Quadratic Functions Lesson #2:
Analyzing the Graph of $y = ax^2 + bx + c$

Recall the following from the previous lesson.

A **quadratic function** is a function of degree 2 which can be written in the form

$f(x) = ax^2 + bx + c$ **or** $y = ax^2 + bx + c,$ where $a, b, c \in R,$ and $a \neq 0.$

The graph of a quadratic function is a parabola. $\vee \wedge$

Basic Characteristics of the Graph of a Quadratic Function

• A parabola has a **maximum**, or **maximum point**, if the parabola **opens down.**

• A parabola has a **minimum**, or **minimum point**, if the parabola **opens up.**

• The vertex of a parabola is the **maximum or minimum point.**

• The maximum or minimum **value** of a quadratic function is the y-coordinate of the vertex.

1. Complete the table of values for the quadratic function $y = x^2 - 6x + 5, x \in R.$

$(-1)^2 - 6(-1) + 5$
$1 + 6 + 5$
$1 - 6 + 5$
$9 - 18 + 5$
$25 - 30 + 5$

2. Plot the points on the grid provided. ✔
Connect the points with a smooth curve and extend the graph. ✔

3. Draw a vertical line representing the **axis of symmetry** of the graph.

4. Fill in the blanks or circle the correct alternative below.

x	y
-1	12
0	5
1	0
2	-3
3	-4
4	-3
5	0
6	5
7	12

• The graph of the quadratic function $y = x^2 - 6x + 5$

is a **parabola**. The vertex is (**3** , **-4**).

• The parabola opens (**up** / down) and therefore has a (maximum / **minimum**) point.

• The (maximum / **minimum**) value of the function is **-4** .

• The x-intercepts of the graph are **1** and **5** .
The y-intercept is **5** .

• The equation of the axis of symmetry of the graph is **X=3** .

• The domain of the quadratic function is **X ∈ R** .

• The range of the quadratic function is **y ≥ -4** .

5. How can the equation of the axis of symmetry of the graph be determined from the x-intercepts of the graph?

average of the x-ints $\frac{1+5}{2} = 3$

y-int
x=0
5

1,5
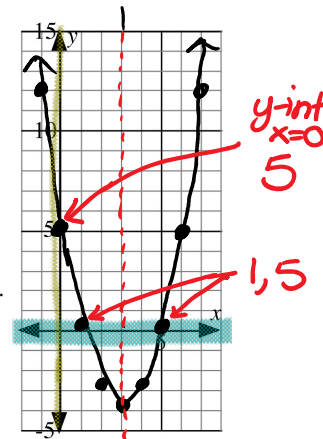
> ### *Using a Graphing Calculator to Determine a Maximum/Minimum Value*

- Verify the calculator window is in standard setting, or `ZStandard`.
- The equation $y = -x^2 + 5x + 14$ will be used to illustrate the maximum feature.

1. Enter the equation into `Y₁`, adjust the window
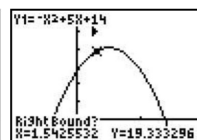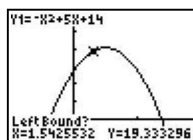   to x: $[-5, 10, 5]$ y: $[-5, 30, 5]$ and press `GRAPH`.
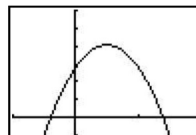
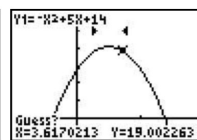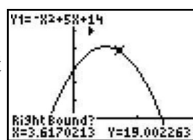2. Access the `CALC` menu by entering `2nd` then `TRACE`.

3. Select `maximum`.

4. On the bottom left of the screen the calculator will ask you for a `LeftBound?`.
   Cursor to the left side of the maximum point and press `ENTER`.
   Notice the arrow above the cursor.

5. On the bottom left of the screen the calculator will ask you for a `RightBound?`.
   Cursor to the right side of the maximum point and press `ENTER`.
   Notice the arrow above the cursor.
   The maximum point will be found between the 2 arrows.

6. On the bottom left of the screen the calculator will ask you to `Guess?`.
   Press `ENTER`.
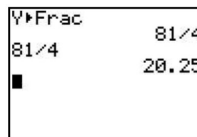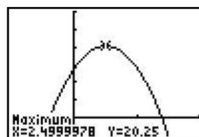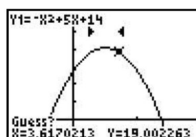
7. The y value will be the maximum value of the function.
   In this case the graph of $y = -x^2 + 5x + 14$ has a **maximum value** of 20.25 or $\dfrac{81}{4}$.

- The graph of $y = -x^2 + 5x + 14$ has a **maximum point** of $(2.5, 20.25)$.
  Even though the x value on the display may read "2.4999991" or "2.5000002,
  it is usually an indication to round off to a reasonable answer (2.5).

- Test the point using the trace feature and entering $x = 2.5$.
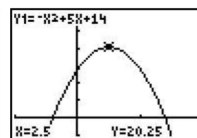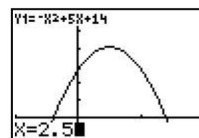
- Use `3:minimum` in `CALC` menu to find the **minimum value** of a function.

Class Ex. #1

Consider the quadratic function $f(x) = 2x^2 - 13x + 15$.   **DESMOS**

a) State an appropriate window setting for displaying the graph of the function on a graphing calculator.

b) Determine the coordinates of the vertex. $(3.25, -6.125)$ or $\left(\dfrac{13}{4}, \dfrac{-49}{8}\right)$

c) Explain how to use the coordinates of the vertex to determine the equation of the axis of symmetry of the graph. State the equation.

$x = 3.25$ axis of symmetry is the x value of the vertex

d) Determine the *x*- and *y*- intercepts of the graph.   y-int 15

x-int 1.5, 5

e) Use the *x*-intercepts to verify the equation of the axis of symmetry in c).

f) State the minimum value of the function.

↳ y value of vertex -6.125

g) State the domain and range of the function.

D: $x \in \mathbb{R}$ R: $y \geqslant -6.125$

Determining the Vertex from the Equation of the Axis of Symmetry

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The coordinates of the vertex of the graph of a quadratic function can be determined from the equation of the axis of symmetry **if** we are given the equation of the quadratic function.

Class Ex. #2

The line $x = 2$ is an axis of symmetry of the graph of $y = 4x^2 - 16x - 9$.
Without using a calculator, algebraically determine the coordinates of the vertex of the graph.

have x value of vertex, sub in and solve for y

$y = 4(2)^2 - 16(2) - 9$
$\quad = 16 - 32 - 9$
$\quad = -25$

vertex
$(2, -25)$

#1-5 due Tuesday
L1 HW ↗

Complete Assignment Questions #1 - #5

Investigating the Parameters a, b, and c in $y = ax^2 + bx + c$

#'s

Investigation 1:

Consider the equation $y = ax^2$.

• State the values of b and c. **=0**

• Investigate what happens to the graph of the function when the value of a is changed (use both positive and negative values for the parameter a). Write down your observations.

Desmos app try   a=-1 ∧   a=1 ∪
                   a=-10 ∧ a=10 ∪
                          a=0.5 ∪

large "a" compress graph, small "a" stretches graph

if a is ⊕ graph opens up, if a is ⊖ graph opens down

Investigation 2:

Consider the equation $y = x^2 + bx$.

a=1
c=0

• State the values of a and c.

• Investigate what happens to the graph of the function when the value of b is changed (use both positive and negative values for the parameter b). Write down your observations.

try b=-1   b=1
     b=-10   b=10

as the value of b changes, the parabola adjusts so that it has one x-int at 0, and another at -b

opp sign of b

Investigation 3:

Consider the equation $y = x^2 + c$.

a=1 b=0   try different c's

• State the values of a and b.

• Investigate what happens to the graph of the function when the value of c is changed (use both positive and negative values for the parameter c). Write down your observations.
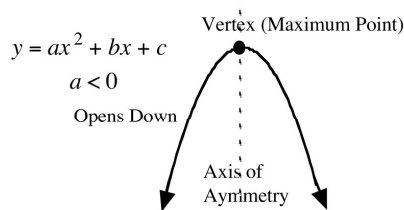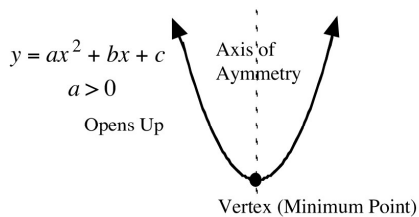
the parabola always has a y-int at c

Summary of the Characteristics of the Graph of $y = ax^2 + bx + c$

The graph of a quadratic function defined by the equation $y = ax^2 + bx + c$ has the following characteristics. Fill in the blanks to complete the summary.

• The shape of the graph is a **parabola** .

• The graph is symmetrical about a vertical line called the **axis of symmetry**

• If $a > 0$, the parabola opens **up** and the vertex is the **minimum** point.
  The y-coordinate of the vertex is the **minimum value** of the function.

• If $a < 0$, the parabola opens **down** and the vertex is the **max** point.
  The y-coordinate of the vertex is the **maximum value** of the function.

• The domain of the graph is **$x \in \mathbb{R}$** .

• If $c > 0$, the graph has a **positive** y-intercept.
  If $c \leq 0$, the graph has a **negative** y-intercept.

y-int = c

$y = ax^2 + bx + c$ Axis of
$a > 0$ Aymmetry

Opens Up

Vertex (Minimum Point)

Vertex (Maximum Point)

$y = ax^2 + bx + c$
$a < 0$

Opens Down

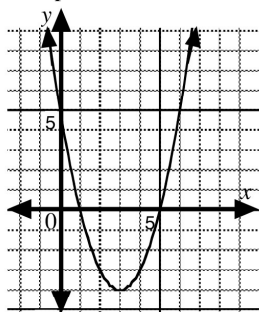Axis of
Aymmetry

Complete Assignment Questions #6 - #15

#1-5 last day
6-8, 10-14 today
#1-8, 10-14 all together

Assignment

1. The quadratic function shown below has integer values for the x- and y-intercepts. Complete the table.

x-intercepts	
y-intercept	
coordinates of the vertex	
equation of the axis of symmetry	
Domain	
Range	

2. Consider the quadratic function $y = -x^2 + 4x + 5, x \in R$.

a) Complete the table of values and graph the quadratic function on the grid.

x	y
–2	
–1	
0	
1	
2	
3	
4	
5	
6	
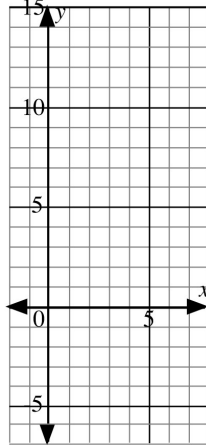
b) Complete the following by filling in the blanks or circling the correct alternative.

- The graph of the quadratic function $y = -x^2 + 4x + 5$ is a _____.
  The vertex is (,).

- The parabola opens (up / down) and therefore has a (maximum / minimum) point.

- The (maximum / minimum) value of the function is _____ .

- The x-intercepts of the graph are _____ and _____ .
  The y-intercept is _____ .

- The domain of the quadratic function is _____.

- The range of the quadratic function is _____.

c) i) Describe two methods for determining the equation of the axis of symmetry from the graph.

ii) Draw the **axis of symmetry** on the grid and state its equation.

3. Consider the quadratic function $g(x) = x^2 + 10x + 16$. State:

    a) an appropriate window setting for displaying the graph of the function
       on a graphing calculator

    b) the coordinates of the vertex   **c)** the equation of the axis of symmetry
                                            of the graph

    d) the x- and y- intercepts of the graph   **e)** the minimum point of the graph

    f) the minimum value of the function   **g)** the domain and range of the function

4. Consider the quadratic function $g(x) = -x^2 + 6x$. State:

    a) the maximum value of the function   **b)** the coordinates of the vertex

    c) the x- and y- intercepts of the graph   **d)** the domain and range of the function

5. The line $x = -3$ is an axis of symmetry of the graph of $y = x^2 + 6x - 7$. Without using a calculator, algebraically determine the coordinates of the vertex of the graph.

6. Consider the graph of $y = -x^2 - 4x + 12$ which has an axis of symmetry with equation $x + 2 = 0$. Without using a calculator, algebraically determine the coordinates of the vertex of the graph.

x = -2

$y = -(-2)^2 - 4(-2) + 12$

$= -4 + 8 + 12 \;\; \boxed{= 16}$

$(-2, 16)$ vertex

7. Without drawing the graph, how can you tell if the graph of the function $f(x) = 2x^2 - 5x - 30$

    a) opens up or down?

    b) has a positive or negative y-intercept?

8. Without drawing the graph, how can you tell if the graph of a specific quadratic function
$y = ax^2 + bx + c$

    a) opens up or down?   **b)** has a positive or negative y-intercept?

9. Write the equation of a quadratic function that opens down and passes through the origin.

Multiple Choice

10. Tony knows that the points $(-3, -8)$ and $(7, -8)$ lie on a parabola.
The equation of the axis of symmetry of the parabola is

    A. $y = -8$

    B. $x = 2$

    C. $x = 4$

    D.   unable to be determined from the given information

11. The equation of the axis of symmetry of the graph of a quadratic function is $x = \dfrac{9}{2}$.
If the graph passes through the point $(-3, 0)$, it must also pass through the point

    A. $\left(-\dfrac{21}{2}, 0\right)$ **B.** $\left(\dfrac{15}{2}, 0\right)$

    C. $(3, 0)$ **D.** $(12, 0)$

12. The maximum value of the quadratic function with equation $y = -x^2 - x + 6$ is

    A. $(-5, 6.25)$ **B.** -5

    C. $\dfrac{25}{4}$ **D.**   none of the above

13. Which of the following quadratic functions has a maximum point and
a negative y-intercept?

    A. $f(x) = \dfrac{1}{2}x^2 + 3x + 7$ **B.** $f(x) = 6x^2 + 3x - 2$

    C. $f(x) = -13x^2 - 8x - 4$ **D.** $f(x) = -16x^2 - 3x + 12$

14. Which of the following statements describes the graph of a quadratic function
$f(x) = ax^2 + bx + c$ if both a and c are positive numbers?

    A.   The graph opens up and has a positive y-intercept.

    B.   The graph opens up and has a negative y-intercept.

    C.   The graph opens down and has a positive y-intercept.

    D.   The graph opens down and has a negative y-intercept.

Numerical Response **15.** The line $x = 6$ is an axis of symmetry of the graph of the quadratic
function $y = x^2 - 5x + c$. If the vertex of the graph is $(6, 18)$, the value of c is _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. x-intercepts 1, 5 y-intercept 5, vertex $(3, -4)$, axis of symmetry $x = 3$,
    Domain $x \in R$, Range $\{y \mid y \geq -4, y \in R\}$

2. a) See below   **b)** • parabola, $(2, 9)$ **c) i)** Determine the number n which is the
                               • down, maximum y-coordinate of the vertex. The equation

x	y
-2	-7
-1	0
0	5
1	8
2	9
3	8
4	5
5	0
6	-7

                              • maximum, 9   of the axis of symmetry is $x = n$.
                              • -1, 5, 5   OR
                              • $x \in R$ Determine the number n which is the
                              • $\{y \mid y \leq 9, y \in R\}$ average of the x-intercepts.
                                                          The equation of the axis of symmetry
                                                        is $x = n$.
                                              ii) $x = 2$.

3. a) $x:[-10, 4, 2]$ $y:[-15, 20, 5]$ **b)** $(-5, -9)$ **c)** $x = -5$ **d)** x-intercepts $-8, -2$, y-intercept 16
    e) $(-5, -9)$ **f)** -9 **g)** Domain $x \in R$, Range $\{y \mid y \geq -9, y \in R\}$

4. a) 9   **b)** $(3, 9)$ **c)** x-intercepts $0, 6$, y-intercept 0   **5.** $(-3, -16)$ **6.** $(-2, 16)$
    d) Domain $x \in R$, Range $\{y \mid y \leq -9, y \in R\}$

7. a) Graph opens up because the coefficient of x^2 (i.e. 2) is positive.
    b) Graph has a negative y-intercept because the constant term (i.e. -30) is negative.

8. a) If $a > 0$, the graph opens up. If $a < 0$, the graph opens down.
    b) If $c > 0$, the y-intercept is positive. If $c < 0$, the y-intercept is negative.

9. Answers may vary (but the parameter c must equal zero) e.g. $y = -2x^2 + 3x$.

10. B   **11.** D   **12.** C   **13.** C   **14.** A   **15.**

1	2		