

4.3

## Trigonometry Lesson #3: The Cosine Law

### Introduction

Consider triangle  $ABC$  in which  $\angle A = 36^\circ$ ,  $AB = 3$  cm and  $AC = 6$  cm. What happens when you try to apply the sine law to determine the length of  $BC$ ?

$$\frac{x}{\sin 36} = \frac{6}{\sin B} = \frac{3}{\sin C}$$

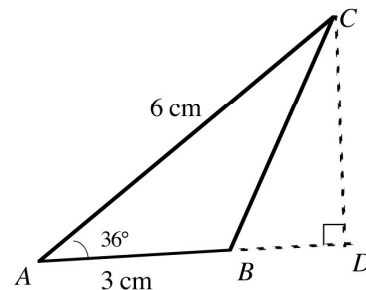
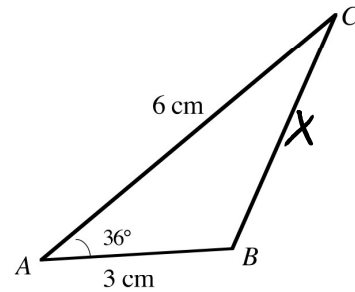
*\* unknown in each part*

In the example above, where we are given the length of two sides and the contained angle, the sine law is **not** applicable.

*we don't have enough info to use Sine law*

We can find the length of  $BC$  by making a right triangle  $BCD$  in the diagram below and using SOHCAHTOA to determine the lengths of  $CD$  and  $AD$ .

Determine the lengths of  $CD$  and  $AD$  to the nearest hundredth of a cm, and show how these lengths can be used to determine the length of  $BC$  to the nearest tenth of a cm.



The method above is time consuming.  
The length of  $BC$  can be determined in one step by using the **cosine law**.

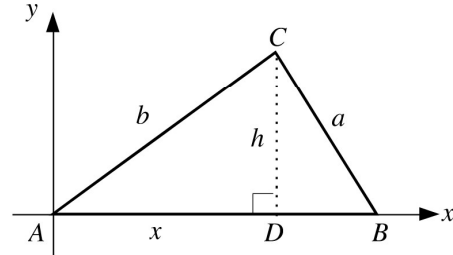
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**The Cosine Law**

In every triangle  $ABC$ ,  $a^2 = b^2 + c^2 - 2bc \cos A$ .

**Proof of the Cosine Law**

- The diagram shows triangle  $ABC$  placed with base  $AB$  on the  $x$ -axis and  $A$  at the origin.
- The line  $CD$  is drawn perpendicular to  $AB$  and is  $h$  units in length.
- $AD = x$  units so  $DB = c - x$  units.



Complete the following work to show that  $a^2 = b^2 + c^2 - 2bc \cos A$ .

In  $\triangle ADC$ ,  $\cos A = \frac{AD}{AC} = \frac{x}{b}$

so  $x =$

In  $\triangle BDC$ ,  $BC^2 = CD^2 + DB^2$

$$a^2 = h^2 + (c - x)^2$$

$$a^2 = h^2 + c^2 - 2cx + x^2$$

$$a^2 = (h^2 + x^2) + c^2 - 2cx$$

$$a^2 = \quad + c^2 - 2c(\quad)$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

By placing  $AC$  and then  $BC$  on the  $x$ -axis, similar equations can be derived.

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



This version of the cosine law can be used in any triangle if we are given the lengths of two sides and the contained angle (SAS).



Consider the  $\triangle ABC$  from Class Ex. #1 in which  $\angle A = 36^\circ$ ,  $AB = 3$  cm, and  $AC = 6$  cm. Determine the length of  $BC$ , to the nearest tenth of a cm, using the cosine law.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

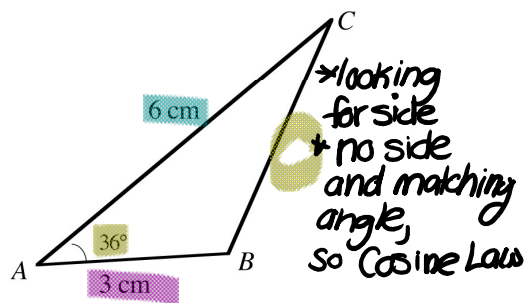
$$a^2 = 6^2 + 3^2 - 2(6)(3)\cos 36$$

$$= 36 + 9 - 36\cos 36$$

$$= 45 - 29.1246118$$

$$\sqrt{a^2} = \sqrt{15.8753882}$$

$$a = 3.984... = 4.0$$

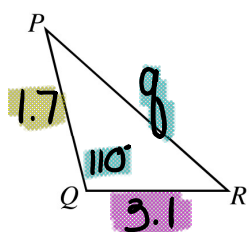


Consider triangle  $PQR$  shown.

a) Complete the cosine law for calculating side  $q$ .

$$q^2 = p^2 + r^2 - 2pr \cos Q$$

b) Calculate, to the nearest tenth of a cm, the length of the third side of  $\triangle PQR$  if  $QP = 1.7$  cm,  $QR = 3.1$  cm, and  $\angle PQR = 110^\circ$ .



$$q^2 = p^2 + r^2 - 2pr \cos Q$$

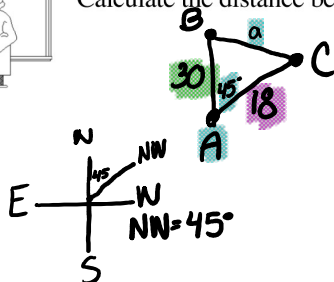
$$q^2 = 3.1^2 + 1.7^2 - 2(3.1)(1.7)\cos 110$$

$$\sqrt{q^2} = \sqrt{16.10489}$$

$$q = 4.0$$



Bellevue is 30 km north of Ayr and Churchville is 18 km northwest of Ayr. Calculate the distance between Bellevue and Churchville to the nearest km.



$$a^2 = b^2 + c^2 - bc \cos A$$

$$a^2 = 18^2 + 30^2 - (18)(30)\cos 45$$

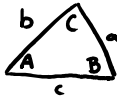
$$\sqrt{a^2} = \sqrt{460.325}$$

$$a = 21 \text{ km}$$

Complete Assignment Questions #1 - #4

**Alternative Form of the Cosine Law**

The equation



$$a^2 = b^2 + c^2 - 2bc \cos A$$

→ finding a side

can be rearranged to the form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

→ finding an angle



**Note** This form of the cosine law can be used to determine any angle in a triangle when we are given the length of all three sides (SSS).



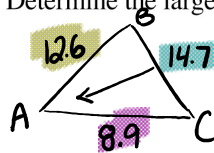
Complete the following for triangle ABC.

a)  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

b)  $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



Determine the largest angle in  $\Delta ABC$  if  $a = 14.7$ ,  $b = 8.9$ , and  $c = 12.6$ .



\* biggest side = biggest angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{8.9^2 + 12.6^2 - 14.7^2}{2(8.9)(12.6)}$$

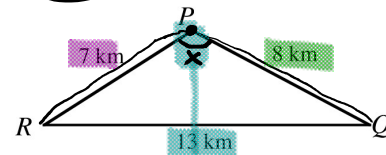
$$\cos A = \frac{21.88}{224.28}$$

$$\cos A = 0.09756$$

$$A = \cos^{-1}(0.09756) = 84^\circ$$



Two ships set sail from port,  $P$ , heading in different directions. The first ship sails 7 km to  $R$  and the second ship sails 8 km to  $Q$ . If the distance between  $R$  and  $Q$  is 13 km, determine the angle between the directions of the two ships.



\* finding angle  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\cos P = \frac{7^2 + 8^2 - 13^2}{2 \cdot 7 \cdot 8} = \frac{49 + 64 - 169}{112} = \frac{-56}{112} = -0.5$$

$$\cos^{-1} \cos P = -0.5$$

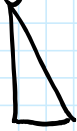
$$P = 120^\circ$$

#1-7

Quiz Tues  
L1-3

Complete Assignment Questions #5 - #11

Right angle



$$a^2 + b^2 = c^2$$

S<sup>o</sup>HCAT<sup>o</sup>  
HTA

No right angle

sine law

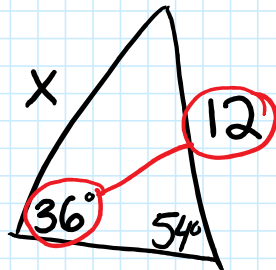
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

↑  
looking for side

$$\text{OR } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

↑  
looking an angle

\*sine law we need a matching side and angle  
cosine if no matching side and angle



\*no right angle

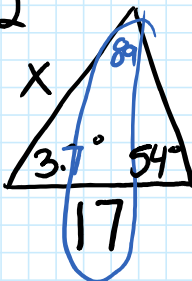
\*matching side + angle

↳ SINE LAW

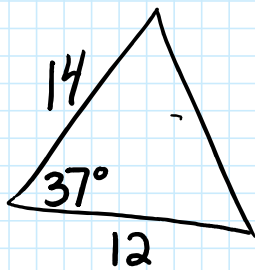
$$\sin 54 \frac{12}{\sin 36} = \frac{x \cancel{\sin 54}}{\cancel{\sin 54}} \text{ * lesson 2}$$

$$\frac{\sin 54 \cdot 12}{\sin 36} = 16.5$$

ex. 2



$$180 - 37 - 54 = 89$$
$$\frac{17}{\sin 89} = \frac{x}{\sin 54}$$



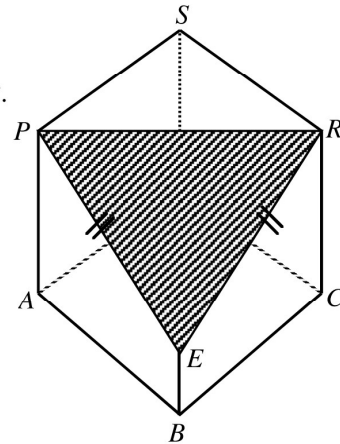
cosine law!

2 sides, 1 mismatched  
angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \text{finding angle}$$

8. The solid in the diagram was formed by removing a corner from a cube of 24 cm. The length of  $EB$  is 6 cm.

a) Calculate, to the nearest tenth, the lengths of  $PE$  and  $PR$ .



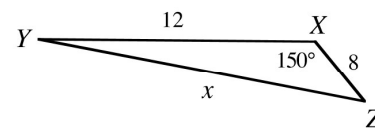
b) Calculate the measure of angle  $PER$  to the nearest degree.

Use the following information to answer the next question.

- $\cos 30^\circ$  can be written as an exact value as  $\frac{\sqrt{3}}{2}$ .
- $\cos 150^\circ$  can be written as  $-\frac{\sqrt{3}}{2}$ .

- Multiple Choice** 9. In the diagram, the value of  $x^2$  is

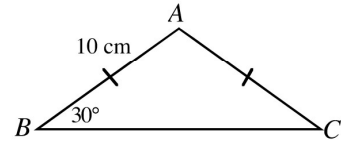
- A. 112
- B. 304
- C.  $208 - 96\sqrt{3}$
- D.  $208 + 96\sqrt{3}$





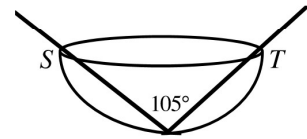
10. The length of  $BC$  in cm is

- A.  $5\sqrt{3}$
- B. 10
- C.  $10\sqrt{3}$
- D. 20



**Numerical Response**

11. The diagram shows a glass bowl with two chop-sticks resting on the rim at points  $S$  and  $T$ . The lengths of the parts of the chop-sticks inside the bowl are 9 cm and 11.5 cm respectively.



The length of  $ST$ , to the nearest tenth of a cm, is \_\_\_\_\_ .  
(Record your answer in the numerical response box from left to right.)

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**Answer Key**

- 1. a)  $s^2 = t^2 + v^2 - 2tv \cos S$       b)  $v^2 = s^2 + t^2 - 2st \cos V$
- 2. a) 12.6 cm      b) 4.2 cm      c) 36.7 cm      3. 28      4. 17.5 m
- 5. a)  $\cos E = \frac{d^2 + f^2 - e^2}{2df}$       b)  $\cos F = \frac{d^2 + e^2 - f^2}{2de}$       6. a)  $41^\circ$       b)  $36^\circ$       c)  $92^\circ$       d)  $138^\circ$
- 7.  $40^\circ$       8. a)  $PE = 30.0$  cm,  $PR = 33.9$  cm      b)  $69^\circ$
- 9. D      10. C      11. 

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