as we shall see, it is easy to determine the vertex of the graph from the equation.

Exploring the Standard Form $y = a(x - h)^2 + k$

a) In the table, complete the columns for the parameters a, h, and k.

b) Use a graphing calculator to determine the vertex of the graph of each function.

c) Write down your observations.

- the vertex is (h, k)
- h inside the bracket is the opposite sign
- k outside the bracket keeps it's sign

Function $y = a(x - h)^2 + k$	a	h	k	Vertex	
$y = x^2$	1	0	0	(0,0)	V
$y = (x - 2)^2$	1	2	0	(2,0)	V
$y = (x + 4)^2$ (−(−4))	1	−4	0	(−4,0)	V
$y = (x - 2)^2 - 3$	1	2	−3	(2,−3)	V
$y = (x + 4)^2 + 5$	1	−4	5	(−4,5)	V
$y = 6(x - 2)^2 - 3$	6	2	−3	(2,−3)	V
$y = -(x + 4)^2 + 5$	−1	−4	5	(−4,5)	∩

d) • In the table below, make up your own functions. Choose two functions with $a > 0$ and two functions with $a < 0$. Complete the table without using a graphing calculator.
• Verify your answers by using a graphing calculator.

Function $y = a(x - h)^2 + k$	a	h	k	Vertex	Axis of Symmetry	Direction of Opening
$y=x^2+5$	1	0	5	(0,5)	x=0	up
$y=-2(x+1)^2$	−2	−1	0	(−1,0)	x=−1	down
$y=(x-5)^2+5$	1	5	5	(5,5)	x=5	up
$y=-(x+3)^2+1$	−1	−3	1	(−3,1)	x=−3	down

* negative a value, opens down

e) Without the aid of a graphing calculator, write the equation of a quadratic function in standard form with the following characteristics:

i) Vertex in quadrant 1, opening up
$y = (x-1)^2 + 1$

ii) Vertex in quadrant 4, opening down
$y = -(x-1)^2 - 5$

iii) Vertex on the x-axis, opening up

iv) Vertex on the y-axis, opening down

Verify your equations using a graphing calculator.

(h,k) a (+) − h (+) − k (+) −
a + (−) h (+) − k + (−) $-(x-1)^2-5$

Quadrant
−+ 2 | 1 ++
−− 3 | 4 +−

Class Ex. #1

Consider the graph of the quadratic function $f(x) = 2(x + 6)^2 - 9$.

a) State the coordinates of the vertex of the graph. (h,k)
(−6,−9)

b) State the equation of the axis of symmetry of the graph. $x = -6$

c) Does the function have a maximum or minimum value? State the value. +a
min. value −9

d) State the domain and range of the function.
D: $x \in \mathbb{R}$
R: $y \geq -9$

e) Determine the y-intercept of the graph. y-int, x=0
$x=0 \quad 2(0+6)^2 - 9$
$= 2(36) - 9 = 72 - 9 = 63$

Number of x-intercepts

y=0

Depending on the position of the vertex and the direction of opening of the parabola, we can determine quadratic functions which have two x-intercepts, one x-intercept, or no x-intercepts.

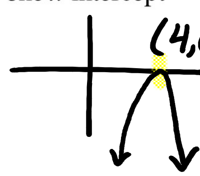
Class Ex. #2

In each case, sketch the graph of a quadratic function with the given number of x-intercepts and state a possible equation for the graph in the form $y = a(x - h)^2 + k$.

a) two x-intercepts

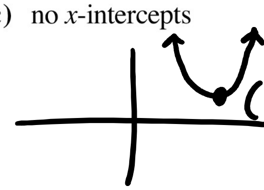
(2,−2)
$y = (x-2)^2 - 2$
(x,y)

b) one x-intercept



(4,0)
$y = -(x-4)^2$

c) no x-intercepts



(5,2)
$y = 2(x-5)^2 + 2$

Summary of the Characteristics of the Graph of $y = a(x - h)^2 + k$

The graph of a quadratic function defined by the equation $y = a(x - h)^2 + k$ has the following characteristics. Fill in the blanks to complete the summary.

- The coordinates of the vertex are (h,k).
- If $a > 0$ (+), the parabola opens up and the vertex is the minimum point. The range of the function is $y \geq k$.
- If $a < 0$ (−), the parabola opens down and the vertex is the maximum point. The range of the function is $y \leq k$.
- The domain of the function is $x \in R$.
- The equation of the axis of symmetry of the graph is $x = h$.

Complete Assignment Questions #1 - #12 #1-4, 6

Assignment

1. Without using a graphing calculator, complete the table below.

Function $y = a(x - h)^2 + k$	a	h	k	**Vertex**	**Axis of Symmetry**	**Direction of Opening**
$y = 2(x - 5)^2 + 1$						
$y = -3(x + 2)^2 - 4$						
$y = (x - 4)^2 - 5$						
$y = -(x - 7)^2$						
$y = x^2 + 9$						
$y = -x^2$						

2. Write the equation of a quadratic function in standard form with the following characteristics:

a) Vertex at (6, 3)

b) Vertex at (–2, 8)

c) Vertex at (3, –5), opening up

d) Vertex at (0, –1), opening down

e) Vertex at (–7, 0), opening up

3. Without the aid of a graphing calculator, write the equation of a quadratic function in standard form with the following characteristics:

a) Vertex in quadrant 2, opening up

b) Vertex in quadrant 3, opening down

c) Vertex on the x-axis, opening down

d) Vertex on the y-axis, opening up

e) Vertex at the origin

f) Opening down with one x-intercept

g) Opening up with no x-intercept

Verify your equations using a graphing calculator.

4. Consider the graph of the quadratic function $f(x) = -5(x - 12)^2 + 15$.

a) State the coordinates of the vertex of the graph.

b) State the equation of the axis of symmetry of the graph.

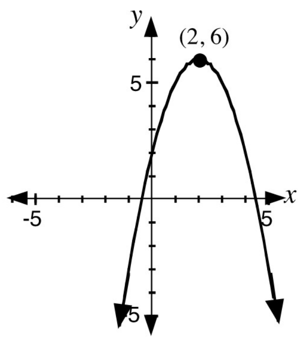
c) Does the function have a maximum or minimum value? State the value.

d) State the domain and range of the function.

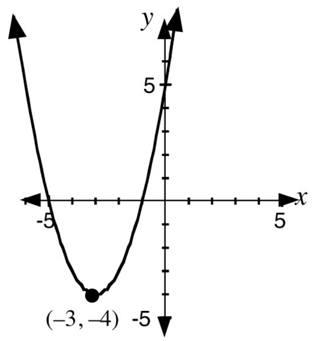
e) Determine the y-intercept of the graph.

5. The quadratic functions represented in the graphs below have equations of the form $y = (x - h)^2 + k$ or $y = -(x - h)^2 + k$.

Graph 1



Graph 2



In each case,

a) **explain** how to determine the equation represented by the graph

b) write the equation in standard form and in general form.

6. Consider the graph of the quadratic function $f(x) = 0.25(x + 3)^2 - 9.75$.
Complete the following:

a) The equation of the axis of symmetry of the graph is _________ .

b) The coordinates of the vertex of the graph are _______ .

c) The range of the function is _____________________ .

d) The minimum value of the function is __________ .

Multiple Choice

7. A parabola has a vertex at (–6, 4) and opens down.
A possible equation of the parabola is

A. $y = 3(x + 6)^2 + 4$

B. $y = -3(x - 4)^2 - 6$

C. $y = -3(x - 6)^2 + 4$

D. $y = -3(x + 6)^2 + 4$

8. The range of the quadratic function $f(x) = -2(x - 3)^2 - 5$ is

A. $\{y \mid y \geq -5, y \in R\}$
B. $\{y \mid y \leq -5, y \in R\}$
C. $\{y \mid y \leq 3, y \in R\}$
D. $\{y \mid y \leq 5, y \in R\}$

9. Which of the following equations does **not** represent a parabola with its vertex on the *x*-axis or the *y*-axis?

A. $y = -0.25(x + 1.5)^2$

B. $y = \frac{1}{2}x^2 + \frac{3}{4}$

C. $y = (x - 1)^2 + 1$

D. $y = -x^2$

10. The graph of a quadratic function has *x*-intercepts –8 and 4, and a minimum value of *k*. Which of the following **could** be the equation of the graph of the function?

A. $y = 2(x + 4)^2 + k$

B. $y = 2(x - 2)^2 + k$

C. $y = 2(x + 6)^2 + k$

D. $y = 2(x + 2)^2 + k$

11. Which of the following statements describes the graph of a quadratic function $f(x) = a(x - h)^2 + k$, if a, h, and k are all negative numbers?

A. The graph opens up with a vertex in quadrant 3.

B. The graph opens down with a vertex in quadrant 3.

C. The graph opens up with a vertex in quadrant 4.

D. The graph opens down with a vertex in quadrant 4.

Numerical Response

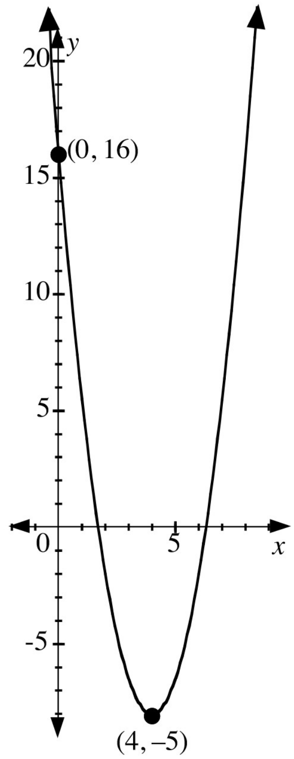
12. The equation of the axis of symmetry of the graph of a quadratic function is $x = 4$. The graph passes through the point (6, 15).
If the equation is of the form $y = (x - h)^2 + k$, then the value of $h + k$ is _____.

(Record your answer in the numerical response box from left to right.)

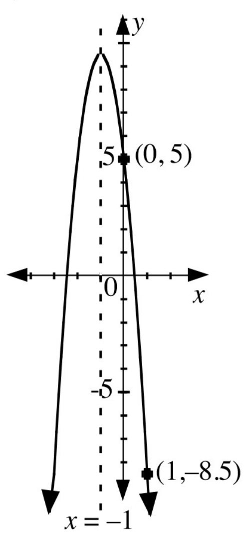
Group Work

In each case, use the given information to determine the equation of the parabola in the form $y = ax^2 + bx + c$.

a)



b)



Answer Key

1.

Function $y = a(x-h)^2 + k$	a	h	k	Vertex	Axis of Symmetry	Direction of Opening
$y = 2(x-5)^2 + 1$	2	5	1	(5, 1)	x = 5	up
$y = -3(x+2)^2 - 4$	−3	−2	−4	(−2, −4)	x = −2	down
$y = (x-4)^2 - 5$	1	4	−5	(4, −5)	x = 4	up
$y = -(x-7)^2$	−1	7	0	(7, 0)	x = 7	down
$y = x^2 + 9$	1	0	9	(0, 9)	x = 0	up
$y = -x^2$	−1	0	0	(0, 0)	x = 0	down

2. Note: Many answers are possible, your equation can be verified on a graphing calculator.
   a) $y = (x-6)^2 + 3$ **b)** $y = (x+2)^2 + 8$ **c)** $y = 3(x-3)^2 - 5$
   d) $y = -3x^2 - 1$ **e)** $y = (x+7)^2$

3. Note: Many answers are possible, your equation can be verified on a graphing calculator.
   a) $y = (x+5)^2 + 1$ **b)** $y = (x+7)^2 - 7$ **c)** $y = -(x-5)^2$ **d)** $y = x^2 + 2$
   e) $y = x^2$ **f)** $y = -2(x-4)^2$ **g)** $y = 2(x-4)^2 + 6$

4. **a)** (12, 15) **b)** $x = 12$ **c)** maximum value, 15
   d) Domain: $\{x \in R\}$, Range: $\{y \mid y \leq 15, y \in R\}$ **e)** −705

5. **a)** For Graph 1: Since the graph opens down, it is of the form $y = -(x-h)^2 + k$. The replacements for h and k are the x- and y-coordinates of the vertex.
   For Graph 2: Since the graph opens up, it is of the form $y = (x-h)^2 + k$. The replacements for h and k are the x- and y-coordinates of the vertex.
   b) Graph 1: $y = -(x-2)^2 + 6$ and $y = -x^2 + 4x + 2$
   Graph 2: $y = (x+3)^2 - 4$ and $y = x^2 + 6x + 5$

6. **a)** $x = -3$ **b)** (−3, −9.75) **c)** $\{y \mid y \geq -9.75, y \in R\}$ **d)** −9.75

7. D 8. B 9. C 10. D 11. B 12.

1	5		

Group Work

a) $y = \frac{21}{16}x^2 - \frac{21}{2}x + 16$ **b)** $y = -4.5x^2 - 9x + 5$